Characteristics of the rotor R4-03 for the O4 NCal system VIR-0664A-22

Eddy Dangelser, Dimitri Estevez, Hubert Kocher, Benoit Mours, Mehmet Ozturk, Antoine Syx

IPHC-Strasbourg

June 29, 2022

Contents

1 Introduction

This note follows the same discussion made on the rotor R4-01 in VIR-0591B-22. The drawings and figures of the rotor can be found in this technical note.

The rotor has been engraved IPHC-R4-03 on one side and sandblasted on the other side.

2 Measurement method

To determine the geometry of the rotor we will use the same method as for R4-01 (see VIR-0591B-22). The thickness was measured using 24*2 = 48 points (see fig. 1), the outer diameter was measured in 4*2 = 8 points and the inner diameter using 4 points.

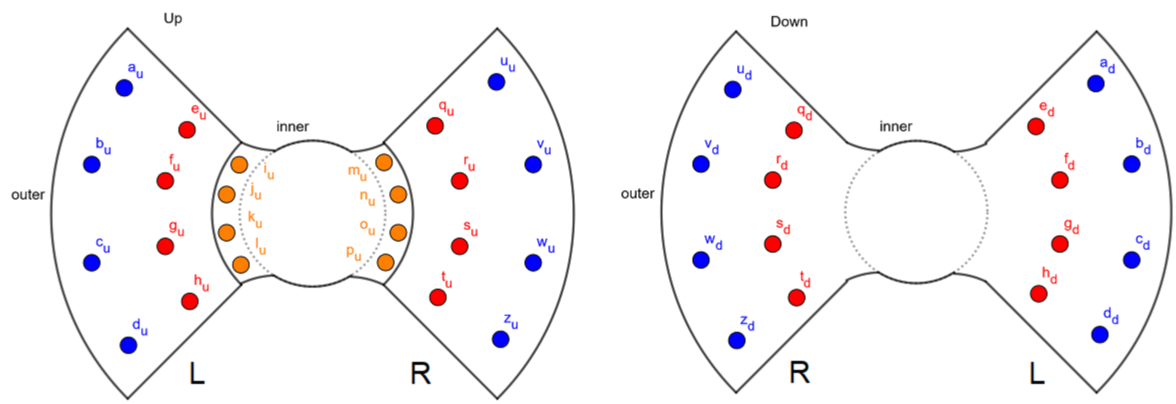

Figure 1: Outline of the faces of the rotor with the measurement points. Left figure is face up, right figure is face down. Sectors have been labelled L for left sector and R for right sector.

The tool used to measure the thickness and the outer diameter is a measuring column "Garant 44 5350_600 HC1" (see VIR-0160A-22) with a given precision of 1.8+L/600 μm (L the measured length in mm). A vernier caliper "TESA-CAL IP67" with a precision of 20 μm was used to measure the inner diameter.

The measuring column was operated on a different metrology table than the first rotor, different measurements were made to check the flatness on a surface roughly equivalent to the rotor layout. The value range from 0 to 2 μm. The rms of the 16 values is 0.9 μm.

2.1 Thermal effects and density

The rotor R4-03 has been machined from the same aluminum block as R4-01 described in VIR-0160A-22. As for R4-01 the results will be expressed at a reference temperature of 23°C. The density of the rotor R4-03 is then 2808.1 $\pm$ 0.2 kg.m^{-3}. This density is measured in air, if the rotor is used under vacuum, the density should be increased by the air density (ρ_{air}=1.3 kg.m^{-3}).

3 Raw measurements of the rotor

This section presents the raw measurements made on the rotor at the ambient temperature of 21.95°C. Table 1 shows the thickness measurements according to the measurement points defined in figure 1. The rotor is laying on the table. The rotor surface as well as the table are not perfectly flat. Some space could be present in between that should be substracted when computing the rotor thickness as discussed later.

