Virgo O3 NCal update
and
“FROMAGE”

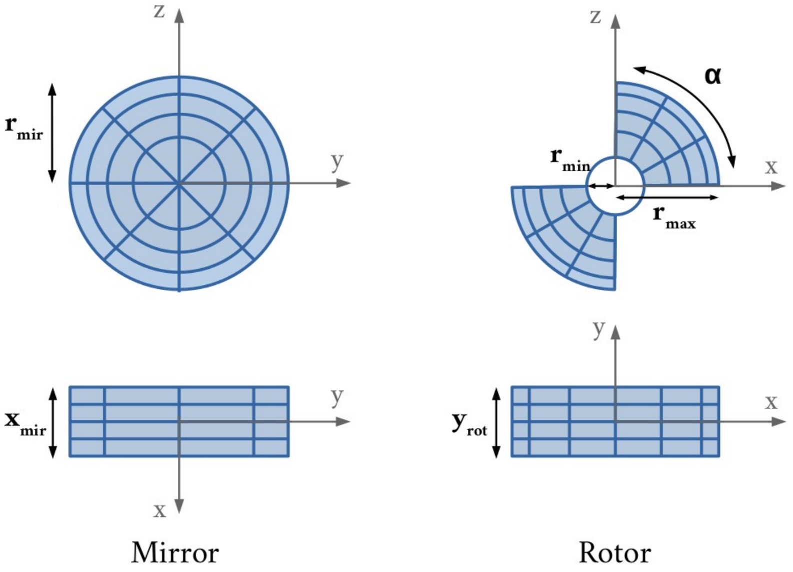
IPHC group
D. Estevez for the calibration team

Numerical model

D. Estevez et al., FROMAGE, VIR-0759A-20
Git repository: https://git.ligo.org/virgo/virgoapp/FROMAGE

FROMAGE: Finite element analysis of ROtating MAsses for Gravitational Effects

1) Discretize the objects into small 3D elements:



$I = (i, j, k)$ $J = (l, p, q)$

2) Compute the Newtonian force of gravity:

$$\vec{F}_\theta = G \sum_I \sum_J \frac{m_{mir,I}\, m_{rot,J}}{d^2_{I,J}} \vec{u}_{I,J}$$

3) Make a full turn with the rotor step by step θ

4) Using a Fourier expansion we extract the contribution to the longitudinal force up to the order N:

$$F(\theta) = \frac{1}{N}\left(C_0 + 2\sum_{k=1}^{N-1} |C_k| \cos(k\theta + \psi_k)\right)$$

Force at k.f_{rotor}

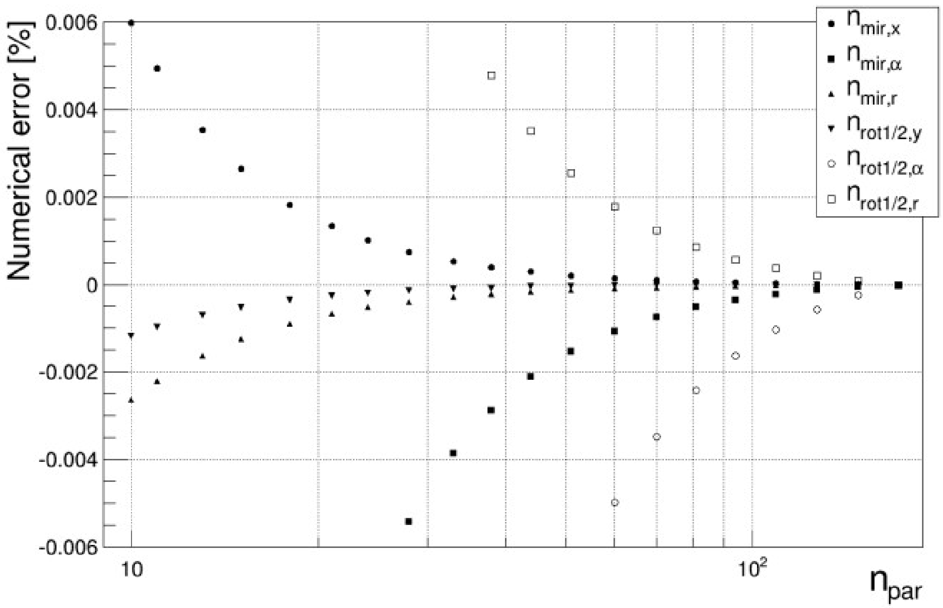
Grid size

Convergence test to choose the grid size with threshold at 0.005%:

