Effect of a rotor misalignment (twist) on the O4 NCal signal VIR-0530A-23

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Contents

1 Introduction

While studying the uncertainties of the NCal signal we investigate the effect of a rotor twist ψ on the amplitude of the signal.

Figure 1 is a top view of a twisted rotor (in red) compared to a non twisted rotor (in black) in the reference frame of the tower. The rotor is at angle ϕ in respect to the beam axis in the plane of the interferometer. The NCal system was initially designed for the rotors to have no twist as this effect was not studied before.

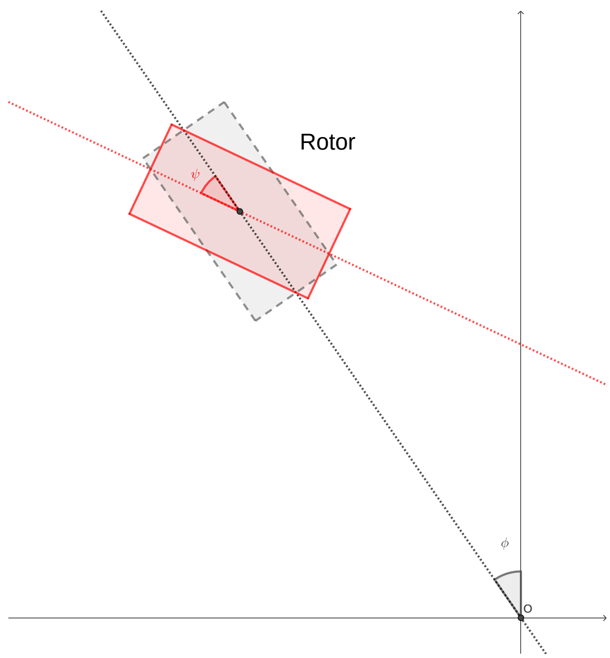


Figure 1: Top view of a rotor in the reference frame of the tower. In black is shown a rotor with no twist and in red is the same rotor with a positive twist ψ. The rotor is at an angle ϕ from the beam axis.

Considering a twist of the rotor in the point mass approximation on the analytical form of the NCal strain and keeping only the second order terms at twice the rotor frequency, we have:

$$h(\phi,\psi) = \frac{G\rho_{rot}\ b\ sin(\alpha)(r_{max}^4 - r_{min}^4)}{32L\pi^2 f_{2rot}^2}\left(9\cos^2\psi\cos\phi + 6\cos\psi\sin\psi\sin\phi\right) \tag{1}$$

As shown in eq. (1) there is a coupling of the twist ψ and the angle ϕ of the rotor. If $\phi = 0°$ then the amplitude is maximum when the rotor is pointing toward the mirror with $\psi = 0°$. Since the O4 NCal system has been designed to operate at an angle $\phi = 34.7°$, we study the coupling effects between ψ and ϕ.

One can notice that $h(\phi,\psi)$ is maximum at the first order for $\psi = \phi/3$ which is expected to be arround $11.6°$ for $\phi = 34.7°$.

2 Predicting the optimal rotor twist ψ using FROMAGE

Using FROMAGE v1r3 we can accurately compute the strain for a given rotor geometry at different angles ϕ and twists ψ. Figure 2 shows the NCal strain variations for a rotor at 1.7 m from the mirror normalized to the strain in $\psi = 0°$ and $\phi = 0°$. We notice that it exists an optimal $\psi \neq 0°$ which maximizes the NCal strain when $\phi \neq 0°$.

