VIRGO	P | J | T | 9 | 4 | 0 | 0 | 7
H. Heitmann 13.3.1994	**Subject :** **Modulation and Interferometer Geometry**

I. Introduction

For keeping the distances between the VIRGO mirrors at the values dictated by the resonance conditions etc., a phase modulation scheme similar to the Pound-Drever technique [1] will be used; the design of this locking scheme has been discussed in detail elsewhere [2]. One common characteristic of the two schemes suggested in [2] is, that the modulation is done between the laser and the interferometer (frontal or Schnupp modulation), and that no modulation occurs inside or after the interferometer. The purpose of the following note is first, to give a collection of the information relevant for decisions concerning modulation frequencies, separation of buildings, length of mode cleaner etc., and second, to give a practical approach to determine these parameters starting from a particular choice of a modulation scheme and given boundary conditions. As examples, some possible configurations are derived; especially it will be shown that there exists a configuration that allows passing all modulation frequencies through the mode cleaner.

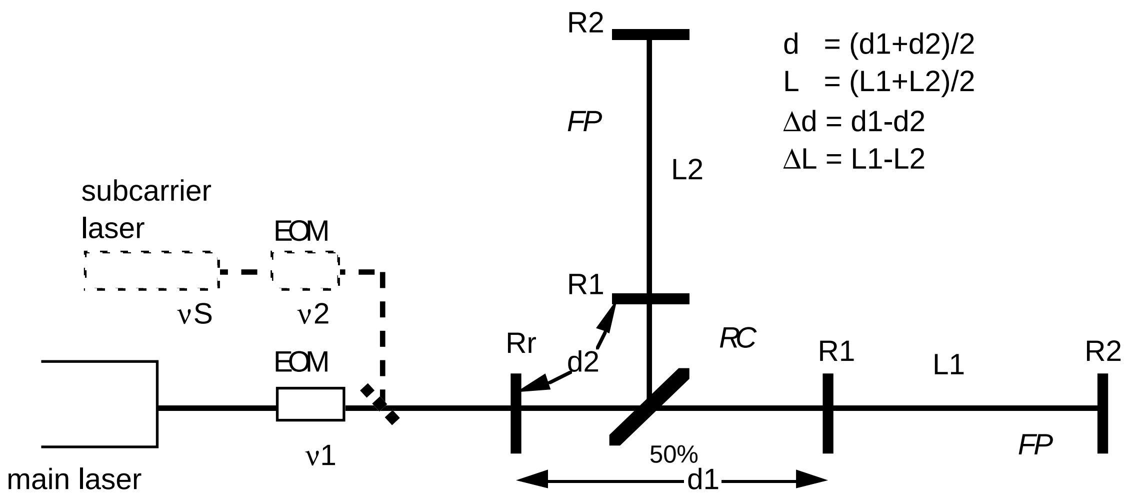

Fig.1 Definition of symbols used in the text. Shown is a frontal modulation configuration (with SSB in dotted lines)

Equal arm cavity lengths ($L_1 = L_2 = L$) are assumed for simplicity. The convention used for the mirror phase shifts is the following: beam impinging from the coated side on a mirror/beam splitter: phase shift π on reflection; beam impinging from the other side and transmissions: 0 phase shift.

II. Constraints imposed by the modulation on lengths/frequencies

General conditions

The error signal giving the deviation of a cavity length from resonance in the Pound-Drever technique is obtained by beating the carrier (= laser frequency) with the modulation sidebands. Therefore it is necessary that at the observation point carrier and sidebands are sufficiently strong. Consequently, all frequencies, carrier and sidebands, must be resonant in the recycling cavity (RC) in order to be enhanced by a certain recycling factor. Another way to state this is, that if the sidebands are not resonant in the RC, they are mostly reflected from the recycling mirror and thus the effective modulation index inside the RC is low.

A second obvious requirement is that the carrier itself is resonant also in the arm (FP) cavities.

The gravitational wave signal must be observed at the modulation frequency in order to get rid of the low frequency laser noise. Therefore some sideband power must be present at the interferometer output in order to create a beat signal with the carrier, if the latter leaks out at the dark fringe due to a GW. This can be achieved by introducing an asymmetry in the recycling cavity arms. The sideband power extracted to the photodiode is maximized, if the percentage coupled out of the recycling cavity (non-perfect dark fringe condition for the sidebands) equals the other losses p in the interferometer (absorption and recycling mirror transmission) :

$$\sin^2 \frac{2\pi\nu\Delta d}{c} = p \quad ;$$

this leads to

$$\Delta d \approx \sqrt{p} \cdot \frac{c}{2\pi\nu} \quad . \qquad (1)$$

So the optimum asymmetry Δd depends on the modulation frequency ν. This condition is not very strict. Especially, the signal-to-noise ratio decreases only very slowly with increasing Δd (see Appendix A).

Simple frontal modulation

The first scheme proposed in [2] uses a laser beam modulated at only one frequency ν; photodiodes located at different points in the interferometer provide 4 independent error signals used to control the lengths of the interferometer (simple frontal modulation).

