CNRS
Centre National de la Recherche Scientifique

INFN
Istituto Nazionale di Fisica Nucleare

Mode spectrum of a ring Fabry-Perot cavity

Matteo Barsuglia, Henrich Heitmann and C.Nary Man

Laser-Optics group
Bat 208, 91405 Orsay cedex - France

VIR-NOT-LAS-1390-120
Issue: 1
Date : June 15, 1998

VIRGO * A joint CNRS-INFN Project
Project Office: INFN-Sezione di Pisa * Via Livornese, 1291-56010 San Piero a Grado, Pisa Italia.
Secretariat: Telephone (39) 50 880 327 or 880 352 * FAX (39) 50 880 350 * e-mail virgo@pisa.infn.it

1 Introduction

It is well known that for a Fabry-Perot cavity formed by two mirrors (linear configuration), the transverse vertical and horizontal modes have the same resonance frequency: they are degenerate. If m and n are respectively the mode numbers in the vertical and horizontal directions, the frequency of the mode (m, n) depends on the sum $m + n$. If the cavity has an odd number of mirrors, the modes with an odd mode number relative to the the ring plane are not degenerate with the modes having the same mode number relative to the plane perpendicular to the ring [2].

In this note we show how this degeneracy breaking arises, and the measurements performed in two different 3-mirrors ring cavities used in VIRGO: the 30 cm *reference* cavity and the 30 m suspended input *mode − cleaner* prototype. The last paragraph of this note presents how to take in account this effect in the configuration design of a ring cavity in order to avoid mode accidental degeneracy.

2 Theory

Let's consider a linear Fabry-Perot cavity. The resonance frequency of the mode TEM_{mn} is given imposing the auto-consistency of the electric field: the phase shift for a round trip has to be equal to a multiple of 2π [1].

$$2kd - 2(m+n+1)arccos\sqrt{\left(1-\frac{d}{R_1}\right)\left(1-\frac{d}{R_2}\right)} = 2\pi q \tag{1}$$

where k is the wave number, d is the cavity length (and then $2d$ is the cavity round trip length), R_1 and R_2 are the two mirror radii of curvature, and q is an integer. Then:

$$\frac{\nu}{\nu_0} = q + \frac{1}{\pi}(m+n+1)arccos\sqrt{\left(1-\frac{d}{R_1}\right)\left(1-\frac{d}{R_2}\right)} \tag{2}$$

where ν_0 is the *free spectral range* (FSR) of the cavity ($\nu_0 = \frac{c}{2d}$). The frequency difference between a mode TEM_{mn} and a mode TEM_{00} is then:

$$\frac{\Delta\nu}{\nu_0} = (m+n)\frac{1}{\pi}arccos\sqrt{\left(1-\frac{d}{R_1}\right)\left(1-\frac{d}{R_2}\right)} \tag{3}$$

Now let's analyse the propagation of a TEM_{mn} beam inside a triangular cavity. For simplicity we will suppose that the beam is linearly $''s''$ polarised (the same discussion in true for the $''p''$ polarisation). We will note *horizontal* the planes parallel to the ring, and *vertical* the planes perpendicular to the ring. The first mode number refers always to the horizontal plane and the second mode number to the vertical plane.

The auto-consistency equation for a beam which propagates in the cavity plane with a field distribution horizontally symmetric with respect to the optical axis is written as:

$$2kd - 2(m+n+1)arccos\sqrt{1-\frac{d}{R}} = 2\pi q \tag{4}$$

(for simplicity we've chosen only one curved mirror)

This is the case of the TEM horizontally even modes: $TEM_{m,n}$ with m even.

Then:

$$\frac{\Delta\nu^{EVEN}}{\nu_0} = q + \frac{1}{\pi}(m+n+1)arccos\sqrt{1-\frac{d}{R}} \tag{5}$$