Grid = (longitudinal cut n_x, angular cut n_α, radial cut n_r)

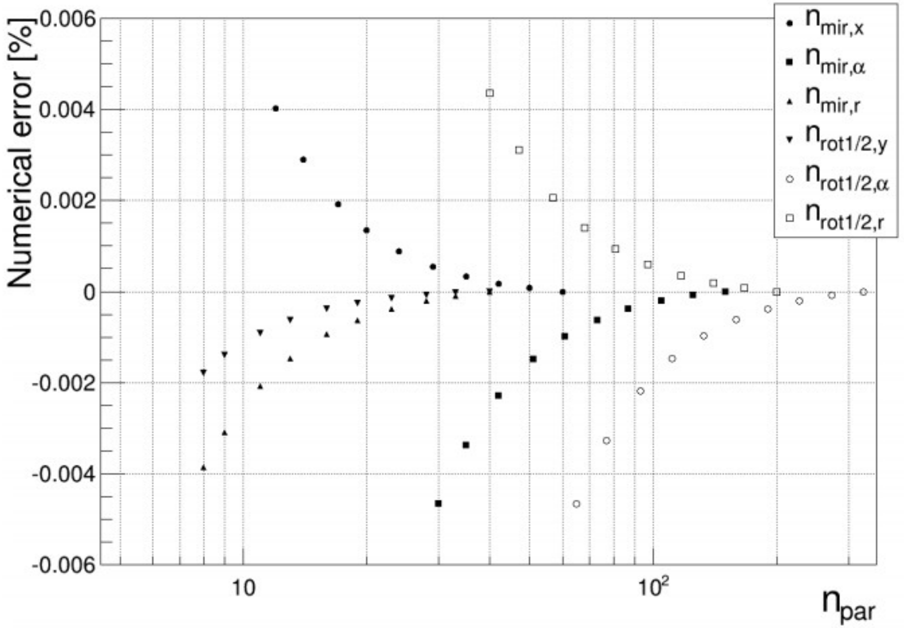
Initial grid

Mirror (10,10,10) ; Rotor (10,10,10)



Optimized grid

Mirror (12,30,8) ; Rotor (8,65,40)



Simulations apply to Virgo NCal03

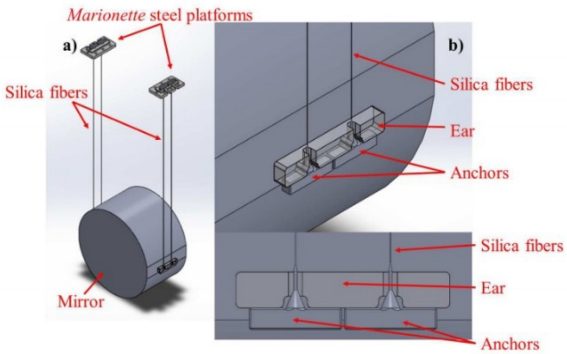
- Simple geometry case:
  - Mirror: full cylinder
  - Rotor: two 90° sectors

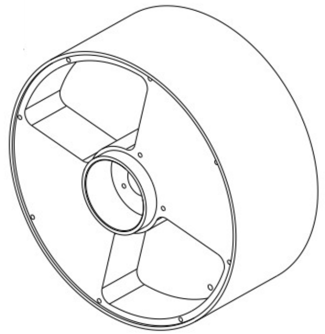
Numerical NCal-induced strain

Fourier Coeff.	Amplitude [h]	Phase [deg]
C_0	7.52397e-16	0
C_1	2.13463e-30 f_{rot}^{-2}	183
C_2	3.35754e-18 f_{2rot}^{-2}	1e-12
Numerical amplitude	3.35752e-18	–
Residuals	0.0007%	–