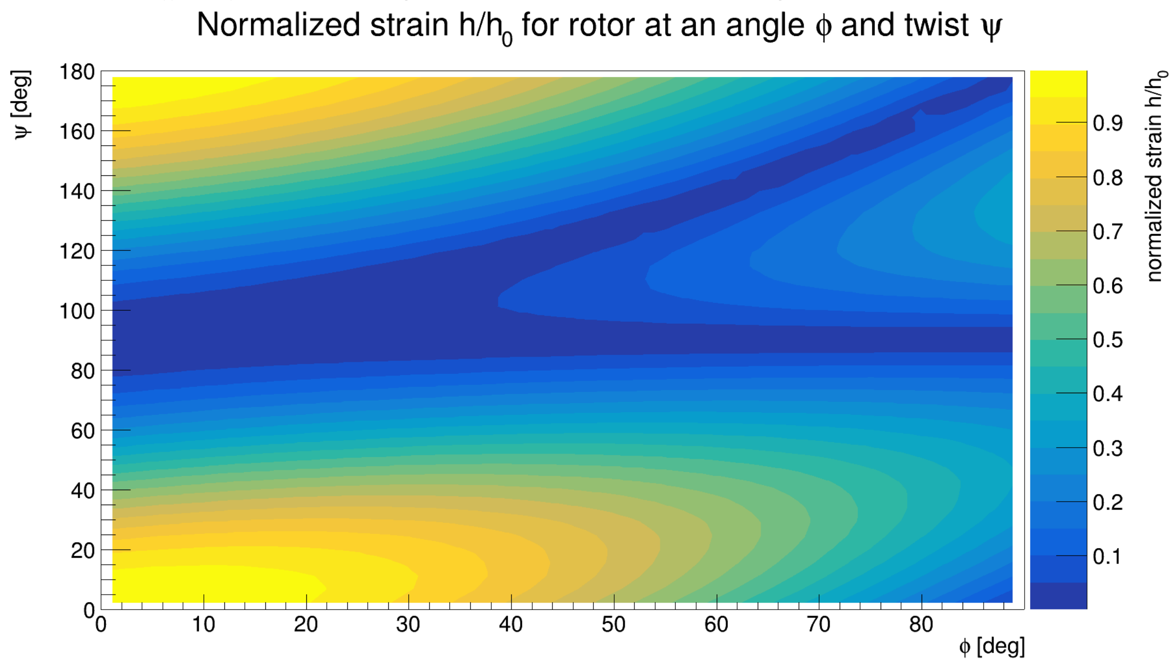


Figure 2: Normalized strain signal of a rotor at 1.7 m from the mirror for different angles ϕ and twists ψ.

In fig. 3 we show the strain variations profiles at $\phi = 0°$ and $\phi = 34.7°$. One can see that the NCal strain is maximal for a twist $\psi = 12.1°$ when $\phi = 34.7°$. As a result, twisting the rotor by $\psi = 12.1°$ allows to reduce the associated uncertainty on the NCal strain as it is less sensitive to small twist variations.

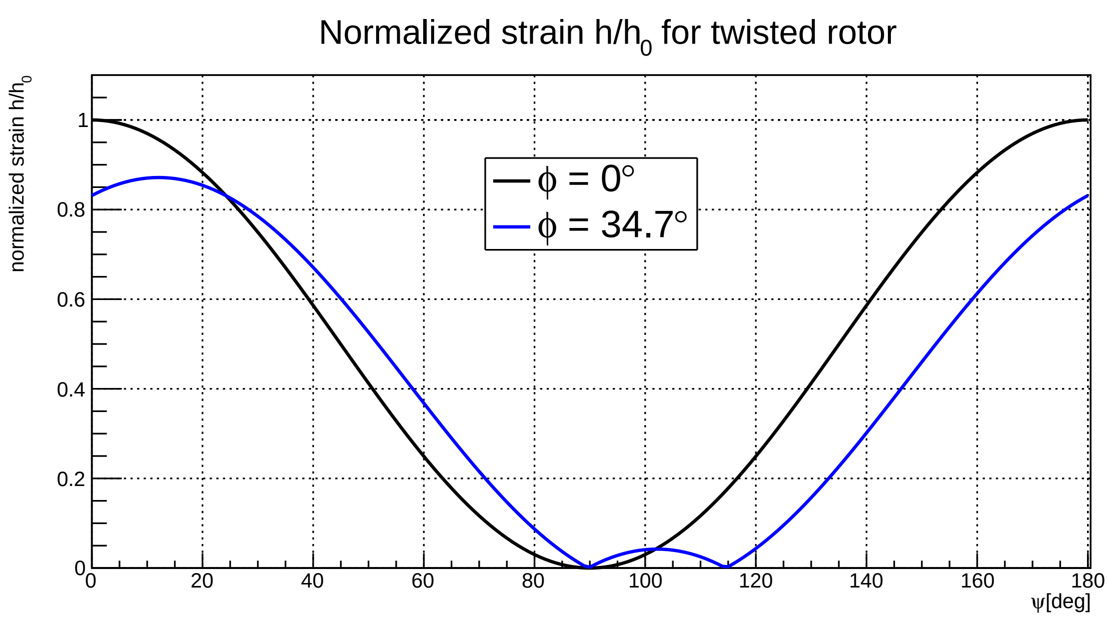


Figure 3: Normalized strain of a rotor at 1.7 m from the mirror for different twists ψ. The black curve shows a rotor at $\phi = 0°$, the blue curve shows a rotor at $\phi = 34.7°$.