The condition for the sideband resonance in the recycling cavity is given by demanding that the phase shift upon a complete roundtrip in the RC is an integer multiple of 2π:

$$\delta_d+\varphi_{asy}+\varphi_L = n \cdot 2\pi \quad (n\in \mathbf{N}) \quad , \qquad (2)$$

with

$$\varphi_L = \mathrm{Arg}(R_1 - r_1r_2\,(1+R_1)\cos\delta_L + R_1R_2 + i \cdot r_1r_2\,(R_1-1)\sin\delta_L)$$

$$\varphi_{asy} = \frac{\pi}{2}\,(1-\mathrm{sign}\cos\frac{2\pi\nu\Delta d}{c}) \quad .$$

$\delta_d = 4\pi\nu d/c$ and $\delta_L = 4\pi\nu L/c$ are the phase shifts encountered by the beams upon a complete roundtrip in the recycling and arm cavities, respectively. R_1,R_2 are the reflectivities of the FP mirrors, φ_L is the phase shift upon reflection of the beam at the entrance mirror of a Fabry-Perot cavity. φ_{asy} takes the values 0 or π and keeps thus track of the phase reversals occuring in the RC because of the asymmetry in the short Michelson interferometer. The important point is that the resonance condition in the recycling cavity depends on the resonance condition in the arm cavities. That means that the spacing between RC resonances is about c/2L for the complete interferometer instead of c/2d for a simple 2 mirror cavity (in our case about 50 kHz instead of 12.5 MHz); so there is a densely spaced comb of recycled frequencies. Numerical simulations show that all these frequencies give useful error signals for simple frontal modulation. However, the RC resonances are broadest if ν is nearly antiresonant in the arm cavities, which reduces the sensitivity to instabilities and fluctuations of ν. The preferred modulation frequencies turn thus out to be the resonances in the close neighborhood of (2n+1)·c/4d (See Appendix B for a numerical example).

SSB modulation

A disadvantage of the simple frontal modulation is that all error signals are dominated by the arm cavity lengths. This is because they are formed by beating the sidebands with the carrier, which is necessarily resonant in the FP's and therefore sees all FP length changes. This may make it difficult to extract the information on the RC lengths. More independent signals for these can be obtained with a second carrier (subcarrier or single sideband, SSB), shifted from the laser frequency by an offset ν_S and itself modulated at ν_2 (secondary modulation), such that subcarrier and its sidebands are non-resonant in the FP's. Then the beat between them at ν_2 is mainly dependent on the RC lengths.

The main modulation at ν_1 now serves only for probing the FP lengths; its sidebands must be recycled as stated above (Eq. 2). This holds also for the subcarrier and the secondary modulation sidebands; moreover, they must not resonate in the arm cavities, as explained above. The subcarrier must even be on exact antiresonance, because otherwise its sidebands don't see symmetric resonance conditions:

$$\frac{2L\nu_S}{c} = n+\frac{1}{2} \quad (n\in \mathbf{N}) \quad . \tag{3}$$

Moreover, in order to sense the RC arm length difference, the subcarrier must have a dark fringe at the output:

$$\frac{2\Delta d\nu_S}{c} = m \quad (m\in \mathbf{N}) \tag{4}$$

The sidebands are resonant in the RC and non-resonant in the FP, if they are approximately an integer number of RC free spectral ranges away from the subcarrier:

$$\frac{2d\nu_2}{c} \approx n \quad (n\in \mathbf{N}, \text{ for } \nu_2 < \frac{c}{4\Delta d}) \tag{5}$$

Special case

If the arm cavity length L is an odd multiple of the recycling cavity length d, then there is a set of special frequencies

$$\nu = \frac{c}{4d} \cdot (2n+j) \qquad n=0,1,2,... , j=0 \text{ or } 1, \cos\frac{2\pi\nu\Delta d}{c} > 0 \ . \tag{6}$$

All these frequencies are recycled (although not with equal efficiency); the ones at j=0 are resonant in the FP cavities, the ones at j=1 are antiresonant. This situation is depicted in Fig. 2:

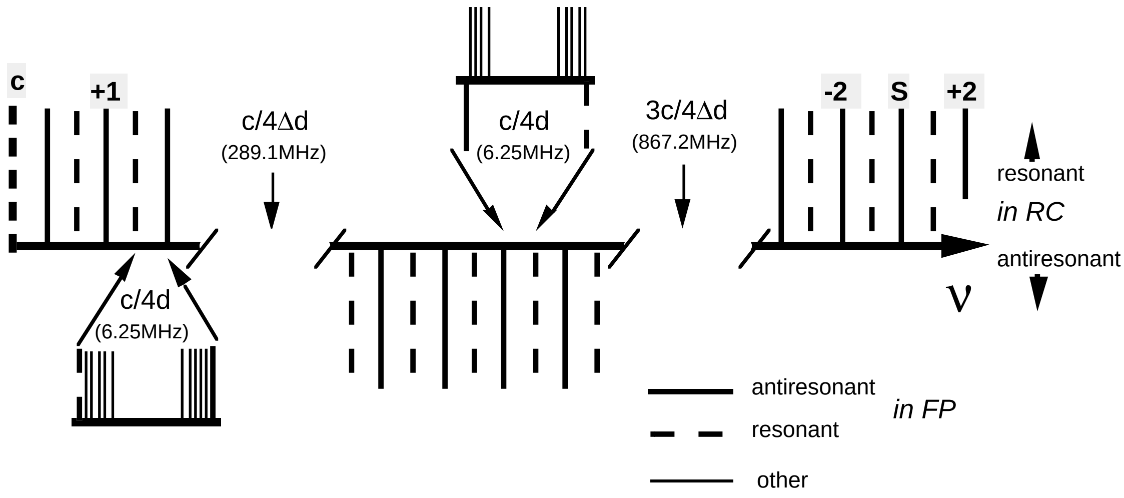

Fig.2 Spectrum of frequencies resonant in the recycling cavity, if the FP length is an odd multiple of the RC length. The numbers given are for d=12 m, L=2988m and Δd=0.2594m. Indicated are also possible positions of carrier (c), one of the main modulation sidebands (+1), subcarrier (S), and the two secondary modulation sidebands (-2,+2), according to the scheme given in the text.