Measurement point	L sector		Measurement point	R sector	
	Up	Down		Up	Down
a	104.275	104.276	q	104.306	104.308
b	104.286	104.287	r	104.314	104.312
c	104.306	104.300	s	104.323	104.322
d	104.319	104.322	t	104.336	104.336
e	104.289	104.290	u	104.301	104.305
f	104.289	104.292	v	104.316	104.331
g	104.300	104.300	w	104.331	104.334
h	104.320	104.315	z	104.347	104.348
i	101.577		m	101.576	
j	101.573		n	101.575	
k	101.574		o	101.575	
l	101.574		p	101.577	

Table 1: Raw measurements of the height in mm for each point at 21.95°C on L and R sectors of R4-03.

Table 2 displays the diameter measurements. The measurements were made on 4*2 diameters (two parts of each diameter, the up and down sides of the rotor).

Measurement point	Up	Down
1	208.022	208.027
2	208.044	208.050
3	208.045	208.046
4	207.997	208.030

Table 2: Raw measurements of the diameter in mm for each point at 21.95°C on R4-03.

Measurements were made on the inner radius r_{min} = 29.00 mm and the up face radius for the counterweight $r_{\text{counterweight}}$ = 39.99 mm. These values were computed using the vernier caliper and are the same for a temperature of 23°C.

4 Extracting the geometrical parameters

4.1 Thickness

We need to correct the possible gap between the rotor and the measuring table. Assuming that the table is flatter than the rotor surface we can extract the gap from the measurement of the top surface considering the plane tangents to the highest points (asking them to be on both sectors). For this rotor these points are h, t, z for the up face and d, t, z for the down face (see fig. 1). Using the measurements in table 1 we can compute a plane equation for each side of the rotor in cartesian coordinates:

$$\text{Up plane equation} : z = 1.69 \times 10^{-4}x - 3.62 \times 10^{-4}y + 104.32 \quad (1)$$

$$\text{Down plane equation} : z = 1.77 \times 10^{-4}x - 4.07 \times 10^{-4}y + 104.32 \quad (2)$$

Using eqs. (1) and (2) the gap can be determined, see table 3. The maximum rms for the gap of each sector is 15 μm.

Measurement point	L sector		Measurement point	R sector	
	Up	Down		Up	Down
a	12	6	q	7	3
b	10	6	r	8	8
c	2	7	s	7	7
d	3	0	t	0	0
e	8	4	u	10	3
f	14	9	v	9	-8
g	11	10	w	6	3
h	0	4	z	0	0

Table 3: Gap computed in μm on up and down sides of both sectors of R4-03.

We can then compute the rotor thickness for each point by removing these gaps. If one of the raw values is lower than the corrected thickness we take this lowest value. The value of each point is shown in table 4 at 23°C.

Measurement point	L sector	Measurement point	R sector
a	104.278	q	104.309
b	104.289	r	104.315
c	104.303	s	104.325
d	104.322	t	104.339
e	104.292	u	104.304
f	104.292	v	104.319
g	104.303	w	104.334
h	104.318	z	104.350
i	101.580	m	101.579
j	101.576	n	101.578
k	101.577	o	101.578
l	101.577	p	101.580

Table 4: Measurements of the thickness in mm for each point at 23°C on L and R sectors of R4-03.

4.2 Radius

Using comparators while the rotor is rotating on its axis we can determine the deformation on both sectors and compute different radii values. Table 5 shows the raw measurements using comparators on L and R sectors. The measurements were made on the up, center and down sides of L and R sectors using three comparators for a total of 5*3*2=30 points (the first and last points are near the edge of the sectors).

Measurement point	L sector			R sector		
	Up	Center	Down	Up	Center	Down
A	0	0	0	0	0	5
B	25	20	20	15	20	25
C	30	30	25	30	25	25
D	30	25	20	30	25	20
E	25	10	0	20	10	0

Table 5: Raw measurements in μm of the comparators for the L and R sectors of R4-03.

The zeroing of the comparators was made arbitrarily close to the edge of the sector. The offsets shown in table 5 are measured relative to this reference.

