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Advanced Virgo design: Optical phase noise originating from the etalon effect driven by thermo-refractive noise

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1 Scope of this document

In the current design process of the Advanced Virgo arm cavities the option of using an etalon as cavity end mirror is under discussion [1], see Figure 1. However, temperature fluctuations inside such an etalon cause optical phase changes, i.e. differential arm length noise. In this document we calculate the noise originating from the etalon effect driven by Brownian thermal noise of the etalon substrate and compare it to the design sensitivity of Advanced Virgo.

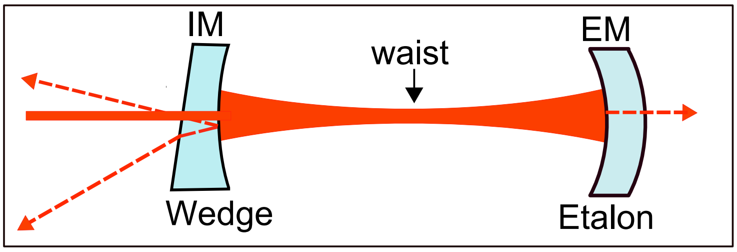

Figure 1: Advanced Virgo arm cavity design as proposed in [1], [2]. Input mirror (IM) and end mirror (EM) are identically curved in order to shift the beam waist to the centre of the cavity. The input mirror features a wedge, while the etalon effect of the end mirror is used to balance the finesse of the two arm cavities. In order to maximize the etalon effect both surfaces of the end mirror have to be curved identically.

2 Optical phase noise of an Advanced Virgo etalon

The overall reflectance of the etalon differs from that of an ordinary mirror not only in amplitude but also in phase. This can be interpreted such that the apparent position of the end mirror is influenced by the etalon tuning. This of course couples etalon fluctuations into optical phase noise, i.e. apparent displacement noise of the test masses at the end of the arm cavities. The apparent position change of the mirror, Δx_E can be computed as:

$$\Delta x_E = \frac{\lambda}{4\pi} \arctan\left(\frac{\Im(\rho_E)}{\Re(\rho_E)}\right), \tag{1}$$

where ρ_E is the overall amplitude reflectance of the etalon and λ describes the wavelength of the light.

Using an actual set of possible Advanced Virgo parameters, as described in [2] we can derive quantitative numbers for the optical phase noise. Figure 2 shows the apparent position change as a function of the etalon tuning for a few examples. The slope of the optical phase change is depending on the actual operating point

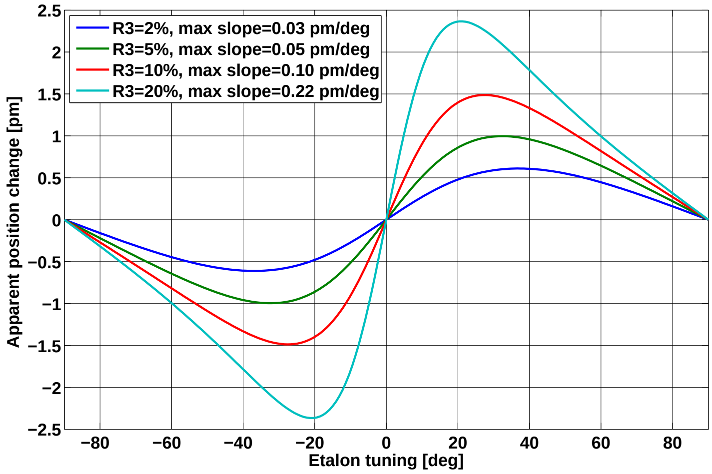

Figure 2: The plot shows the apparent position change of the end mirror as a function of the etalon tuning. The maximum slope around the zero tuning point is reported in the label.

of the etalon as well as on the refelctivities of the etalon coatings. Considering an etalon back surface coating of a power reflectance of $R_3 = 0.2$ and an etalon operating point with the strongest slope, we get a slope of the optical phase noise of $n_p = 0.22\,\mathrm{pm/deg}$. We can also express n_p as an unitless conversion factor describing the ratio of optical path length change inside the etalon and resulting length change of the arm cavity. 1 deg of tuning is equivalent to $\lambda/360$, where $\lambda = 1064\,\mathrm{nm}$. Thus we can derive:

$$n_p = \frac{0.22\,\mathrm{pm}}{2.96\,\mathrm{nm}} = 7.4 \times 10^{-5} \qquad (2)$$

This means that an optical path length change of x will only cause a change of $7.4 \times 10^{-5}x$ in the apparent position of the etalon.