So there is a system of RC resonances that alternatingly enter the FP's or not, and which are all a multiple of a common basis frequency $\nu_b = c/4d$. (The frequencies ν for which $\cos(\Delta d 2\pi\nu/c) < 0$ are antiresonant in the recycling cavity). The asymmetry Δd must be chosen within the neighborhood of the optimum value obtained by Eq. 1. ν_S must fulfill Eq. 6 and the dark fringe condition Eq. 4, which gives

$$\Delta d = d \cdot \frac{2m}{2k+1} \quad (k \in \mathbf{N}, m \in 2\mathbf{N}) \ , \qquad (7)$$

where $(2k+1) = \nu_S/\nu_b$. ν_1 must be an odd multiple, ν_2 an even multiple of ν_b; this assures that they fulfill the recycling and antiresonance requirements. The fact that all frequencies are at multiples of ν_b makes it possible to do all the modulation before the mode cleaner, provided that its length is an even multiple of d.

Interactions of the servo loops for SSB modulation

Once the automatic length control is working, the RC servo system will tune Δd such that there is a dark fringe for the subcarrier. However, the exact dark fringe condition can in general not be met for the carrier at the same time, since $\lambda_s = c/\nu_S$ ($\approx$ 1m) will be no exact multiple of the laser wavelength λ. Thus for the carrier there is a small deviation $\delta(\Delta d)$ from the dark fringe condition of the order $\lambda \cdot \lambda/\lambda_s \approx 10^{-6}\,\lambda$. The FP servo system will reestablish the dark fringe for the carrier by introducing an opposite detuning $\delta(\Delta L) = \pi/2F \cdot \delta(\Delta d) \approx 10^{-7}\text{-}10^{-8}\,\lambda$ in the FP, where F is the FP finesse; this will not change the situation for the subcarrier, since it is (to first order) not affected by the FP lengths. The same is true for d and L: The RC servo system will tune the average RC length d such that there is an exact resonance for the subcarrier. This will in general not coincide with an exact resonance for the carrier. Therefore the FP servo system will correct L by a tiny amount δL in the opposite sense and thus tune the RC resonance peak also to the carrier frequency.

III. Stepwise procedure for determining frequencies and lengths

Due to the mutual entanglement of all parameters, it is best to observe certain sequences when determining the modulation frequencies and interferometer lengths. This section gives a practical approach to finding the required parameters depending on the modulation scheme to be used and other boundary conditions, and derives numerical values for some possible configurations.

SSB modulation

1. If the modulation frequencies must pass through the mode cleaner
The mode cleaner will have a finesse of about 1000; at a length of 150m its free spectral range (FSR) is 1 MHz, and the bandwidth about 1000 Hz. This means, that the laser frequency and all important sidebands must coincide with a MC resonance within a few 100 Hz. This constraint can be relaxed by making the MC shorter; on the other hand, the resonances will get rarer then, since their spacing increases. So instead of hoping that one can find a set of three frequencies (subcarrier, main and secondary modulation) that accidentally fall on MC resonances, it is better to make use of the equally-spaced frequency pattern explained above (Fig. 2) by having a well-defined ratio between the RC and FP lengths.

The recycling cavity defines the length on the basis of which all other lengths and the frequencies are determined; so any value can be chosen. By choosing a bigger or smaller RC length, one can compress or expand the spectrum of possible frequencies. Let's choose for example 12m, giving a FSR of 12.5 MHz, and a basis frequency of 6.25 MHz. Next, the FP cavity length must be an odd multiple of this. If we want a length of about 3 km, the closest possible values are 2988 and 3012 m. Let's take 2988 m. Before we can determine the asymmetry, we must know the approximate frequency of the main modulation, which must be an odd multiple of the basis frequency. The possibilities are essentially 6.25 or 18.74 MHz; at higher frequencies the speed of the photodiodes becomes critical. 6.25 MHz is the basis frequency of which all other frequencies are multiples; therefore this is not a good choice for the main modulation, because its sidebands at higher harmonics might disturb the signal at the secondary modulation. 12.5, 25.0, ... MHz are possible, but more sensitive to modulation frequency instabilities. So let's take 18.74 MHz. At this frequency the sideband power coupled out to the interferometer exit is maximum, if the asymmetry is about 0.25 m (for losses ≈ 0 the interferometer, since the sideband is non-resonant, and 1% recycling mirror transmission). This gives as coarse values for the SSB frequency fulfilling the dark fringe condition 0, 600, 1200, ... MHz. Among these frequencies, every second falls in a band where the multiples of the basis frequency are antiresonant in the RC (see Fig. 2); so the lowest possible SSB frequency lies near 1200 MHz (m=2 in Eq. 4). This value must still be corrected, since because of the resonance requirements we need an odd multiple of the basis frequency. Let's take 185 ν_b=1155 MHz. With the dark fringe condition we can now determine the precise value of the asymmetry $\Delta d=12\text{m} \cdot 2{\cdot}2/185 = 0.2594\text{m}$. If the secondary modulation sidebands are an even multiple of the basis frequency away from the subcarrier, they meet the same resonance conditions; the lowest possible value of 12.5 MHz seems most appropriate.

Now all interferometer parameters are fixed, and the modulation frequencies can pass through the mode cleaner, if the latter is transparent for the basis frequency; so its length must be an even multiple of the RC length (24, 48, 72, ... m).

For the numbers given above, the subcarrier with more than 1 GHz lies quite high. This value comes from the small asymmetry due to the high main modulation frequency. The SSB could be generated by a laser which is locked to the main laser by observing the beat note between the two and comparing it to a high frequency oscillator. At the detection side no GHz speed is required, since only the beat between subcarrier and its sidebands (12.5 MHz) is observed. If the SSB frequency seems too high, it can be reduced by choosing a bigger asymmetry (see Appendix A). For example, if the asymmetry is increased by a factor of two, the SSB frequency decreases by 50%; then the signal-to-noise ratio decreases by 2%, since asymmetry and main modulation frequency are no longer matched.