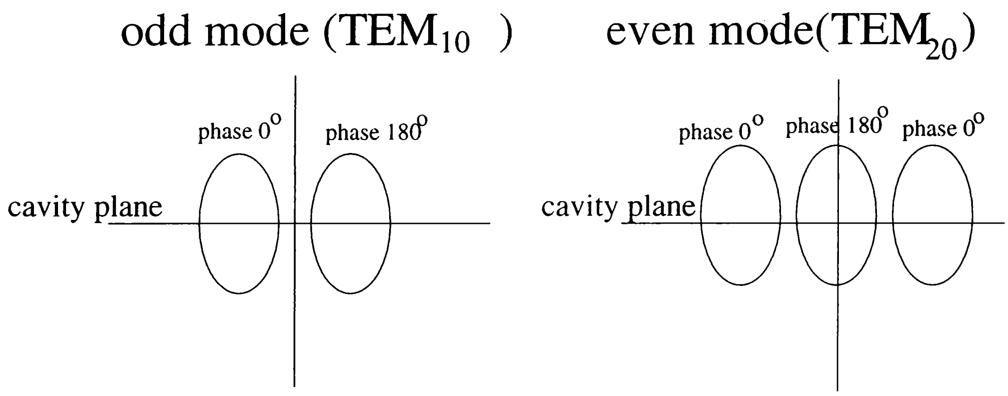

Figure 1: The odd modes are anti-symmetric to respect with the optical axis. The horizontally even modes are symmetric

A beam with a field distribution horizontally anti-symmetric with respect to the optical axis takes a geometrical π phase shift after a round trip (see fig. 2 and fig.1), then we have to include this phase shift in the auto-consistency equation:

$$\pi + 2kd - 2(m+n+1)arccos\sqrt{1-\frac{d}{R}} = 2\pi q \tag{6}$$

This is the case of the TEM horizontally odd modes: TEM_{mn} with m odd.
Then:

$$\frac{\Delta\nu^{ODD}}{\nu_0} = \frac{1}{2} + q + \frac{1}{\pi}(m+n+1)arccos\sqrt{1-\frac{d}{R}} \tag{7}$$

(see fig. 2 and fig.1)
The difference between an horizontally odd mode and the TEM_{00} is given by:

$$\frac{\Delta\nu^{ODD}}{\nu_0} = -\frac{1}{2} + \frac{1}{\pi}(m+n)arccos\sqrt{1-\frac{d}{R}} \tag{8}$$

and the difference between an horizontally even mode and the TEM_{00} is:

$$\frac{\Delta\nu^{EVEN}}{\nu_0} = (m+n)\frac{1}{\pi}arccos\sqrt{1-\frac{d}{R}} \tag{9}$$

Then there are two different patterns of modes: a *normal* one, for the horizontally even modes, and a pattern shifted of one half FSR to respect to the first, for the horizontally odd modes. The resulting spectrum (for the first 3+3 modes, and $d/L \approx 0.2$), is given in fig. 3.

This discussion can be extended easily to a ring cavity with an odd number of mirrors.

3 Measurements: reference cavity

The following measurements are obtained sweeping the laser frequency and looking at the transmission of the reference cavity with a photodiode and a CCD camera in order to identify the modes. If the alignment is perfect only the TEM_{00} resonates. When we misalign the input beam a TEM_{01} or a

