Coating detectability on thin disk-shaped substrates.

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Abstract. Coating material loss angle ϕ_{coat} measurement is fundamental to make an evaluation of coating thermal noise. The measurement can be performed through a differential measurement of the mechanical losses before the coating deposition (that is, just substrate's losses) and after (that is, substrate and coating losses). This kind of measurement suffer, beyond the measurement error due to the experimental set-up, from the dependence from the substrate's loss angle, that indeed can vary over time, giving an additional sensitivity error on coating's loss angle estimation. In this note, the two source if error are considered, defining limits to make coating's loss angle estimation feasible.
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1. Sensibility condition

The sensibility is the ability to see in terms of mechanical loss angle the effect of coating deposition upon a substrate over the possible variation of the substrate itself. For this reason, the sensibility condition depends from the substrate's mechanical loss angle or quality factor variation during time: $\phi_{sub} \to \phi'_{sub}$ or $Q_{sub} \to Q'_{sub}$.
To keep under control this phenomenon, a condition must be imposed. The relative error on the coating mechanical loss angle evaluation must be small, that is:

$$\frac{|\Delta\phi_{coat}|}{\phi_{coat}} = \frac{|\phi_{coat}^{meas} - \phi_{coat}^{true}|}{\phi_{coat}} \leq \eta, \tag{1}$$

where η is a small quantity and $\Delta\phi_{coat}$ is the difference between the measured ϕ_{coat}^{meas} and true ϕ_{coat}^{true} values of coating loss angle.
The only available informations are the one regarding loss angle measurements performed before $\phi_1 = \phi_{sub}$ and after coating deposition $\phi_2 = \phi_{sub+coat}$. The latter one depends on substrate loss angle variation, giving the uncertainty on coating loss angle estimation:

$$\phi_2 = \phi_{sub} + D\phi_{coat}^{meas} = \phi'_{sub} + D\phi_{coat}^{true} = \phi_{sub} + \Delta\phi_{sub} + D\phi_{coat}^{true}, \tag{2}$$

where $\Delta\phi_{sub} = \phi'_{sub} - \phi_{sub}$ is the variation of the substrate's loss angle.
From eq. (2), $\phi_{coat}^{meas} = \phi_{coat}^{true} + \Delta\phi_{sub}/D$ and putting this in eq. (1):

$$\frac{|\Delta\phi_{coat}|}{\phi_{coat}} = \frac{\left|\phi_{coat}^{true} + \frac{\Delta\phi_{sub}}{D} - \phi_{coat}^{true}\right|}{\phi_{coat}} = \frac{|\Delta\phi_{sub}|}{D\phi_{coat}} \leq \eta \qquad \longrightarrow \qquad |\Delta\phi_{sub}| \leq \eta D\phi_{coat}, \tag{3}$$

or in terms of substrate's quality factors Q:

$$\left| \frac{1}{Q'_{sub}} - \frac{1}{Q_{sub}} \right| \leq \eta D \phi_{coat} \tag{4}$$

Therefore, there are the tow following chances:

$$\begin{cases} \text{if } \; Q_{sub} > Q'_{sub} & \rightarrow \quad \frac{1}{Q'_{sub}} - \frac{1}{Q_{sub}} > 0 \quad \rightarrow \quad Q'_{sub} \geq \frac{1}{\frac{1}{Q_{sub}} + \eta D \phi_{coat}} = Q_s^- \\ \text{if } \; Q_{sub} < Q'_{sub} & \rightarrow \quad \frac{1}{Q'_{sub}} - \frac{1}{Q_{sub}} < 0 \quad \rightarrow \quad Q'_{sub} \leq \frac{1}{\frac{1}{Q_{sub}} - \eta D \phi_{coat}} = Q_s^+ \end{cases} \tag{5}$$

In Fig. 1 the two limits are shown in a plot Q'_{sub} VS Q_{sub}, for a fused silica Corning7980 substrate of 1 mm thickness with a fused silica film coating of 100 nm.