2. If only the main modulation must pass through the mode cleaner

The configuration developed above can of course be used also in this case; but here one has a greater freedom in choosing the modulation frequencies and lengths, since their values need not have fixed ratios to each other. However, the situation becomes more complex, because there is in general no simple resonance scheme like in Fig.2 for guidance.

Again, one can start by setting the RC length e.g. to 12m. Since this time the modulation frequencies do not have to be multiples of a common basis frequency, one can take for the main modulation any one of the many RC resonances spaced at ≈ 50 kHz intervals. In practice, however, modulation frequencies close to the antiresonance in the FP's are better, since they have kHz instead of Hz linewidthes. Thus the main modulation frequency will be a few FP free spectral ranges around 6.25, 18.74, ... MHz. Let's choose ≈ 6.25 MHz. This gives an optimum asymmetry of about 0.76 m, which in turn imposes by the dark fringe condition a subcarrier frequency of a multiple of 197 MHz; due to the asymmetry, odd multiples experience a 180° phase jump during a RC roundtrip, while even multiples do not. The exact frequency value is given by the resonance conditions: One demands resonance in the RC and simultaneously antiresonance in the FP. The combination of the two conditions fixes the subcarrier frequency to an even multiple of the RC FSR if there is a phase jump, else to an odd multiple. If we prefer the lowest possible frequency around 197 MHz, this gives a value of e.g. 187.4 or 200 Mhz. Let's take 187.4 MHz; then the asymmetry must be fine tuned to 0.8m in order to get a dark fringe. For antiresonance in the FP the frequency must be an odd multiple of half the FP FSR, which fixes the FP length to ..., 2999.6, 3000.4, ... m. Let's take 3000.4 m. With this value the exact main modulation frequency can be determined by solving Eq. 2. If we want a broad resonance, we can choose between about a dozen frequencies from 6 to 6.5 MHz, e.g. 6.22 or 6.27 MHz. For the secondary modulation, the frequencies must again be searched in a not too broad range around multiples of the RC FSR (12.5, 25, ...) MHz. The exact values are again given by the peak of the RC resonance (e.g. 12.49 MHz). The mode cleaner must be transparent for the main modulation, which allows lengths of 23.9, 47.8, ... m.

This scheme has the advantage that the modulation frequencies have no relation with each other, so there will be no interference between them due to higher order harmonics. Therefore it was possible to use the frequency at ≈6.25 MHz for the main modulation, which allows a lower SSB frequency. Since there are no a priori fixed length relations, there is also a greater freedom for the FP cavity length, which can be chosen in finer steps (0.8 m instead of 24 m). On the other hand, it may turn out that 6.25 MHz is too low for the main modulation, since at this frequency the laser might not yet be quiet enough.

Simple frontal modulation

In the case of the simple frontal modulation, one sacrifices the requirement that the demodulated signals are well decoupled. This greatly simplifies the constraints for the parameters, since only one modulation frequency is needed now. So one can determine the RC and FP lengths (say, 12 and 3000 m), and then choose for the modulation a frequency that is recycled while preferably being non-resonant in the FP (around 6.25, 18.74, ... MHz), e.g. 6.27 MHz. Finally, the signal is optimized by adjusting the asymmetry to the modulation frequency (0.8 m).

IV. Conclusion

The following table summarizes the frequency schemes given above. Of course these are just examples, and other configurations are possible according to requirements.

N°	1	2	3	4
RC length [m]	12			11.94
FP length [m]	3000	3000.4	2988	4000
asymmetry [m]	0.8		0.2594	?
MC length	23.9/47.8/...	23.9/47.8/...	24/48/...	23.88
Main modulation [MHz]	6.268…	6.267…	18.737…	18.84
Subcarrier [MHz]	---	187.37...	1155.45...	250
Sec. modulation [MHz]	---	12.490…	12.491…	12.54
Ref.	[2]	[2]		[3]

Tab.1 Some possible sets of numerical values. 1: Simple frontal modulation, 2: SSB modulation, 3: SSB modulation, where all frequencies pass through the mode cleaner. 4: Scheme proposed for LIGO (there is also a ternary modulation at 31.4 MHz). $c \equiv 299792458$ m/s.

As it could be seen, there exists a scheme in which all the modulation frequencies in the case of SSB modulation can pass through the mode cleaner, so that the beam jitter and small misalignments are filtered out also for the subcarrier and its sidebands. Whether this is necessary or not remains to be investigated. If not, some flexibility can be gained.

[1] R.W. Drever, J.L. Hall, F.V. Kowalski, J. Hough, G.M. Ford, A.J. Manley, H. Ward "Laser phase and frequency stabilization using an optical resonator". Appl. Phys. B **31**, 1983

[2] R. Flaminio, H. Heitmann "Interferometer locking scheme". VIRGO note PJT 93021, 1993

[3] J. Giaime, "The effects of harmonics of modulation frequencies in an asymmetry GW-readout interferometer with a subcarrier for auxiliary readout." LIGO internal report 1993

[4] J.-Y. Vinet "Fonction de transfert optique d'un interféromètre à recyclage standard". VIRGO note 27.01.1993