To compute the radius per measurement point we use the following process: First we compute the mean deformation for one comparator. Then we remove this mean deformation to each measurement of this comparator. The corrected shift value is added to the mean radius of 104.019 mm computed using table 2 at 23°C. This process is repeated for each comparator. The final radius for each point are shown in table 6.

Measurement point	L sector			R sector		
	Up	Center	Down	Up	Center	Down
A	103.996	104.002	104.002	103.996	104.002	104.007
B	104.021	104.022	104.022	104.011	104.004	104.027
C	104.026	104.032	104.027	104.026	104.027	104.027
D	104.026	104.027	104.022	104.026	104.027	104.022
E	104.021	104.012	104.002	104.016	104.012	104.002

Table 6: Radius measurements in mm at 21.95°C for the L and R sectors of R4-03.

5 Characterization of the rotor using a simple model

5.1 Theoretical model of the rotor

As explained in VIR-0591B-22 the theoretical strain of the mirror at 2f using FROMAGE v1r2 with the rotor design parameters:

$$\text{strain}(2\text{f}) = \frac{2.1198 \times 10^{-18}}{(2f_{rot})^2}$$

This strain value will be compared to models based on the measurements of the rotor.

5.2 Thickness

A simple model can be used to determine a mean value for the thickness and its uncertainty.

As shown on fig. 1, a total of 16 points were used to compute the thickness of each sector. In this case we will not consider the inner points so that we obtain uniform sectors.

For the simple model we take the thickness as the mean value of table 4: 104.312 mm at 23°C. Since we have a limited number of measurement points, to be conservative we take the thickness uncertainty as the rms of table 4 (19.5 μm) to which we add linearly the metrology table uncertainty (0.9 μm) and the tool uncertainty (2.0 μm). Therefore, for this simple model, the thickness is 104.312 $\pm$ 0.022 mm.

5.3 Radius

For the simple model we take the radius as the mean value of table 6: 104.019 mm at 23°C. Using a linear sum of the rms of table 6 (11.2 μm) and the tool uncertainty (2.2 μm) we take an uncertainty of 13.3 μm on the mean radius.

We have to point out that we do not take into account the fact that the sectors might not be centered on the same axis. Therefore the uncertainty might be underestimated. We will then consider each sector individually later.

5.4 Expected NCal signal and uncertainties

The geometry used to describe the rotor as a simple model is represented in figure 2.

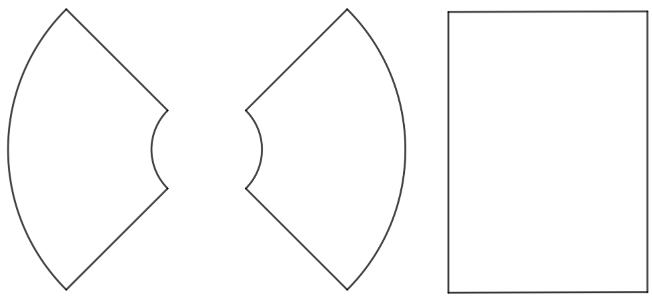

Figure 2: Simple model geometry used to describe the rotor. Left is a front view, right is a side view.

Using the analytical equation of the strain at 2f (see eq.8 in Newtonian calibrator tests during the Virgo O3 data taking) we compute with our parameters (d = 1.7 m and an angle ϕ = 34.7°):
strain(2f) $= \dfrac{2.1195 \times 10^{-18}}{(2f_{rot})^2}$.

Using FROMAGE on this geometry we compute the following 2f strain on the mirror at a distance of 1.7m and an angle of 34.7°:
strain(2f) $= \dfrac{2.1196 \times 10^{-18}}{(2f_{rot})^2}$

Comparing the theoretical model strain with the simple model at 2f using FROMAGE we obtain a relative deviation of 0.009%.

Comparing the analytical strain at 2f with FROMAGE we obtain a relative deviation of 0.005%.

The uncertainties considered for this model are displayed in table 7.