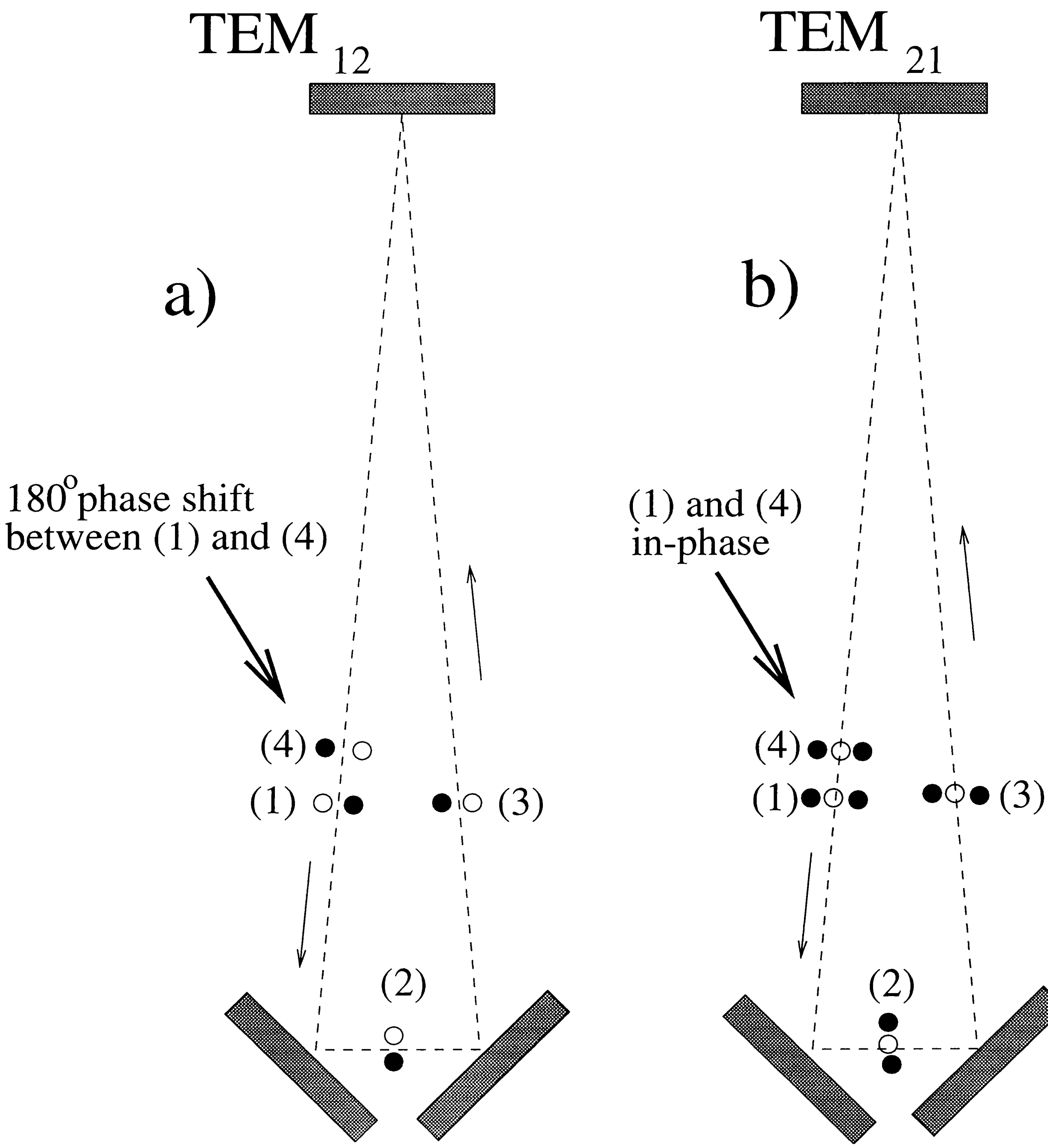

Figure 2: Propagation in the ring plane for $m + n = 3$, for a beam horizontally anti-symmetric (a) and for a beam horizontally symmetric (b). After a round trip, if the beam horizontally symmetric resonates in the cavity, the beam horizontally anti-symmetric can't resonate because it takes a π phase shift due to the geometry