Appendix A: Constraints for the choice of the asymmetry

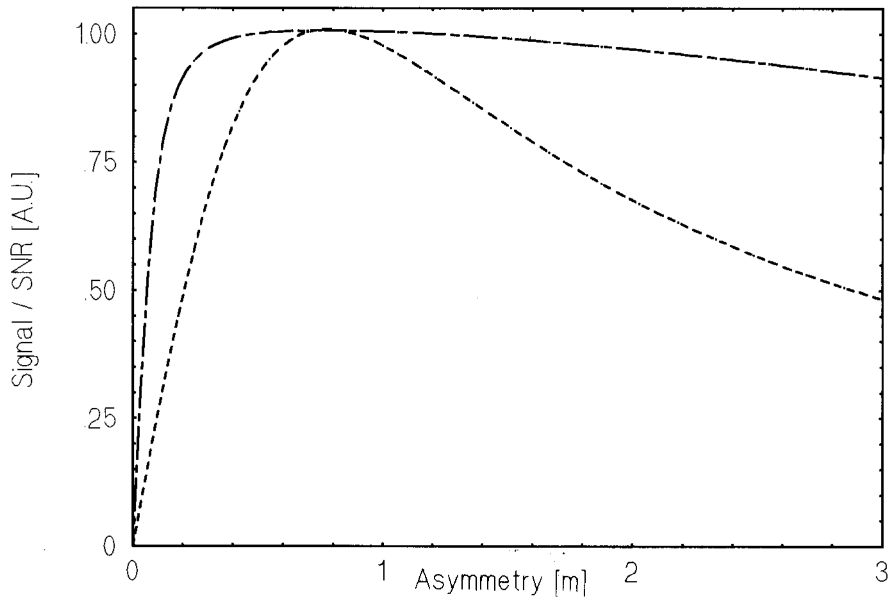

Fig.3 Demodulated interferometer output signal (lower curve) and signal-to-noise ratio (upper curve) for simple frontal modulation in the case of varying asymmetry. Optimum value = 0.8 m.

In order to help assess the influence of the asymmetry on the signal, here is the result of a numerical simulation of the interferometer for varying asymmetry. It was assumed simple frontal modulation (scheme 1 in Tab. 1 (modulation at 6.27 MHz); contrast defect = 10^{-4}, recycling mirror transmission = 1% etc.). The lower curve in Fig. 3 shows the size of the demodulated signal at the interferometer output. With increasing asymmetry, the efficiency for sideband extraction from the recycling cavity first increases; at the same time, the recycling gain for the sideband decreases, as more power is coupled out. So the signal strength decreases after reaching a maximum. The upper curve is the signal-to-noise ratio; it shows a much weaker dependence on the asymmetry. This is because the noise considered here, the shot noise of the photodiode current, is mainly due to the sideband power for the given configuration (the carrier power on the photodiode is 100mW as opposed to 700 mW for both sidebands). So if the sideband power decreases, the shot noise and the signal decrease together, which leads to a weak overall dependency around the maximum. If one allows for a 2% decrease in SNR, one can vary the asymmetry between 0.4 m and 1.6 m around the optimum value 0.8 m from Eq. 1.

Appendix B: Frequency list

This Appendix gives as an example the list of frequencies resonant in the recycling cavity for the case of simple frontal modulation with the FP length being an *even* multiple of the RC length (scheme 1 in Tab. 1); so here there is no "nice" resonance frequency around 6.25 MHz. The table contains for each RC resonance the frequency, the linewidth, the recycling gain and the power enhancement in the FP cavity. As it can be seen, the only broad resonances occur quite near to the FP antiresonance, and around 6.25 MHz there are altogether about a dozen resonances with kHz width. Furthermore one sees that for increasing frequencies the recycling tends to get worse, because more and more power is lost through the asymmetry. Accordingly, the recycling gain of the subcarriers in scheme 2 and 3 (187.4 and 1155 MHz) is high (about 375), since no power is coupled out to the dark fringe, and no power is lost by resonance in the FP's.