R4-03 rotor parameter simple model (23°C)			NCal 2f signal uncertainty	
name	value	uncertainty	formula	value (%)
Density ρ (kg.m^{-3})	2808.1	0.2	$\delta\rho/\rho$	0.007
Thickness b (mm)	104.312	2.2×10^{-2}	$\delta b/b$	0.021
r_{max} (mm)	104.019	1.3×10^{-2}	$4\delta r_{max}/r_{max}$	0.051
G (m^3.kg^{-1}.s^{-2})	$6.674\,30 \times 10^{-11}$	1.5×10^{-15}	$\delta G/G$	0.002
Temperature T (°C)	23	3	$\left\lvert\frac{\partial h}{\partial T}\right\rvert \frac{\Delta T}{h}$	0.014
Total uncertainty from the rotor (quadratic sum)				0.058

Table 7: Uncertainties on the amplitude of the calibration signal at 2f from the R4-03 rotor simple model geometry. The uncertainties from the opening angles are not included.

6 Characterization of the rotor using an advanced model

6.1 Thickness

A more advanced model can be used considering the deformations on the surfaces of the sectors for better accuracy. Each measurement point of table 4 can be considered as a sub-sector with its own thickness.

The uncertainty on this value is more complex to evaluate. As a conservative approach we use the maximum rms of the deviation to a plane for each sector (4.8 μm see section 4.1) to which we add linearly the uncertainty on the flatness of the measurement table (0.9 μm) as well as the measurement tool (2.0 μm). The total uncertainty on the thickness is 7.6 μm.

6.2 Radius

On fig. 1 we divided the external sectors in 4 sub-sectors for each sector (blue points). We convert the point of table 6 to the grid of fig. 1 by averaging the two closest values and converting them to 23°C. The results are shown in table 8. We notice that the L sector is on average 2 μm larger than the R sector.

Radius	L sector			R sector		
	Up	Center	Down	Up	Center	Down
1	104.011	104.015	104.015	104.006	104.006	104.020
2	104.026	104.030	104.028	104.021	104.018	104.030
3	104.029	104.032	104.028	104.029	104.030	104.028
4	104.026	104.022	104.015	104.024	104.022	104.015

Table 8: Radius measurements (in mm at 23°C) for the L and R sectors of R4-03.

The maximum rms of the radii for each sector is 8.4 μm. The tool uncertainty is 2.2 μm. Like for the thickness we use a linear sum and find the uncertainty on both radii to be 11 μm.

6.3 Counterweight

A counterweight has been designed to reduce the unbalance of the rotor. This counterweight is made of aluminum 2017 ($\rho_{\mathrm{Al}_{2017}} = 2790$ kg.m^{-3}) and the geometry is shown in fig. 3.

The dimensions of the counterweight are: an inner radius of 10 mm, an outer radius of 40 mm and a material cut with a chord of 50.44 mm to balance the rotor (see hatched are on fig. 3).

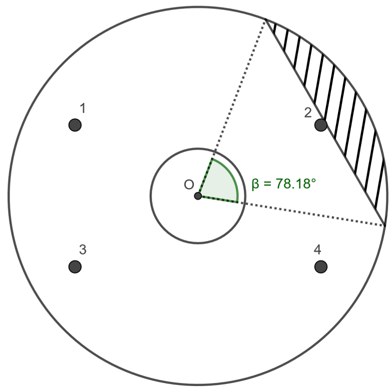

Figure 3: Outline of the counterweight for R4-03. The hatched area represents the material removed.

6.4 Expected NCal signals and uncertainties

6.4.1 Advanced geometry including chamfers and counterweight

The geometry used to describe the rotor as an advanced model is represented in figure 4. The external parts of the sectors are divided in 3 sub-sectors each to correspond to the different radii determined. In addition we include the counterweight, the screws and screw holes.