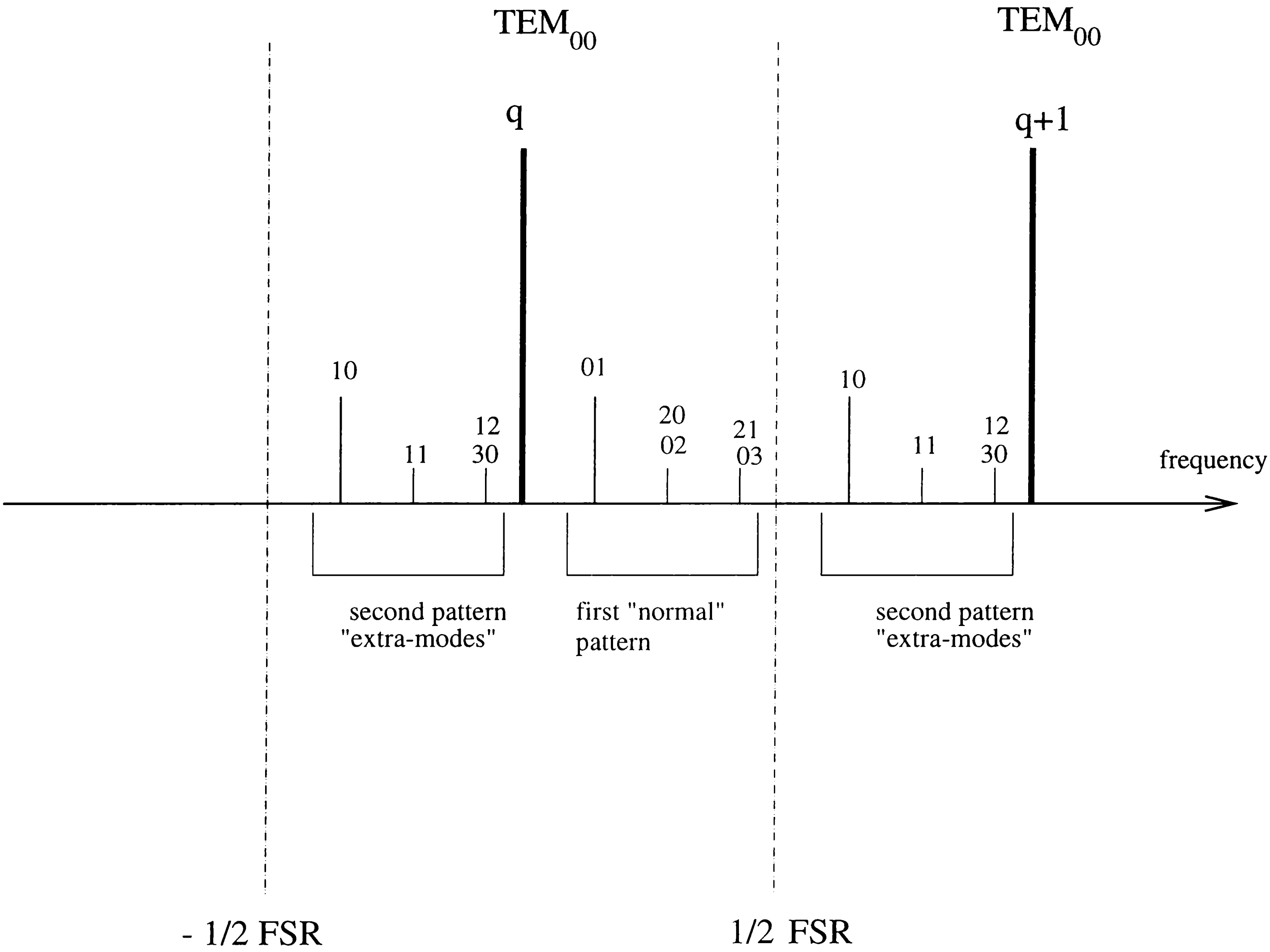

Figure 3: Mode spectrum of a ring triangular cavity for $d/R \approx 0.2$

TEM_{10} can resonate. We can calibrate the frequency scale using a $6.25MHz$ modulation (we can see the two sidebands around the TEM_{00}), and assuming that the frequency sweep is linear. From fig. 4 we observe that for a vertical misalignment a TEM_{01} appears, with a frequency:

$$\Delta\nu_{TEM_{01}-TEM_{00}} = 126MHz$$

When we misalign horizontally the cavity the TEM_{10} doesn't appear at the same frequency, but at:

$$\Delta\nu_{TEM_{10}-TEM_{00}} = -102MHz$$

Considering that the FSR of the cavity is 475 MHz, we expect a theoretical value for $\Delta_{TEM_{10}-TEM_{00}}$:

$$\Delta\nu^{Th}_{TEM_{10}-TEM_{00}} = -\frac{1}{2}\cdot 475 + 126 = -111.5MHz$$

The error between the expected value (-102) and the theoretical one (-111.5) is probably due to the non linearity of the sweep (we can observe the non-linearity of the sweep looking at the distance between the carrier and the sidebands for the two modes 00 and 01. This distance is not the same)

4 Measurements: mode-cleaner cavity

The following measurements are obtained sweeping the laser frequency and looking at the transmission of the $mode-cleaner$ cavity. In this case the alignment is imperfect in both vertical and horizontal direction, and we can easily see several modes. In this case we don't have a clear identification of the peaks with a CCD camera, as with the reference cavity, because of the seismic excitation, which sweeps the cavity length over some FSR (we identify only the TEM_{01} peak).