Now that we understood the correlation between the twist and the rotor to beam axis angle we can check if the distance affects the signal of a twisted rotor (higher order terms neglected in eq. (1)). Figure 4 shows the maximum normalized signal obtained using FROMAGE for a rotor at $\phi = 34.7°$ and at distances of 1.3 m, 1.7 m, 2.1 m and 2.5 m which are the possible NCal O4 distances from the mirror.

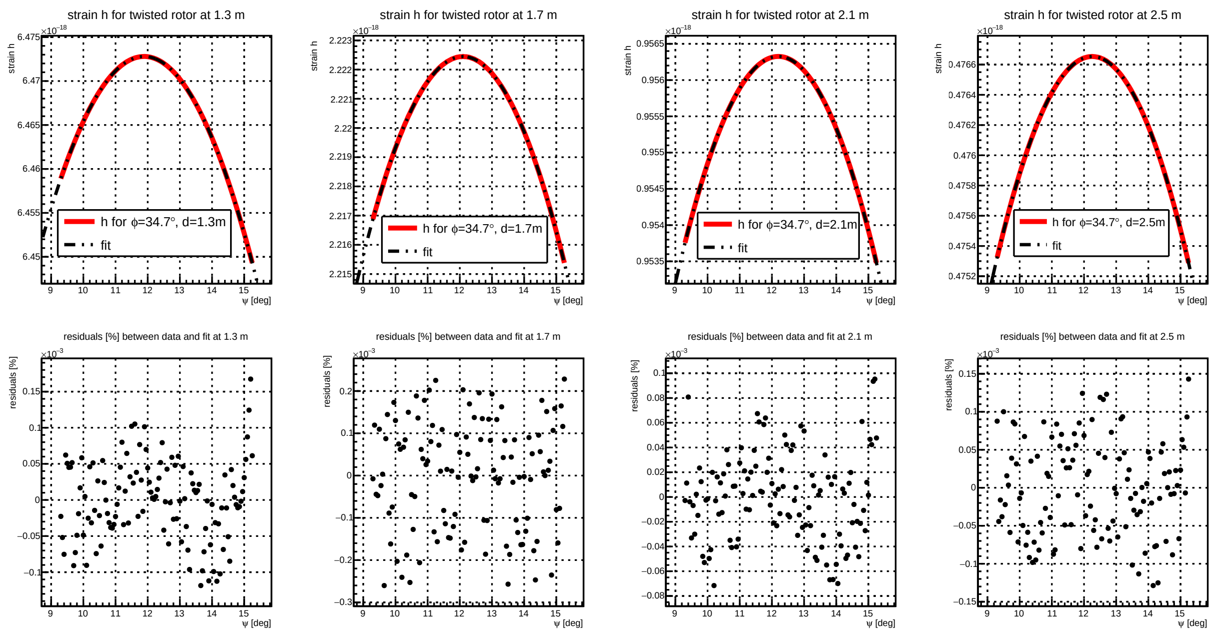


Figure 4: Top plots show the maximum signals for different NCal to mirror distances at $\phi = 34.7°$, the red curves show the FROMAGE simulations and the black curve show quadratic fits. Bottom plots show the fit residuals which are usually smaller than $10^{-4}\%$.

Using the fit results from fig. 4 we compute the twist ψ_{max} associated to the maximum signal at $\phi = 34.7°$ for each NCal to mirror distance. Unlike eq. (1) which is at the second order and does not show any distance dependancy, the computation with FROMAGE which includes the effects of all higher order terms predicts a small dependancy with the distance presented in table 1.

Distance (m)	1.3	1.7	2.1	2.5
ψ_{max}	11.89°	12.10°	12.20°	12.26°

Table 1: Twist ψ_{max} for each NCal to mirror distance associated to the maximum signal at $\phi = 34.7°$.