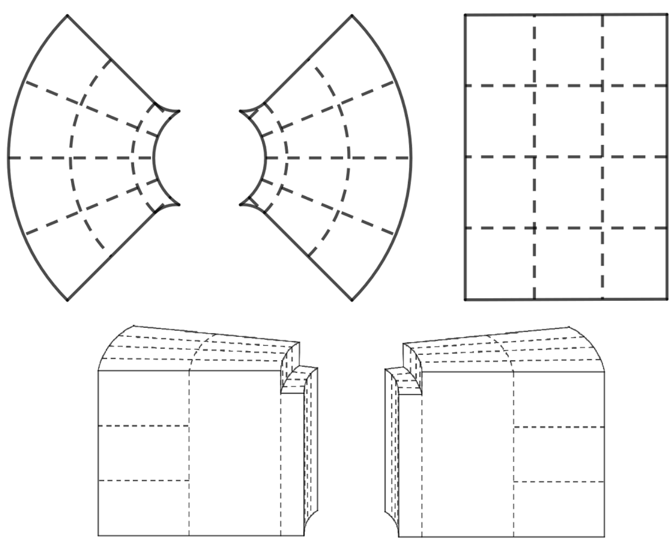

Figure 4: Advanced model geometry used to describe the rotor. Top left is a front view, top right is a side view (external sub-sectors) and bottom is a tilted view of the sectors. Only the 4 external part sectors are divided in 3 sub-sectors each. The chamfers are visible on the inner radius.

Using FROMAGE on this geometry gives the following strains:

- strain(1f) $= \dfrac{1.2747 \times 10^{-20}}{(1 f_{rot})^2}$
- strain(2f) $= \dfrac{2.1188 \times 10^{-18}}{(2 f_{rot})^2}$

- strain(3f) $= \dfrac{4.7818 \times 10^{-23}}{(3f_{rot})^2}$

The relative deviation to the simple model at 2f is 0.038%.

6.4.2 Opening angles and sectors asymmetry

The counterweight is supposed to compensate the unbalance of the rotor at 1f. As the 1f signal from section 6.4.1 is a bit more than a factor 6 smaller than R4-01 we will consider an error of 0.047%/6=0.008% on the 2f signal for R4-03 due to the opening angle and asymmetry.

6.4.3 Uncertainties

Comparing the advanced model of the rotor + chamfers + screw holes (strain(2f) = $2.1188 \times 10^{-18}/(2f_{rot})^2$) to the simple model gives a relative deviation of 0.038%. This deviation will be considered as our modelling uncertainty.

The uncertainties considered for this full model are displayed in table 9.

R4-03 rotor parameter advanced model (23°C)			NCal 2f signal uncertainty	
name	mean value	uncertainty	formula	value (%)
Density ρ (kg.m^{-3})	2808.1	0.2	$\delta\rho/\rho$	0.007
Thickness b left sector (12 sub-sectors) (mm) Thickness b right sector (12 sub-sectors) (mm)	104.300 104.324	1.8×10^{-2}	$\delta b/b$	0.007
r_{max} left sector (12 ext sub-sectors) (mm) r_{max} right sector (12 ext sub-sectors) (mm)	104.023 104.021	1.1×10^{-2}	$4\delta r_{max}/r_{max}$	0.040
G (m^3.kg^{-1}.s^{-2})	$6.674\,30 \times 10^{-11}$	1.5×10^{-15}	$\delta G/G$	0.002
Temperature T (°C)	23	3	$\left\lvert\dfrac{\partial h}{\partial T}\right\rvert \dfrac{\Delta T}{h}$	0.014
Modelling Uncertainty				0.038
Opening angle and asymmetry uncertainty				0.008
Total uncertainty from the rotor (quadratic sum)				0.059

Table 9: Uncertainties on the amplitude of the calibration signal at 2f from the R4-03 rotor advanced model geometry at 23°C.