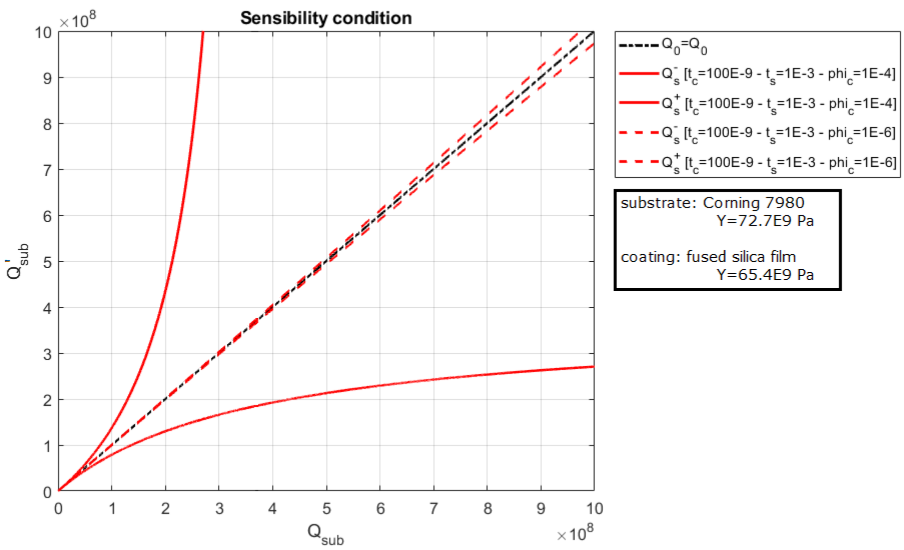

Figure 1. Sensibility condition for a fused silica Corning7980 substrate of 1 mm thickness with a fused silica film coating of 100 nm (and $\eta \sim 0.1$). The variation of the coating loss angle afflict significantly the sensibility condition, reducing drastically the variation of substrate loss angle variation area.

2. Detectability condition

Like in every measurement set-up, also GeNS [1] has an intrinsic uncertainty in the measured loss angle, ϵ. The typical uncertainty of the mechanical losses measured with GeNS are of the order of $\epsilon \sim 5\%$.

The detectability depends on the quality factor Q_{sub} of the substrate, in relation to Q'_{sub} and ϕ_{coat}, trough the uncertainty of the measurement ϵ. To be able to see the variation between before and after coating deposition, it must be:

$$\frac{\Delta\phi}{\phi_{sub}} = \frac{\phi_2 - \phi_1}{\phi_1} = \frac{\phi_{sub+coat} - \phi_{sub}}{\phi_{sub}} = \geq \epsilon \tag{6}$$

Using eq. (2), one can obtain that:

$$... = \frac{\phi_{sub} + \Delta\phi + D\phi_{coat} - \phi_{sub}}{\phi_{sub}} = \frac{\Delta\phi_{sub} + D\phi_{coat}}{\phi_{sub}} \geq \epsilon \tag{7}$$

from which:

$$\Delta\phi_{sub} = \left|\phi'_{sub} - \phi_{sub}\right| \geq \epsilon\phi_{sub} - D\phi_{coat} \tag{8}$$

One obtains a behaviour in ϕ'_{sub} (or Q'_{sub}):

$$\phi'_{qub} \geq (\epsilon + 1)\phi_{sub} - D\phi_{coat} \qquad \longrightarrow \qquad \frac{1}{Q'_{sub}} \geq \frac{\epsilon + 1}{Q_{sub}} - D\phi_{coat} \tag{9}$$

and there are two cases:

$$\begin{cases} \text{if } \dfrac{\epsilon+1}{Q_{sub}} - D\phi_{coat} \geq 0 & \longrightarrow \quad Q_{sub} \geq \dfrac{\epsilon+1}{D\phi_{coat}} \quad \longrightarrow \quad Q'_{sub} \leq \dfrac{1}{\frac{\epsilon+1}{Q_{sub}} - D\phi_{coat}} \\ \text{if } \dfrac{\epsilon+1}{Q_{sub}} - D\phi_{coat} \leq 0 & \longrightarrow \quad Q_{sub} \leq \dfrac{\epsilon+1}{D\phi_{coat}} \quad \longrightarrow \quad Q'_{sub} \geq \dfrac{1}{\frac{\epsilon+1}{Q_{sub}} - D\phi_{coat}} \end{cases} \tag{10}$$

In Fig. 2 some examples of the curves described by eq. (10) are shown, for substrate and coating materials like before.