Moreover the distances between the modes and the overall spectrum reconstruction agree with the model (see fig. 5)

5 Accidental mode degeneracy of a ring cavity

We recall that, with a linear cavity, in order to avoid accidental mode degeneracy, the ratio d/R must not satisfy the following relation:

$$(m+n)arccos\sqrt{1-\frac{d}{R}} = integer\ numbers \tag{10}$$

For a ring cavity with an odd number of mirrors, we have seen above that there is a second pattern shifted one half FSR away from the first; then we can deduce easily that in order to avoid accidental mode degeneracy the ratio d/R must not satisfy also the relation:

$$(m+n)arccos\sqrt{1-\frac{d}{R}} = half\ integer\ numbers \tag{11}$$

References

[1] H.Kogelnik and T.Li, "Laser beams and resonators", Applied Optics **10**, 1150 (1966)

[2] C. Mathis et al., "Resonances and instabilities in a bidirectional ring laser", Phisica D, **96**, 242 (1996)

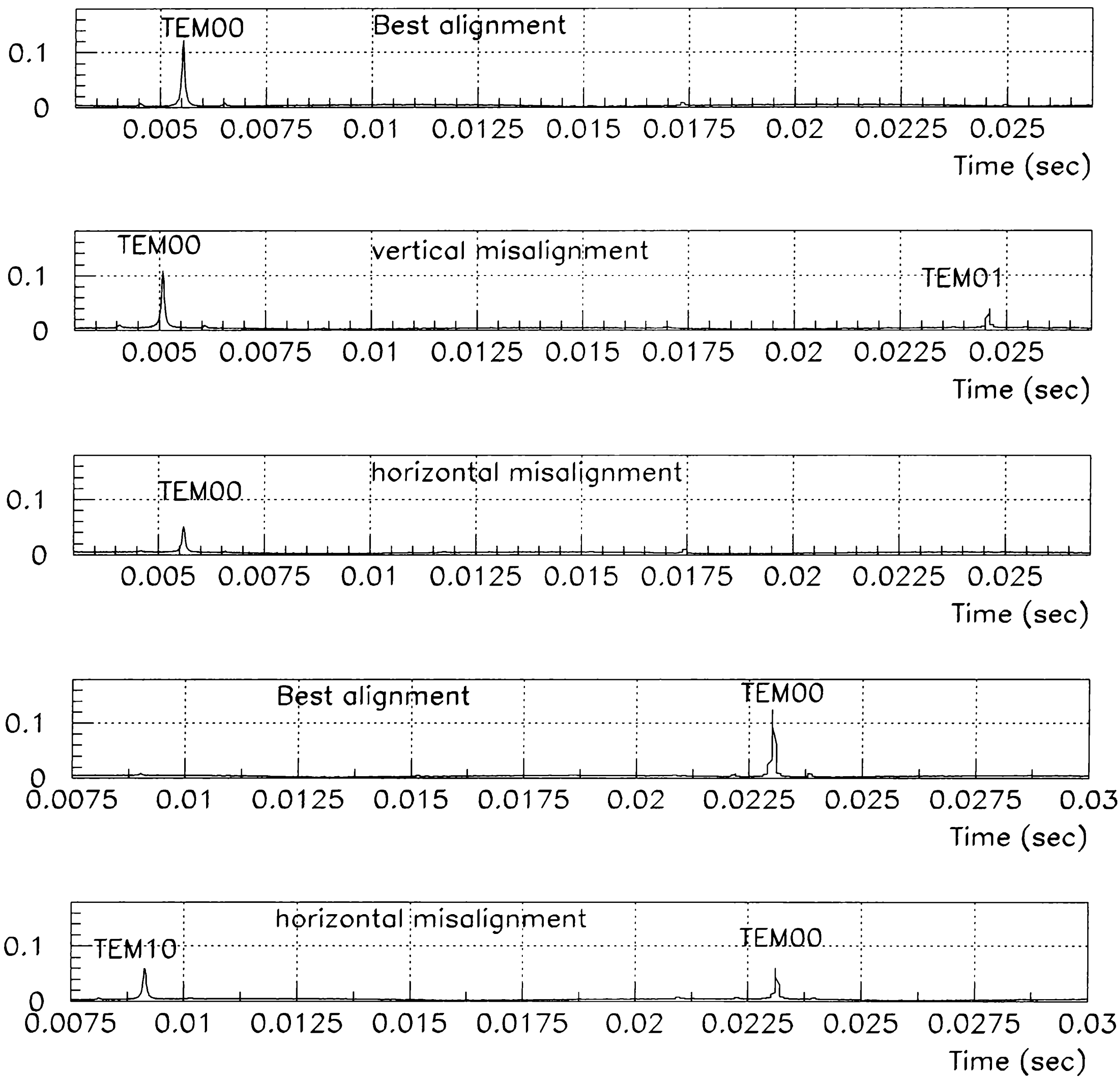

Figure 4: Modes of the reference cavity. The frequency sweeping coefficient is positive: the frequency increases with the time. The "best alignment" and "horizontal misalignment" spectra are split in two plots for technical reasons