0.0	3.3	99.0	24.54	49957.3	3.3	99.0	24.54	99914.6	3.3	99.1	24.53	149871.9	3.3	99.1	24.51
199829.1	3.3	99.2	24.48	249786.4	3.3	99.2	24.45	299743.7	3.3	99.3	24.41	349700.9	3.3	99.4	24.36
399658.1	3.3	99.6	24.30	449615.3	3.3	99.7	24.23	499572.5	3.3	99.9	24.16	549529.6	3.3	100.0	24.08
599486.8	3.3	100.2	23.99	649443.9	3.3	100.4	23.90	699400.9	3.3	100.7	23.80	749357.9	3.3	100.9	23.69
799314.9	3.4	101.1	23.57	849271.8	3.4	101.4	23.45	899228.7	3.4	101.7	23.32	949185.6	3.4	102.0	23.18
999142.3	3.4	102.3	23.04	1049099.1	3.4	102.6	22.88	1099055.7	3.5	103.0	22.73	1149012.4	3.5	103.3	22.56
1198968.9	3.5	103.7	22.39	1248925.4	3.5	104.1	22.22	1298881.8	3.5	104.5	22.03	1348838.1	3.6	104.9	21.84
1398794.3	3.6	105.3	21.65	1448750.5	3.6	105.8	21.45	1498706.6	3.6	106.2	21.24	1548662.5	3.7	106.7	21.03
1598618.4	3.7	107.2	20.81	1648574.2	3.7	107.7	20.58	1698529.9	3.8	108.2	20.35	1748485.4	3.8	108.7	20.12
1798440.9	3.8	109.2	19.88	1848396.2	3.9	109.7	19.64	1898351.4	3.9	110.3	19.39	1948306.4	3.9	110.8	19.14
1998261.3	4.0	111.4	18.88	2048216.1	4.0	112.0	18.62	2098170.7	4.1	112.5	18.35	2148125.2	4.1	113.1	18.08
2198079.5	4.2	113.7	17.81	2248033.6	4.2	114.3	17.54	2297987.6	4.3	114.9	17.26	2347941.3	4.4	115.4	16.97
2397894.9	4.4	116.0	16.69	2447848.2	4.5	116.6	16.40	2497801.3	4.6	117.2	16.11	2547754.3	4.6	117.8	15.81
2597706.9	4.7	118.4	15.52	2647659.3	4.8	119.0	15.22	2697611.5	4.9	119.6	14.92	2747563.4	5.0	120.2	14.62
2797515.0	5.1	120.7	14.32	2847466.3	5.1	121.3	14.01	2897417.3	5.3	121.9	13.71	2947368.0	5.4	122.4	13.40
2997318.4	5.5	122.9	13.09	3047268.3	5.6	123.5	12.79	3097218.0	5.7	124.0	12.48	3147167.2	5.9	124.5	12.17
3197116.0	6.0	124.9	11.86	3247064.3	6.1	125.4	11.56	3297012.2	6.3	125.8	11.25	3346959.6	6.5	126.2	10.94
3396906.5	6.6	126.6	10.64	3446852.9	6.8	127.0	10.33	3496798.7	7.0	127.3	10.03	3546743.8	7.2	127.6	9.73
3596688.4	7.4	127.9	9.43	3646632.3	7.7	128.2	9.13	3696575.4	7.9	128.4	8.83	3746517.8	8.2	128.6	8.54
3796459.4	8.5	128.8	8.25	3846400.2	8.8	128.9	7.96	3896340.0	9.1	129.0	7.67	3946278.9	9.5	129.0	7.39
3996216.7	9.8	129.0	7.11	4046153.4	10.2	129.0	6.83	4096089.0	10.7	128.9	6.55	4146023.2	11.1	128.8	6.28
4195956.1	11.6	128.7	6.02	4245887.6	12.1	128.5	5.75	4295817.5	12.7	128.2	5.50	4345745.6	13.4	128.0	5.24
4395672.0	14.0	127.6	4.99	4445596.3	14.8	127.2	4.75	4495518.5	15.6	126.8	4.51	4545438.4	16.5	126.4	4.27
4595355.7	17.4	125.9	4.04	4645270.2	18.5	125.3	3.82	4695181.7	19.7	124.7	3.60	4745089.8	21.0	124.0	3.38
4794994.2	22.4	123.4	3.17	4844894.5	24.0	122.6	2.97	4894790.3	25.8	121.8	2.77	4944681.0	27.7	121.0	2.58
4994566.2	30.0	120.2	2.40	5044445.0	32.5	119.2	2.22	5094316.7	35.3	118.3	2.05	5144180.3	38.6	117.3	1.88
5194034.8	42.3	116.3	1.72	5243878.8	46.5	115.2	1.57	5293710.8	51.5	114.1	1.42	5343528.8	57.2	113.0	1.29
5393330.4	64.0	111.9	1.15	5443113.0	72.0	110.7	1.03	5492872.8	81.5	109.5	0.91	5542605.3	93.1	108.2	0.80
5592304.9	207.1	106.9	0.70	5641964.2	224.5	105.6	0.60	5691573.4	246.1	104.3	0.52	5741119.6	273.7	103.0	0.44
5790585.3	409.2	101.6	0.36	5839946.4	455.8	100.3	0.30	5889168.8	618.1	98.9	0.24	5938204.0	803.1	97.5	0.19
5986982.4	1020.8	96.1	0.14	6035404.1	1385.2	94.8	0.11	6083333.5	1912.6	93.4	0.08	6130609.5	2510.8	92.0	0.06
6177113.0	3150.3	90.7	0.05	6222919.1	3627.1	89.3	0.04	6268432.1	3696.6	88.0	0.04	6314238.6	3217.5	86.7	0.05
6360742.3	2602.3	85.3	0.06	6408018.6	1984.6	83.9	0.08	6455948.1	1526.6	82.6	0.11	6504370.0	1134.4	81.2	0.14
6553148.4	895.0	79.7	0.19	6602183.6	693.7	78.3	0.24	6651406.0	519.3	76.9	0.30	6700767.1	463.5	75.5	0.36
6750232.8	420.9	74.2	0.44	6799779.1	287.9	72.8	0.52	6849388.3	261.8	71.4	0.60	6899047.5	240.9	70.0	0.70
6948747.1	223.8	68.7	0.80	6998479.7	209.8	67.4	0.91	7048239.5	98.2	66.1	1.03	7098022.0	88.4	64.8	1.15
7147823.7	80.1	63.5	1.29	7197641.7	73.0	62.2	1.42	7247473.6	66.8	61.0	1.57	7297317.6	61.5	59.7	1.72
7347172.1	56.9	58.5	1.88	7397035.8	52.8	57.3	2.05	7446907.5	49.2	56.2	2.22	7496786.3	45.9	55.0	2.40