Starting from this worst case scenario ($n_p = 7.4 \times 10^{-5}$), we will calculate in the next Section the required temperature stability of the etalon substrate for achieving the advanced Virgo design sensitivity.

3 Temperature requirement for an Advanced Virgo etalon substrate

Temperature fluctuations of the etalon substrate can change the optical path length of the etalon, i.e. the etalon tuning by two different mechanisms:

- Thermo refractive: change of the index of refraction, $\frac{dn}{dT}$.
- Thermal expansion: change of geometrical length, $\frac{dL}{dT}$.

The effect of thermal expansion of fused silica is with $\frac{dL}{dT} = 5.1 \cdot 10^{-7}\,\mathrm{K}$ [5] more than one order of magnitude smaller than the thermo-refractive effect $\frac{dn}{dT} = 10.9 \cdot 10^{-6}/\mathrm{K}$. Therefore we will only consider the contribution of thermo-refractive noise in this document.

As soon as the effective[1] temperature fluctuations of the etalon exceed a certain level they will limit the sensitivity of Advanced Virgo, $\tilde{h}_{\mathrm{adv}}$ (linear spectral desnsity). We can derive a requirement for the effective temperature

[1] We are not interested in the average temperature of the full etalon, but only the part, i.e. cylinder, that is actually sensed by the laser beam.

stability of an Advanced Virgo etalon, $\tilde{T}_{\rm req}(f)$ (linear spectral density) using the following equation:

$$\tilde{h}_{\rm adv}(f) = \tilde{T}_{\rm req}(f) \cdot \frac{dn}{dT} \cdot l_{\rm eta} \cdot n_p \cdot \frac{1}{L}, \tag{3}$$

where $\frac{dn}{dT}$ describes the temperature dependent change of index of refraction of fused silica, $l_{\rm eta}$ is the geometrical length (thickness) of the etalon and L is the length of a Advanced Virgo arm cavity. Using the actual Advanced Virgo parameters $\frac{dn}{dT} = 10.9 \cdot 10^{-6}$/K, $l_{\rm eta} = 0.2\,$m, $L = 3 \cdot 10^3\,$m and the Advanced Virgo design sensitivity, $\tilde{h}_{\rm adv}(f)$ as shown in [3] we can calculate the $\tilde{T}_{\rm req}(f)$. The resulting spectrum is shown in Figure 3 (blue trace). Over a wide range of the detection band the effective temperature fluctuations inside the etalon substrate have to be below $10^{-10}\,$K/$\sqrt{\rm Hz}$.

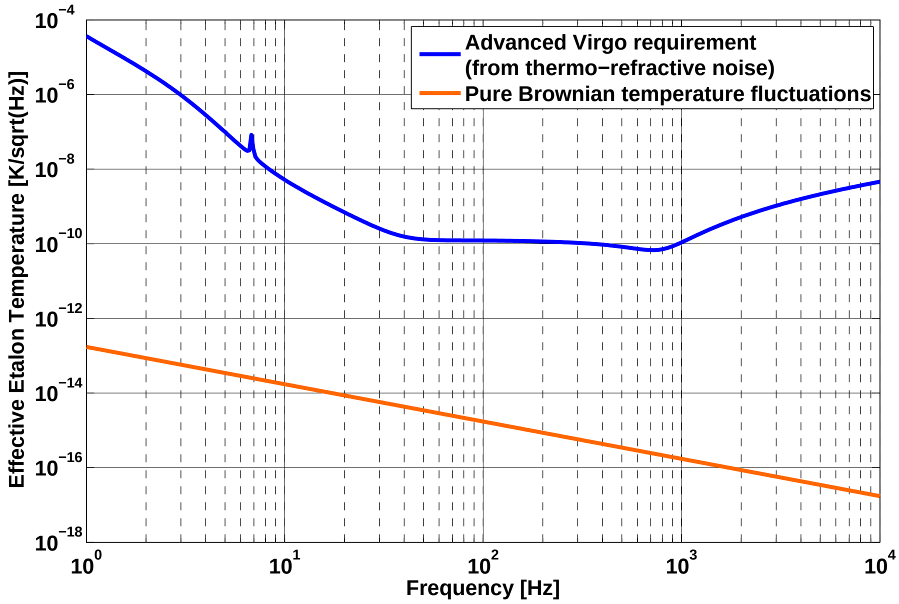

Figure 3: Requirement for the effective etalon temperature in Advanced Virgo, derived from etalon thermo-refracting noise coupling to differential arm length noise (blue trace). The pure Brownian temperature fluctuations are at least four orders of magnitude below the requirement for the Advanced Virgo etalons.