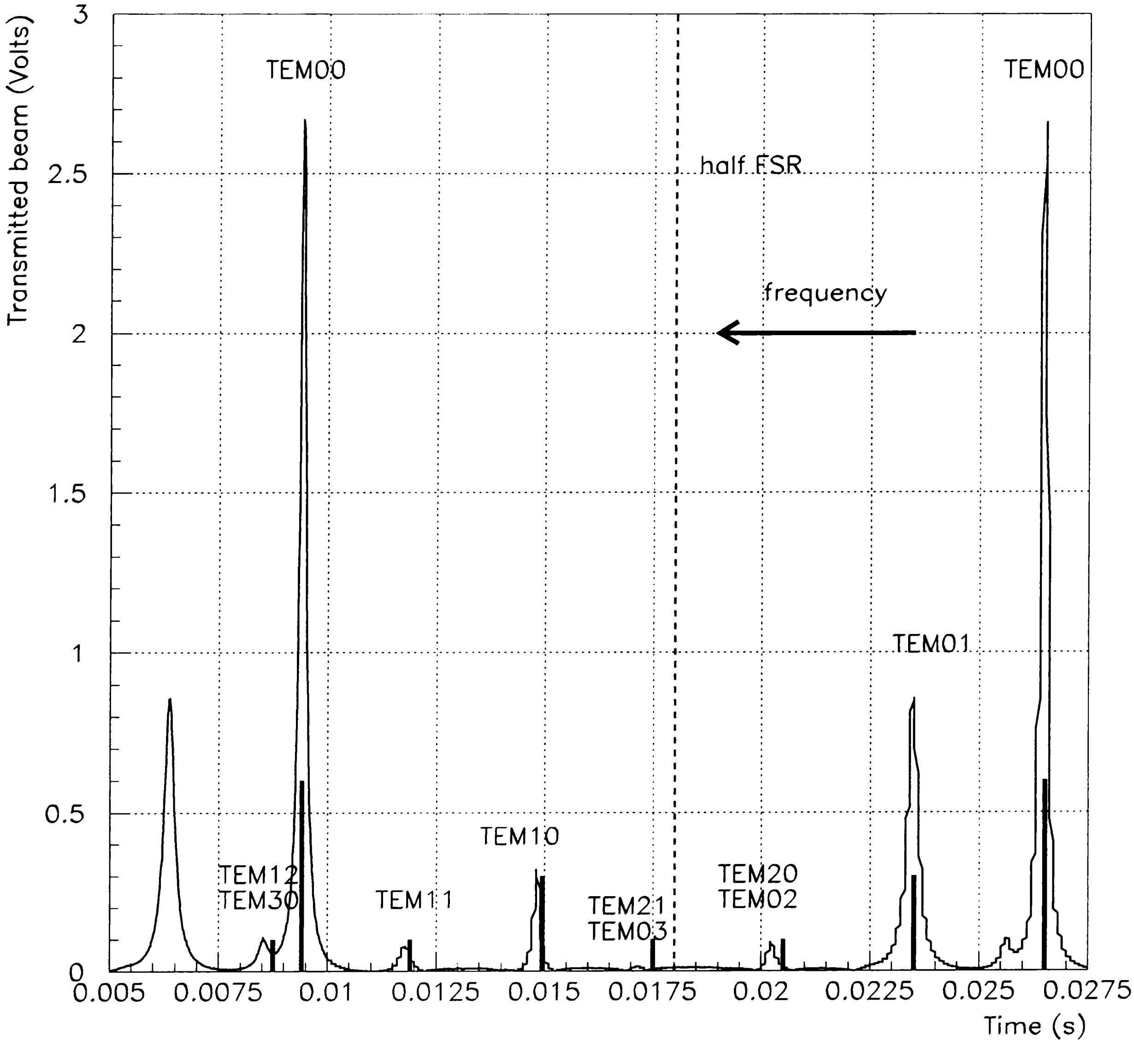

Figure 5: Transmission spectrum of the 30 m input mode-cleaner prototype. The thick right lines are the theoretical values