7546671.4	43.1	53.9	2.58	7596562.2	40.5	52.8	2.77	7646457.9	38.2	51.7	2.97	7696358.3	36.1	50.6	3.17
7746262.7	34.3	49.5	3.38	7796170.8	32.5	48.5	3.60	7846082.2	31.0	47.5	3.82	7895996.7	29.6	46.5	4.04
7945914.1	28.3	45.5	4.27	7995833.9	27.1	44.6	4.51	8045756.1	26.0	43.7	4.75	8095680.5	24.9	42.7	4.99
8145606.8	24.0	41.9	5.24	8195535.0	23.1	41.0	5.50	8245464.9	22.3	40.1	5.75	8295396.3	21.6	39.3	6.02
8345329.2	20.9	38.5	6.28	8395263.5	20.2	37.7	6.55	8445199.0	19.6	36.9	6.83	8495135.7	19.0	36.1	7.11
8545073.6	18.5	35.4	7.39	8595012.4	18.0	34.7	7.67	8644952.3	17.5	34.0	7.96	8694893.0	17.1	33.3	8.25
8744834.6	16.7	32.6	8.54	8794777.0	16.3	31.9	8.83	8844720.2	15.9	31.3	9.13	8894664.0	15.5	30.7	9.43
8944608.6	15.2	30.1	9.73	8994553.8	14.9	29.5	10.03	9044499.6	14.6	28.9	10.33	9094445.9	14.3	28.3	10.64
9144392.8	14.1	27.8	10.94	9194340.2	13.8	27.2	11.25	9244288.1	13.6	26.7	11.56	9294236.5	13.4	26.2	11.86
9344185.3	13.1	25.7	12.17	9394134.5	12.9	25.2	12.48	9444084.1	12.8	24.7	12.79	9494034.1	12.6	24.2	13.09
9543984.4	12.4	23.8	13.40	9593935.1	12.2	23.3	13.71	9643886.1	12.1	22.9	14.01	9693837.4	11.9	22.5	14.32
9743789.0	11.8	22.1	14.62	9793740.9	11.7	21.7	14.92	9843693.1	11.5	21.3	15.22	9893645.5	11.4	20.9	15.52
9943598.2	11.3	20.6	15.81	9993551.1	11.2	20.2	16.11	10043504.2	11.1	19.8	16.40	10093457.6	11.0	19.5	16.69
10143411.1	10.9	19.2	16.97	10193364.9	10.8	18.8	17.26	10243318.8	10.7	18.5	17.54	10293272.9	10.6	18.2	17.81
10343227.2	10.6	17.9	18.08	10393181.7	10.5	17.6	18.35	10443136.3	10.4	17.3	18.62	10493091.1	10.4	17.1	18.88
10543046.0	10.3	16.8	19.14	10593001.1	10.3	16.5	19.39	10642956.2	10.2	16.3	19.64	10692911.6	10.2	16.0	19.88
10742867.0	10.1	15.8	20.12	10792822.6	10.1	15.5	20.35	10842778.2	10.0	15.3	20.58	10892734.0	10.0	15.0	20.81
10942689.9	10.0	14.8	21.03	10992645.9	10.0	14.6	21.24	11042601.9	9.9	14.4	21.45	11092558.1	9.9	14.2	21.65
11142514.3	9.9	14.0	21.84	11192470.7	9.9	13.8	22.03	11242427.1	9.9	13.6	22.21	11292383.5	9.9	13.4	22.39
11342340.1	9.8	13.2	22.56	11392296.7	9.8	13.0	22.73	11442253.3	9.8	12.9	22.88	11492210.1	9.8	12.7	23.04
11542166.9	9.8	12.5	23.18	11592123.7	9.8	12.4	23.32	11642080.6	9.9	12.2	23.45	11692037.5	9.9	12.0	23.57
11741994.5	9.9	11.9	23.69	11791951.5	9.9	11.7	23.80	11841908.6	9.9	11.6	23.90	11891865.7	9.9	11.4	23.99
11941822.8	10.0	11.3	24.08	11991779.9	10.0	11.2	24.16	12041737.1	10.0	11.0	24.23	12091694.3	10.0	10.9	24.30
12141651.5	10.1	10.8	24.36	12191608.8	10.1	10.7	24.41	12241566.0	10.2	10.5	24.45	12291523.3	10.2	10.4	24.48
12341480.6	10.2	10.3	24.51	12391437.8	10.3	10.2	24.53	12441395.1	10.3	10.1	24.54	12491352.4	10.4	10.0	24.54
12541309.7	10.5	9.9	24.54	12591267.0	10.5	9.8	24.53	12641224.3	10.6	9.6	24.51	12691181.6	10.6	9.5	24.48
12741138.8	10.7	9.4	24.45	12791096.1	10.8	9.4	24.41	12841053.3	10.9	9.3	24.36	12891010.5	10.9	9.2	24.30
12940967.7	11.0	9.1	24.23	12990924.9	11.1	9.0	24.16	13040882.1	11.2	8.9	24.08	13090839.2	11.3	8.8	23.99
13140796.3	11.4	8.7	23.90	13190753.3	11.5	8.6	23.80	13240710.3	11.6	8.6	23.69	13290667.3	11.7	8.5	23.57
13340624.2	11.8	8.4	23.45	13390581.1	11.9	8.3	23.32	13440538.0	12.1	8.3	23.18	13490494.8	12.2	8.2	23.04
13540451.5	12.3	8.1	22.88	13590408.2	12.5	8.0	22.73	13640364.8	12.6	8.0	22.56	13690321.3	12.8	7.9	22.39
13740277.8	12.9	7.8	22.21	13790234.2	13.1	7.8	22.03	13840190.5	13.3	7.7	21.84	13890146.7	13.4	7.6	21.65
13940102.9	13.6	7.6	21.45	13990059.0	13.8	7.5	21.24	14040014.9	14.0	7.4	21.03	14089970.8	14.2	7.4	20.81
14139926.6	14.4	7.3	20.58	14189882.3	14.6	7.3	20.35	14239837.8	14.9	7.2	20.12	14289793.3	15.1	7.1	19.88
14339748.6	15.3	7.1	19.64	14389703.8	15.6	7.0	19.39	14439658.8	15.9	7.0	19.14	14489613.7	16.1	6.9	18.88
14539568.5	16.4	6.9	18.62	14589523.1	16.7	6.8	18.35	14639477.6	17.0	6.8	18.08	14689431.9	17.3	6.7	17.81
14739386.0	17.7	6.7	17.53	14789339.9	18.0	6.6	17.26	14839293.7	18.4	6.6	16.97	14889247.2	18.8	6.5	16.69
14939200.6	19.2	6.5	16.40	14989153.7	19.6	6.4	16.11								