Main contribution to the total amplitude:
- C_2: signal at $2f_{rotor}$

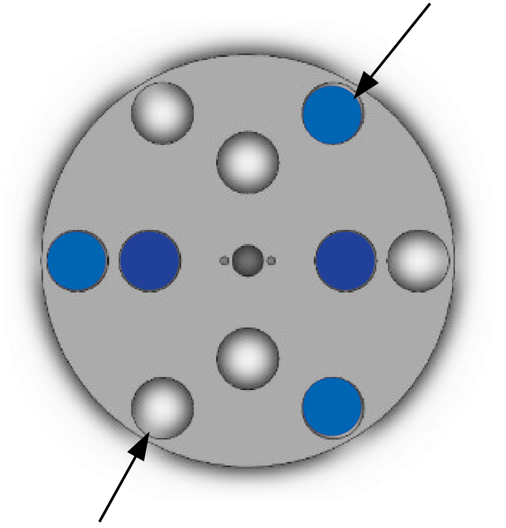
- More realistic geometry case:
  - Mirror: full cylinder + flats + ears and anchors
  - Rotor: four 90° sectors, fillets, screws and holes





Amplitude at $2f_{rotor}$: 3.35721e-18 $(f_{2rot})^{-2}$
Change of -0.01% compared to the simpler case

Simulations apply to LIGO prototype -v1



Using FROMAGE

Moments	Force $[N_{pk}]$	Strain
2f	1.96333e-11	3.13505e-18 f_{2rot}^{-2}
3f	9.3326e-12	1.49023e-18 f_{3rot}^{-2}
4f	4.9853e-14	7.96051e-21 f_{4rot}^{-2}
6f	6.80053e-15	1.08591e-21 f_{6rot}^{-2}

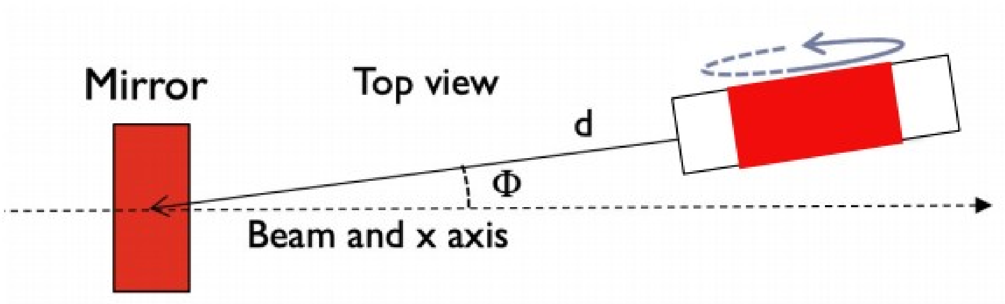
Relative difference with LIGO model:

- 2f → ~0.02%
- 3f → ~0.02%
- 4f → ~0.03%
- 6f → ~0.07%

Results from LIGO NCal model provided by M.P. Ross

→ Nice cross-check of FROMAGE

Analytical model



Extended rotor and **point** mirror model at 2f:
→ previous model

$$a(f_{2rot}) = \frac{9G\rho_{al}\ b\ \sin(\alpha)(r_{max}^4 - r_{min}^4)}{32\pi^2 f_{2rot}^2 d^4} \cos(\phi)\Big[1 + \frac{25}{54d^2}\frac{(r_{max}^6 - r_{min}^6)}{(r_{max}^4 - r_{min}^4)}\Big]$$

Extended rotor and **extended** mirror model at 2f:
→ our new analytical model

$$a(f_{2rot}) = \frac{9G\rho_{al}\ b\ \sin(\alpha)(r_{max}^4 - r_{min}^4)}{32\pi^2 f_{2rot}^2 d^4} \cos(\phi)\Big[1 + \frac{25}{54d^2}\frac{(r_{max}^6 - r_{min}^6)}{(r_{max}^4 - r_{min}^4)} + \Big(\frac{405}{72}\sin^2(\phi) - \frac{5}{2}\Big)\Big(\frac{r_{mir}}{d}\Big)^2 + \Big(\frac{405}{216}\cos^2(\phi) - \frac{225}{216}\Big)\Big(\frac{x_{mir}}{d}\Big)^2\Big]$$

Uncertainty on the numerical model

- Study done for two NCals:
  - NCal_N (near) at d_N = 1.267 m and Φ=34.71°
  - NCal_S (far) at d_S = 1.947 m and Φ=34.71°

