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Radiation pressure effects in the Advanced Virgo IMC yaw direction

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P_{in}	180 [W]	
IMC cavity length	L	144 [m]
c	$3 \cdot 10^8$ [m/s]	
g	$1 - L/R = 0.2$	$R = 180$ [m]
IMC end mirror θ_y	resonant frequency	1.315 [Hz]
injection bench θ_y	resonant frequency	0.01 [Hz]
IMC end mirror	mass	1.4 [kg]
suspended injection bench	mass	145 [kg]
IMC end mirror	radius	0.07 [m]
IMC Finesse		1100
Mirror density	$2.201 \cdot 10^3$ [kg/m^3]	

Table 1: Optical and mechanical parameters [1]. The mechanical resonant frequencies of the IMC end mirror suspension listed above are the measured ones.

1 Introduction

In order to evaluate the effects of the radiation pressure in the Advanced Virgo IMC the Sidles-Sigg computation has been applied [2].
Along the pitch direction the IMC can be considered as a simple Fabry-Perot cavity, as it has been analysed in the previous Virgo note [3], while on the yaw direction the behaviour is a little bit more complex since the Sidle-Sigg computation has to be extended to the triangular cavity geometry.

2 Radiation pressure in a triangular cavity - Advanced Virgo IMC

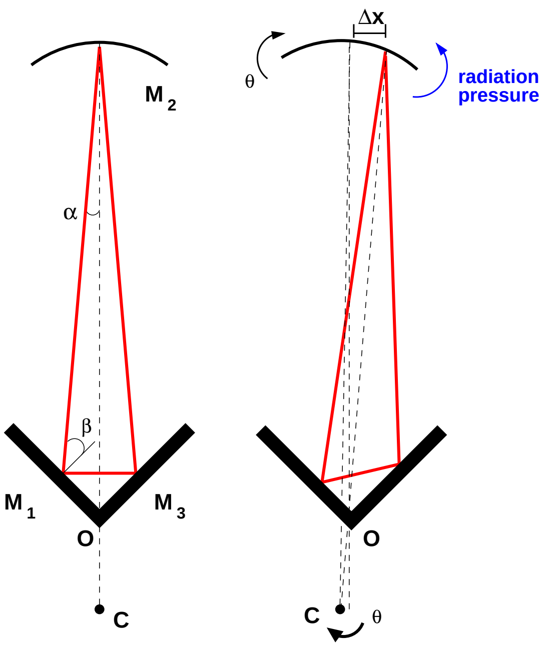

Figure 1: Effect on the translation of the beam in a triangular cavity due to the misalignment of the terminal mirror. The beam will meet the terminal mirror following the straight line which connects the center of the dihedron, O, and the center of curvature, C.

The amount of the radiation pressure effect depends on the beam position on the mirrors. The beam produces a torque proportional to the lever, which is the displacement of the beam with respect to the rotation axis on the mirror surface, and to the power stored in the cavity. The evaluation of the additional optical stiffness requires then the computation of the position of the beam on the IMC mirrors.
The IMC cavity is formed by three mirrors, M_1 and M_3 which form the dihedron, plus the curved terminal mirror M_2, but in order to have a more intuitive approach the misalignment of the mirrors can be written in the following base:

- $(\theta_1 + \theta_3)/\sqrt{2}$ which corresponds to Common mode for the M_1 and M_3 mirrors
- $(\theta_1 - \theta_3)/\sqrt{2}$ which corresponds to Differential mode for the M_1 and M_3 mirrors
- θ_2 which is the M_2 mirror misalignment

At this point some geometrical considerations will reduce the number of degrees of freedom of the system to only the terminal mirror mode.
Considering that the Advanced Virgo dihedron is a monolithic structure it is trivial that the differential misalignment mode can be neglected. Moreover since the M_1 and M_3 mirrors are fixed at a right angle, as a corner cube, a misalignment of the dihedron does not produce any effect on the displacement of the beam on the terminal mirror, thus any torque will be introduced.
This fact simplify also the computation of the resonant frequency since the system is reduced to a one d.o.f. system and the strong differences in the inertia of the mirrors can be neglected.
The effect of the radiation pressure on the yaw direction depends then only on the misalignment of the terminal mirror, the optical torque can be written as:

$$T = \frac{2P}{c}\cos(\alpha)\Delta x \tag{1}$$

where P is the stored power, c is the light speed, α is the IMC opening angle, and Δx is the displacement of the beam on the terminal mirror.
The translation of the beam on the M_2 mirror can be geometrically computed [4] knowing that the resonant mode passes trough the center of curvature of the terminal mirror, C, and the center of the dihedron, O, as it is shown in Figure 2, and is equal to:

$$\Delta x = -\left(L/g\right)\theta \tag{2}$$

Moreover since g is always positive and for a long IMC, such the Advanced Virgo IMC, α can be approximated to zero, it turns out that a triangular cavity is intrinsically stable, which means that the radiation pressure will restore the alignment for all the possible geometrical configurations of the IMC, which it is also clearly shown by the Figure 2.
The resonant frequency of the terminal mirror grows then as the square root of the stored power as:

$$\omega = \sqrt{\omega_0^2 + \frac{2P}{Ic}\frac{L}{g}} \tag{3}$$

3 Resonant frequency behaviour in the Advanced Virgo IMC

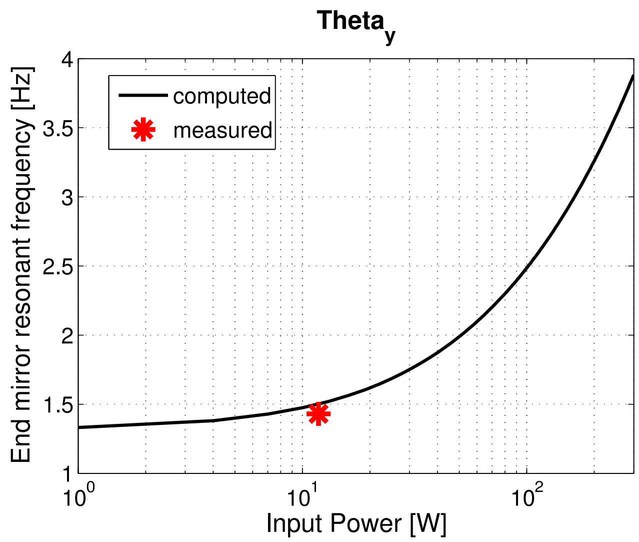

Figure 2: Resonant frequency of the IMC terminal mirror as a function of the input power. The behaviour obtained with the analytic computation result to be quite in agreement with the measured resonant frequency, red star.

In order to validate the results obtained in the previous section the computed resonant frequency has been compared with the measured resonant frequency of the IMC yaw mode. As it is shown in Figure 2 there is a good agreement between the computed and measured resonant frequencies.

4 Conclusion

In this note the effect of the radiation pressure on the Advanced Virgo IMC has been studied by extending the Sidles-Sigg computation to a triangular cavity configuration.
It has been shown that a triangular cavity is intrinsically stable, since the radiation pressure acts to restore the alignment, increasing the resonant frequency as the square root of the stored power. Moreover in the case of a monolithic dihedron, such as the Advanced Virgo case, the system can be reduced to one-single degree of freedom system, where only the terminal mirror misalignment produces radiation pressure effects and the dihedron misalignment can be completely neglected.

References

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