A Appendix

```
### This is a cfg file for a more realistic geometry of the mirror and the Virgo NCal for O4 (2022)

### ALL THE OBJECTS ARE DEFINED IN THE MIRROR'S FRAME (O,x,y,z),
### with O the center of the mirror, x axis along the ITF's beam toward the beam-splitter,
### y axis orthogonal to x in the plane of the ITF,
### z axis orthogonal to the plane of the ITF upward

### MIRROR DEFINITION

GRID_SIZE 12 30 8

CYLINDER 2202. 0 0.175 0.2 360 0 0 0

GRID_SIZE 1 1 1

# Defining the flats on the edge of the mirror
CUT_CYL 2202. 0.175 0.2 0.05 0 0
CUT_CYL 2202. 0.175 0.2 0.05 0 180

# Defining the ears and anchors of the mirror
CUBOID 2202. 0.090 0.010 0.015  0     0.1782 -0.0125
CUBOID 2202. 0.090 0.010 0.015  0    -0.1782 -0.0125
CUBOID 2202. 0.039 0.008 0.008 -0.02 -0.1772 -0.024
CUBOID 2202. 0.039 0.008 0.008 -0.02  0.1772 -0.024
CUBOID 2202. 0.039 0.008 0.008  0.02 -0.1772 -0.024
CUBOID 2202. 0.039 0.008 0.008  0.02  0.1772 -0.024

### ROTOR DEFINITION: CYLINDER DENSITY INNER_RADIUS OUTER_RADIUS THICKNESS OPEN_ANGLE r z theta

ROTOR_CYLINDRICAL 1.7 34.7 0 0

### COUNTERWEIGHT 2790.
GRID_SIZE 16 65 40
CYLINDER 2790. 0.010 0.040 0.003 360 0 0.049569 0
GRID_SIZE 1 1 1
CUT_CYL 2790. 0.040 0.002 0.05044 0 330

### SCREW HOLES
GRID_SIZE 4 4 4
CYLINDER -2808.1 0 0.0015 0.012 360 0.03 0.04478993833032 150
CYLINDER -2808.1 0 0.0015 0.012 360 0.03 0.04478843828784 30
CYLINDER -2808.1 0 0.0015 0.012 360 0.03 0.04478943831616 210
CYLINDER -2808.1 0 0.0015 0.012 360 0.03 0.04478993833032 330

### SCREWS COUNTERWEIGHT
CYLINDER 4810. 0 0.0015 0.003 360 0.03 0.04956861126452 150
CYLINDER 7600. 0 0.0015 0.00665 360 0.03 0.04746493833032 150

CYLINDER 4810. 0 0.0015 0.003 360 0.03 0.04956861126452 30
CYLINDER 7600. 0 0.0015 0.00665 360 0.03 0.04746493833032 30

CYLINDER 4810. 0 0.0015 0.003 360 0.03 0.04956861126452 210
CYLINDER 7600. 0 0.0015 0.00665 360 0.03 0.04746493833032 210
```

```
CYLINDER 4810. 0 0.0015 0.003 360 0.03 0.04956861126452 330
CYLINDER 7600. 0 0.0015 0.00665 360 0.03 0.04746493833032 330

# TRES RAPIDE
#GRID_SIZE 4 4 4
# RAPIDE
#GRID_SIZE 8 17 14
# LENT
GRID_SIZE 8 65 40

### L sector

## Inner part
OUTER_FILLET 2808.1 0.029 0.101580 -0.002721 0.01 -11.25 146.25
CYLINDER 2808.1 0.029 0.03999 0.101580 22.5 0 -0.002721 146.25
CYLINDER 2808.1 0.029 0.03999 0.101576 22.5 0 -0.002725 168.75
CYLINDER 2808.1 0.029 0.03999 0.101577 22.5 0 -0.002721 191.25
CYLINDER 2808.1 0.029 0.03999 0.101577 22.5 0 -0.002721 213.75
OUTER_FILLET 2808.1 0.029 0.101577 -0.002721 0.01 11.25 213.75

## Middle part
CYLINDER 2808.1 0.03999 0.07199 0.104292 22.5 0 0 146.25
CYLINDER 2808.1 0.03999 0.07199 0.104292 22.5 0 0 168.75