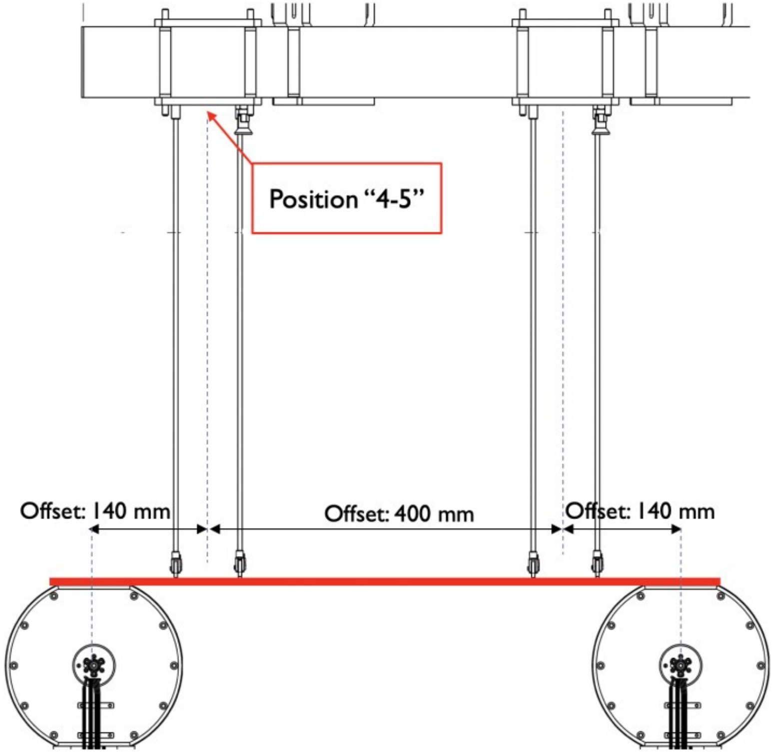
Parameter	Relative impact [%] (d_N, ϕ)	Relative impact [%] (d_S, ϕ)
Numerical grid	$\lesssim 0.01$	$\lesssim 0.01$
Torque (±0.5 mm)	0.05	0.03
Geometry approx. NCalO3	0.01	$\lesssim 0.02$
Analytical/Numerical	0.12	0.03
Total (quadratic sum)	~ 0.13	~ 0.05

We don't know how far our numerical model is from reality…
Comparison between LIGO and Virgo numerical models could help

Virgo NCalO3 layout

NCal_N at d_N = 1.267 m and Φ=34.71°
NCal_S at d_S = 1.947 m and Φ=34.71°

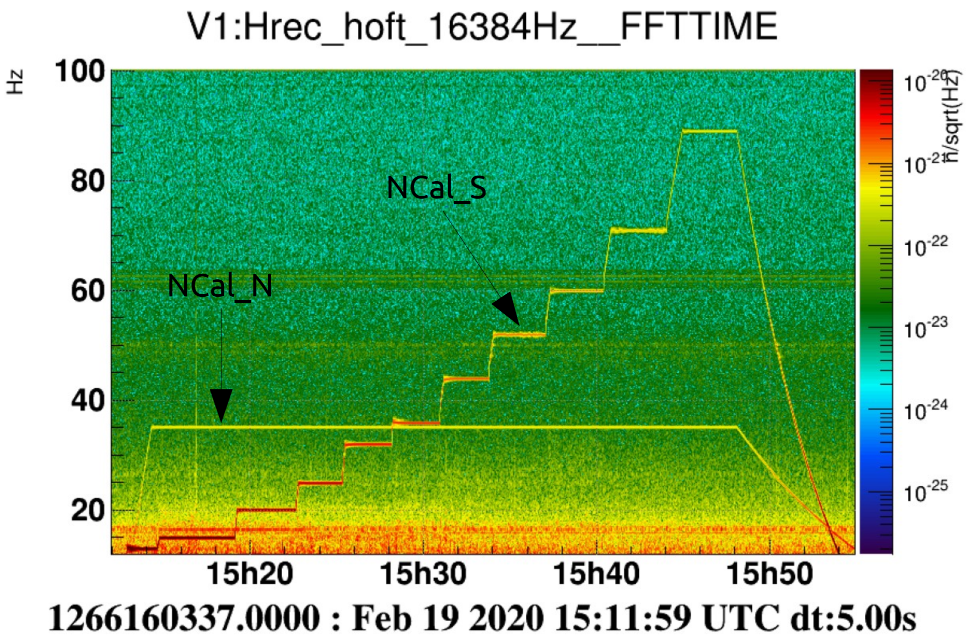
- Maximal rotor speed: 100 Hz (signal at 200 Hz)
- Known fixed distance between the NCals
  - Extract the distance to the mirror