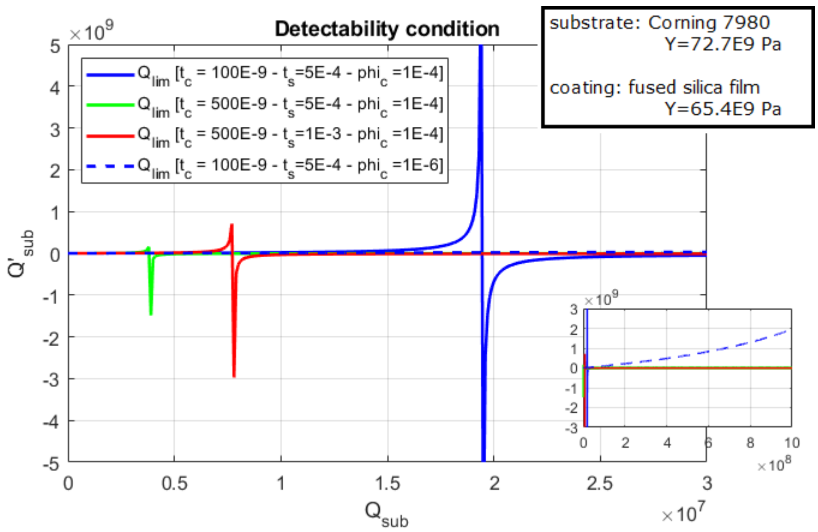

Figure 2. Detectability condition for a fused silica Corning7980 substrate with a fused silica film coating and $\epsilon \sim 0.05$. Two coating and two substrate thickness are shown, together with tow coating loss angle values. In the case $\phi_{coat} \sim 1E-6$, the curve changes lot, growing up much further, as can be seen in the small plot bottom-right. Vertical lines corresponds to $Q_{sub} = \frac{\epsilon+1}{D\phi_{coat}}$

3. Singularities

There are two singular values that came from the two conditions shown before:

$$\begin{cases} Q_s = \dfrac{1}{\eta D\phi_{coat}} & \text{given by sensibility} \\ Q_r = \dfrac{\epsilon+1}{D\phi_{coat}} & \text{given by detectability} \end{cases} \tag{11}$$

If, like in our case, $\eta \sim \epsilon$, it is always $Q_s < Q_r$. This mean that the condition is the one in figure.

Therefore, the detectability is imposed simply by $Q_{sub} \geq \bar{Q}$, and from sensibility request.

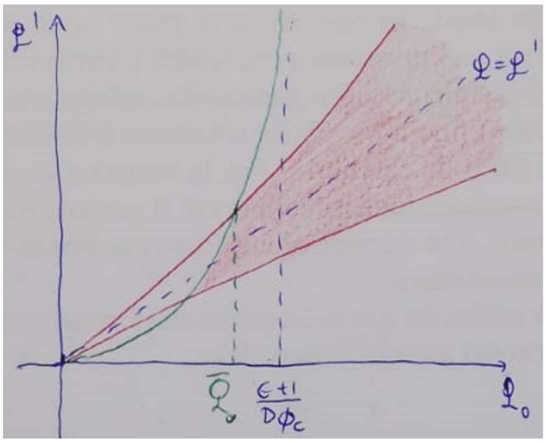

Figure 3. Superimposition of the sensibility condition (red lines) and detectability condition (green light).

It is possible to obtain the critical value $\bar{Q}_{sub}$ like:

$$\frac{1}{\frac{1}{Q_s ub} - \eta D\phi_{coat}} = \frac{1}{\frac{\epsilon+1}{Q_{sub}} - D\phi_{coat}} \tag{12}$$

from which:

$$\bar{Q}_{sub} = \frac{\epsilon}{1-\eta} \frac{1}{D\phi_{coat}} \tag{13}$$

For a system composed by a Corning7980 substrate of 0.5 mm thickness plus a fused silica film coating of 500 nm thickness with a mechanical loss angle of the order of $\phi_c \sim 1E-6$, the reference value is $\bar{Q}_{sub} \sim 2.06 \times 10^7$.

References

[1] Cesarini E., Lorenzini M., et al., *A "gentle" nodal suspension for measurements of the acoustic attenuation in materials*, Rev. of Sci. Instruments 80, 053904 (2009)