CYLINDER 2808.1 0.03999 0.07199 0.104303 22.5 0 0 191.25
CYLINDER 2808.1 0.03999 0.07199 0.104318 22.5 0 0 213.75

## Outer part
CYLINDER 2808.1 0.07199 0.104011 0.0347593176893333 22.5 0 0.0347593176893333 146.25
CYLINDER 2808.1 0.07199 0.104015 0.0347593176893333 22.5 0 0 146.25
CYLINDER 2808.1 0.07199 0.104015 0.0347593176893333 22.5 0 -0.0347593176893333 146.25

CYLINDER 2808.1 0.07199 0.104026 0.03476298445984 22.5 0 0.03476298445984 168.75
CYLINDER 2808.1 0.07199 0.104030 0.03476298445984 22.5 0 0 168.75
CYLINDER 2808.1 0.07199 0.104028 0.03476298445984 22.5 0 -0.03476298445984 168.75

CYLINDER 2808.1 0.07199 0.104029 0.0347676512586667 22.5 0 0.0347676512586667 191.25
CYLINDER 2808.1 0.07199 0.104032 0.0347676512586667 22.5 0 0 191.25
CYLINDER 2808.1 0.07199 0.104028 0.0347676512586667 22.5 0 -0.0347676512586667 191.25

CYLINDER 2808.1 0.07199 0.104026 0.03477398477136 22.5 0 0.03477398477136 213.75
CYLINDER 2808.1 0.07199 0.104022 0.03477398477136 22.5 0 0 213.75
CYLINDER 2808.1 0.07199 0.104015 0.03477398477136 22.5 0 -0.03477398477136 213.75

### R sector

## Inner part
OUTER_FILLET 2808.1 0.029 0.101579 -0.002743 0.01 11.25 33.75
CYLINDER 2808.1 0.029 0.03999 0.101579 22.5 0 -0.002743 33.75
CYLINDER 2808.1 0.029 0.03999 0.101578 22.5 0 -0.002744 11.25
CYLINDER 2808.1 0.029 0.03999 0.101578 22.5 0 -0.002744 348.75
CYLINDER 2808.1 0.029 0.03999 0.101580 22.5 0 -0.002742 326.25
OUTER_FILLET 2808.1 0.029 0.101580 -0.002742 0.01 -11.25 326.25

## Middle part
CYLINDER 2808.1 0.03999 0.07199 0.104309 22.5 0 0 33.75
CYLINDER 2808.1 0.03999 0.07199 0.104315 22.5 0 0 11.25
```

```
CYLINDER 2808.1 0.03999 0.07199 0.104325 22.5 0 0 348.75
CYLINDER 2808.1 0.03999 0.07199 0.104339 22.5 0 0 326.25

## Outer part
CYLINDER 2808.1 0.07199 0.104006 0.03476798460144 22.5 0 0.03476798460144 33.75
CYLINDER 2808.1 0.07199 0.104006 0.03476798460144 22.5 0 0 33.75
CYLINDER 2808.1 0.07199 0.104020 0.03476798460144 22.5 0 -0.03476798460144 33.75

CYLINDER 2808.1 0.07199 0.104021 0.03477298474304 22.5 0 0.03477298474304 11.25
CYLINDER 2808.1 0.07199 0.104018 0.03477298474304 22.5 0 0 11.25
CYLINDER 2808.1 0.07199 0.104030 0.03477298474304 22.5 0 -0.03477298474304 11.25

CYLINDER 2808.1 0.07199 0.104029 0.03477798488464 22.5 0 0.03477798488464 348.75
CYLINDER 2808.1 0.07199 0.104030 0.03477798488464 22.5 0 0 348.75
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CYLINDER 2808.1 0.07199 0.104024 0.0347832294330357 22.5 0 0.0347832294330357 326.25
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CYLINDER 2808.1 0.07199 0.104015 0.0347832294330357 22.5 0 -0.0347832294330357 326.25

### GENERAL PARAMETERS

STEP 22.5 16

ARM_LENGTH 3000

SIGNAL 3
```