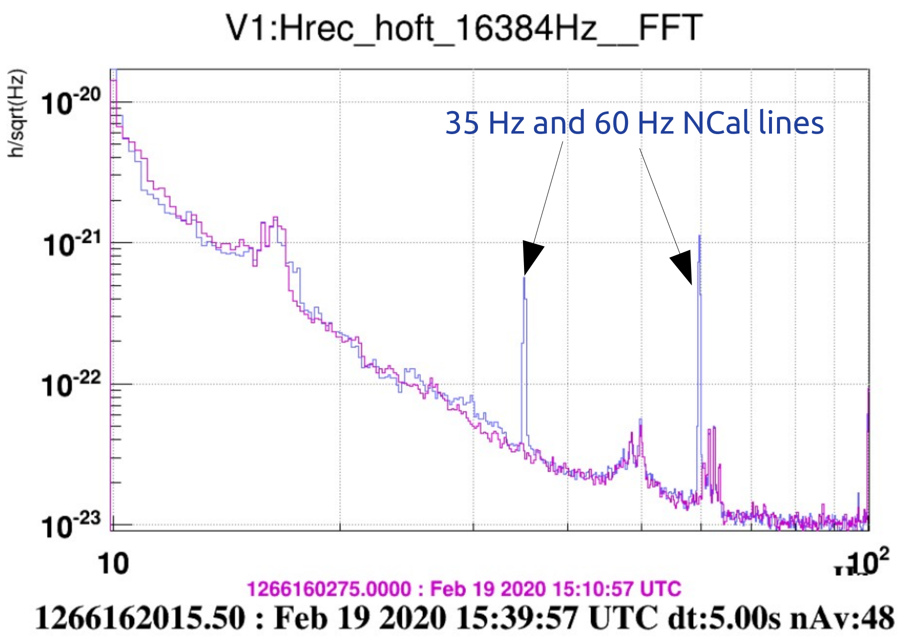


NCal_N NCal_S

O3 data taking

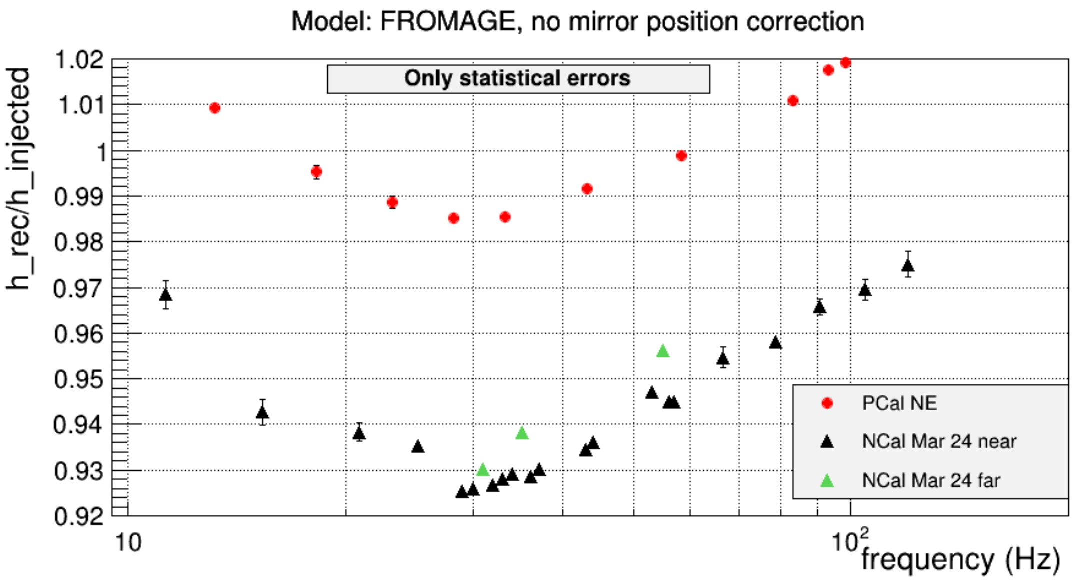
Example from February 2020: clean NCal signal