Tab.2 List of modulation frequencies resonant in the RC as calculated numerically using the formulas derived in [4] (adapted to the phase convention used). The values given for each resonance are the frequency (carrier=0) and the linewidth [Hz], and the power enhancement in the RC and FP [relative to the power incident on the corresponding cavity]. L=3000m, d=12m, Δd=0.8m, R_r=0.99, R_1=0.85,R_2=0.9999, c=299792458 m/s. The mirror losses are respectively 0, $3\cdot10^{-4}$, 0.

Appendix C: Stability of modulation frequencies

Subcarrier

Static detunings and slow drifts

Once the feedback loops are working, we can assume that they assure the RC resonance and dark fringe condition for carrier and subcarrier. The feedback using the subcarrier acts on the RC lengths, while the main feedback controls the FP lengths. If the subcarrier frequency ν_S deviates from the ideal value by $\delta\nu_S$, its RC resonance and dark fringe condition are restored by the "short" Michelson loops by changing RC length d and asymmetry Δd:

$$\delta d = \frac{\delta\nu_S}{\nu_o+\nu_S} \cdot (d+L/g) \quad ; \quad \delta(\Delta d) = \frac{\delta\nu_S}{\nu_o+\nu_S} \cdot (\Delta d+\Delta L/g) \ . \qquad (C1)$$

Here $g=2F/\pi\approx 25$ is the power enhancement in the FP, with F its finesse, and ν_o the laser frequency. Since this affects the carrier RC resonance and dark fringe condition, the "long" Michelson feedbacks change L and ΔL in the opposite sense, as explained before:

$$\delta L = -\frac{\delta d}{g} \quad ; \quad \delta(\Delta L) = -\frac{\delta(\Delta d)}{g} \ . \qquad (C2)$$

These actions detune the FP resonances:

$$\nu_{FP}(\nu_o)-\nu_o \approx -\frac{\delta L}{L} \cdot \nu_o = \delta\nu_S \cdot \frac{1}{g}\left(\frac{d}{L} + \frac{1}{g}\right) \qquad (C3)$$

$$\nu_{FP}(\nu_S)-\nu_S \approx -\delta\nu_S \qquad (C4)$$

For a good resonance/antiresonance in the FP, we demand

$$\nu_{FP}-\nu_S < \varepsilon \cdot \frac{c}{2L} \cdot \frac{1}{2F} \quad (\text{e.g. } \varepsilon = 0.1) \ ; \qquad (C5)$$

then Eqs. C4 with C5 give the strongest upper limit on the static deviation with $\delta\nu_S<50$ Hz.

Frequency noise

The "short" Michelson feedback system translates subcarrier frequency fluctuations $\tilde{\nu}_S$ to RC asymmetry fluctuations $\tilde{\Delta d}$; those are interpreted by the main detector in the same way as GW signals. This led in [2] to an upper limit for \o(Δd;~) of $1.5\cdot 10^{-17}$ and $5\cdot 10^{-19}$ m/$\sqrt{\text{Hz}}$ for 10 and 100 Hz, respectively. With

$$\tilde{\Delta d} = \frac{\tilde{\nu}_S}{\nu_o+\nu_S} \cdot (\Delta d+\Delta L/g) \qquad (C6)$$

it follows $\tilde{\nu}_S < 5\cdot 10^{-3}$ and $2\cdot 10^{-4}$ Hz/$\sqrt{\text{Hz}}$, respectively, for $\Delta d=0.8$m. (As usual, the FP arm asymmetry was assumed to be negligible, which is however not necessarily the case; especially, the finesses of the two cavities may be different.)

Main and secondary modulation

Static detunings and slow drifts

If the optical and electrical paths from the frequency generator to the demodulator experience different delays, then a changing ν (ν_1 or ν_2) introduces a changing phase shift $\varphi(\delta\nu)$ between photocurrent and local oscillator. If the phase was initially compensated such that $\varphi(0)=0$, then one must demand $\varphi(\delta\nu) < \varepsilon\pi/2$ with e.g. $\varepsilon = 0.1$ in order to have a good demodulation efficiency. If the demodulation must at the same time reject a strong quadrature signal (like the L_1,L_2 dependence for simple frontal modulation) by a factor s (e.g. $s=10^{-3}$), then the much stronger condition $\varphi(\delta\nu) < s$ must hold. The phase shift is given by

$$\varphi(\delta\nu) = \varphi_{el}(\delta\nu)-\varphi_{opt}(\delta\nu) = \delta\nu \, \frac{2\pi}{c} \, (l_{el} - l_{opt}) \ , \qquad (C7)$$

where $l_{opt} \approx 4/p \cdot (d+L/g) \approx 27$km $\{4/p \cdot (d+Lg) \approx 4900km\}$ is the effective optical delay length for the sideband non-resonating {resonating} in the FP, p being its losses. If the wire length between oscillator and demodulator is negligible, then it follows $\delta\nu < 280$ {1.5} Hz and 5 {0.03} Hz, respectively, for the two cases given above.

Frequency noise

Frequency jitter of the modulation sidebands has to first order no influence on the signal, because, if the optical and electrical delay are the same, the phases of photocurrent and local oscillator change synchronously; thus the fluctuations cancel in the demodulated output. In the case of unequal delays a varying phase shift is introduced. Again, the effect of a varying phase between local oscillator and signal is to change the demodulation efficiency. So, if a gravitational signal is present, its detected amplitude will be slightly modulated according to the instantaneous frequency deviation of the oscillator. If the fluctuating modulation is represented by $\cos(2\pi\nu+m\alpha(t))$, then the instantaneous frequency is $\nu+\delta\nu(t)$ with the jitter $\delta\nu(t)=m\dot{\alpha}(t)/2\pi$. Demanding $\varphi(\delta\nu(t)) < \varepsilon\pi$ leads to the same tolerances for $\delta\nu(t)$ as for $\delta\nu$ above.