4 Actual temperature fluctuations of an Advanced Virgo test mass

The temperature fluctuations of the etalon for frequencies within the detection band are likely to be driven by pure Brownian noise. Using Equation 5.2 from [4] we can describe the effective temperature fluctuations (amplitude spectral density)), $\tilde{T}_{\rm mirror}(f)$ of the etalon to be:

$$\tilde{T}_{\rm mirror}(f) = \sqrt{\frac{4k_b T^2 \kappa}{(\rho C)^2 l_{\rm eta}} \frac{1}{\pi R_b^4 (2\pi f)^2}}, \tag{4}$$

where k_b is the Boltzmann constant, T the temperature of the Etalon, κ the thermal conductivity, ρ the density of the etalon, C the heat capacity and R_b the radius of the laser beam inside the etalon substrate[2]. If we put in the material properties of fused silica, $\kappa = 1.38\,$W/m K, $\rho = 2200\,$kg/m^3, $C = 740\,$J/kg K, a temperature

[2]Please note, that the authors of [4] define the beam radius in the rather unconventional way: "intensity of beam decreases as e^{-1} at distance R_b from center". That means that we have to correct our 'normal' beam radius ($1/e^2$) by dividing by a factor $\sqrt{2}$.

of $T = 290\,\mathrm{K}$ and from [6] a beam radius of $R_b = 0.06\,\mathrm{m}/\sqrt{2}$ we can finally calculate effective temperature fluctuations in the etalon. Figure 3 shows the spectrum of the Brownian driven effective temperature of the etalon substrate. The pure Brownian temperature fluctuations turn out to be at least four orders of magnitude below the requirement for all frequencies of interest.

5 Summary and Discussion

We derived a requirement for the effective temperature stability of the end mirror etalons of advanced Virgo, by considering the coupling of thermo-refractive noise of the etalon into differential arm length noise. In order to avoid spoiling the Advanced Virgo sensitivity we have to keep the temperature fluctuations of the etalon below $10^{-10}\,\mathrm{K}/\sqrt{\mathrm{Hz}}$ for most frequencies within the detection band. We compared this requirement to the actually present temperature fluctuations, assuming, that these are for frequencies in the detection band entirely driven by Brownian fluctuations. It turns out that the Brownian temperature fluctuations are at least four orders of magnitude below the temperature requirement of the etalon.

This analysis confirms that the optical phase noise of the potential end mirror etalons will not be limiting the advanced Virgo sensitivity. However, keep in mind that neither laser induced temperature fluctuations nor temperature fluctuations driven by the Advanced Virgo environment are taken into account for the analysis presented here.

References

[1] S. Hild: "Update on the Advanced Virgo Arm Cavity Design", Presentation at the Advanced Virgo R&D-review meeting, April 2008, http://wwwcascina.virgo.infn.it/collmeetings/presentations/2008/2008-04/Advanced/hild_210408.ppt 1

[2] S. Hild et. al., "Using the etalon effect for in-situ balancing of the Advanced Virgo arm cavities", in preparation. 1

[3] The Advanced Virgo Team, "Advanced Virgo Conceptual Design", Virgo note VIR-042A-07 3

[4] Braginski V. B and Vyatchanin S. P "Corner reflectors and quantum-non-demolition measurements in gravitational wave antennae" *Physics Letters A* **324** (2004) 345-360 3

[5] *"Quarzglas fuer die Optik, Daten und Eigenschaften"*, Heraeus Quarzglas GmbH & Co.KG, Hanau (1999). 2

[6] S. Hild and A. Freise, "Advanced Virgo Design: Preliminary Revision of the Beam Size and the Test Mass Curvatures", Virgo note VIR-038B-08 4