Check hrec with the NCalO3

- Frequency dependent shape agrees with PCal data
- 50 Hz h(t) reconstruction issue spotted
  - Points not shown in this plot
- Absolute value is ~5% off PCal data
- Difference between NCal_N and NCal_S data
  - expected distance to the mirror may be wrong

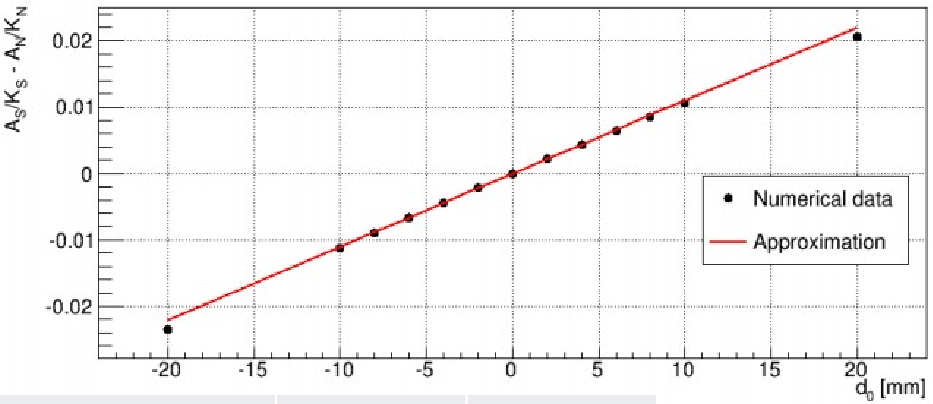


Extract the absolute distance

$$A_i = C_i(d_i + d_0)^{-4}$$

$$A_i \approx K_i(1 - 4\frac{d_0}{d_i})$$

$$\frac{A_S}{K_S} - \frac{A_N}{K_N} = 4d_0\frac{d_S - d_N}{d_S \cdot d_N} \approx 1.103\ d_0$$



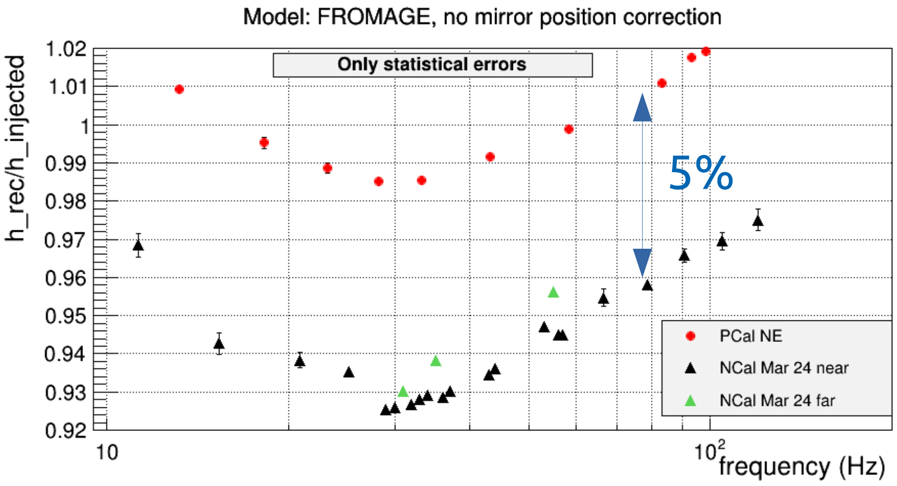
NCal FAR (S)		NCal near (N)					
freq. (Hz)	**hrec/hinj**	Freq low	hrec/hinj	Freq high	hrec/hinj	**<hrec/hinj>**	**d_0 (mm)**
31	**0.930**	30	0.9259	32	0.928	**0.9268**	**2.6**
35	**0.938**	34	0.9292	36	0.9285	**0.92885**	**8.5**
55	**0.9562**	54	0.9472	56	0.9449	**0.94605**	**9.2**

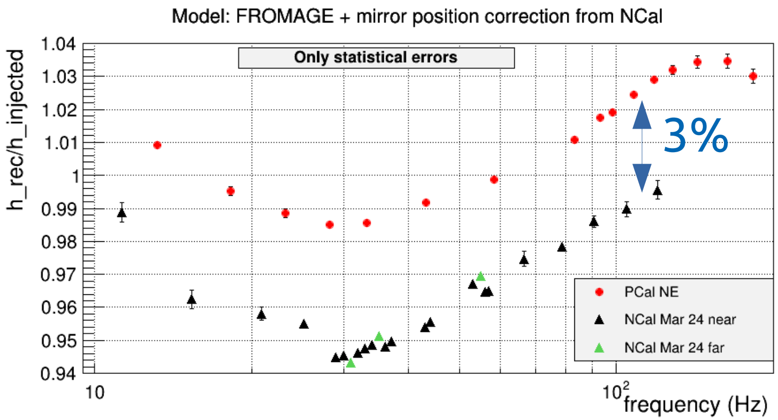
	d_0 (mm)	h_{rec}/h_{inj} near (%)	h_{rec}/h_{inj} far (%)
Mean value	6.8	2.1	1.4
Error due to point dispersion	4.2	1.3	0.9
Error due to rotor differences	4.6	1.6	0.9

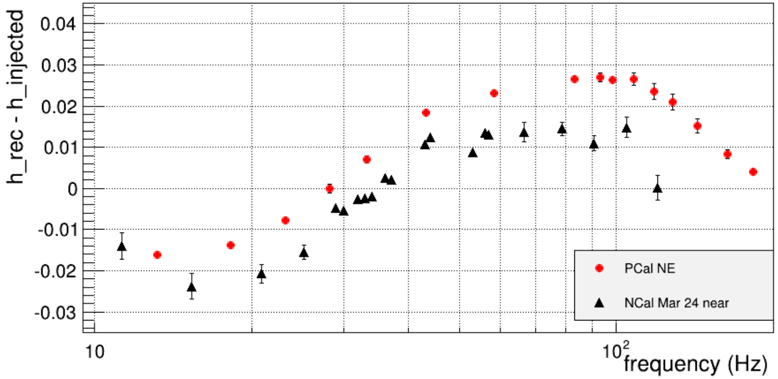
Assuming identical rotors

Assuming 0.5% difference in rotor geometry

Check hrec with measured distance







Phase:

- Good agreement for the shape
- NCal phase origin not accurate

NCalO3 uncertainty

Rotor geometry

Parameter			Relative impact	
name	value	uncertainty	formula	value (%)
density ρ (SI)	2805	5	$\delta\rho/\rho$	0.18
thickness b (mm)	74	0.2	$\delta b/b$	0.27
r_{max} (mm)	95	0.1	$4\delta r_{\mathrm{max}}/r_{\mathrm{max}}$	0.42
Total			quadratic sum	0.53

Total uncertainty budget

	NCal_N	NCal_S
Rotor	0.53	0.53
Mirror distance	1.33	0.87
Mir. Dist. Syst. Rot.	1.62	0.93
Angle	0.23	0.23
Vertical position	0.06	0.03
Model	0.13	0.05
Total	2.2	1.4

Note: Virgo PCal systematic uncertainties O3b are 1.39% NE PCal, 1.73% WE Pcal
Pcal is the reference method for O3 calibration

Plan for O4

- 1 close NCal for high frequency check (red)
  - Same distance as O3 Ncal_N (~1.27 m)
- 3 NCal for mirror position control (green)
  - Same mirror distance for reduced model uncertainties (~1.7 m)
- 1 Far NCal for permanent line (blue)
  - At 2.1 m or 2.5 from the mirror
  - Reduced systematic uncertainties
- New rotor geometry
  - Stay close to current geometry
  - Remove some parts and new design → gain in amplitude
- R&D made at IPHC (Strasbourg - France)