Since it is much easier to use the same twist on every NCal we study the deviation of the maximum amplitude around a fixed ψ. Using $\psi = 12.1°$ the twist at $d = 1.7$ m as a fixed twist value we can compute the deviation for each case from fig. 4 around this twist for a given uncertainty $\delta\psi$. We expect the uncertainty $\delta\psi$ to be much lower than $0.1°$. Using this value as our uncertainty we compute the deviations shown in table 2 for each NCal to mirror distance around $\psi = 12.1°$. The results show that the relative amplitude deviation is of the order of $10^{-3}\%$ taking a fixed twist. If the rotors are not twisted this deviation is 100 times greater.

Introducing a twist on the rotor will increase the signal and reduce the uncertainty associated to this parameter. We should be able to use the same twist value on every rotor since for each distance this value is close enough to keep a stronger signal (see fig. 4 and table 1) and reduced uncertainties (see table 2).

Distance (m)	1.3	1.7	2.1	2.5
Relative amplitude deviation (%)	1.67×10^{-3}	3.20×10^{-4}	9.68×10^{-4}	1.33×10^{-3}

Table 2: Relative amplitude deviations for different NCal to mirror distances at a fixed $\psi = 12.1° \pm \delta\psi = 0.1°$.

3 Estimating the optimal rotor twist ψ for the O4 NCal setups

3.1 Measurements of ϕ for each setup

In the previous section we were using the design ϕ value but the actual orientation of the NCal setup is slightly different. Using the coordinates of the reference holes on the reference plates determined in VIR-0343B-22, we can compute the angle ϕ of each setup to the beam axis and estimate the uncertainty.

The coordinates of the reference holes and their uncertainties are shown in table 3.

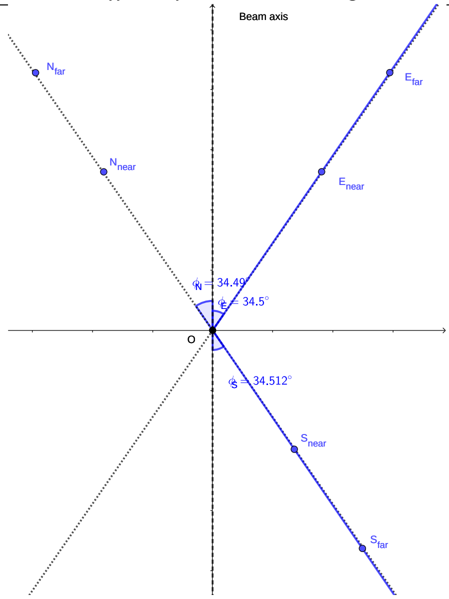
Hole position	x (mm)	y (mm)
North near	-905.29 ± 0.36	1317.84 ± 0.57
North far	-1471.86 ± 0.52	2141.85 ± 0.45
East near	906.25	1318.6
East far	1472.69 ± 0.06	2142.78 ± 0.09
South near	679.06 ± 1.14	-988.67 ± 0.79
South far	1245.32 ± 1.27	-1812.88 ± 0.70

Table 3: Coordinates and uncertainties considered for each reference hole in the NE tower reference frame. The East reference hole is fixed.

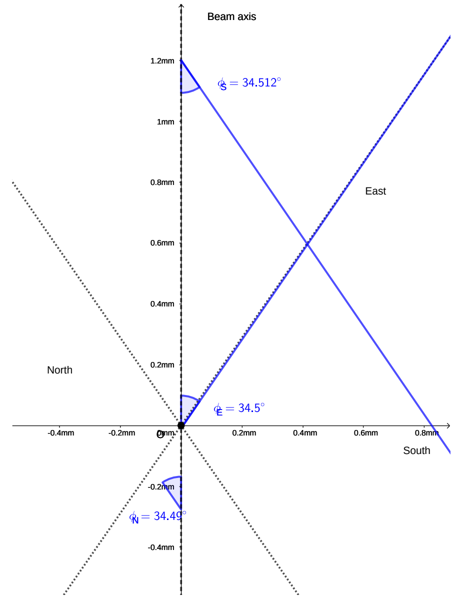
We can use an online geometry software to place the points and compute the angles. Using Geogebra we draw fig. 5. As expected due to mecanical uncertainties the setups are not placed at the same ϕ angle relative to the beam axis, the average measured value is $34.5°$ instead of the $34.7°$ design value. The setups are therefore not pointing at the center of the referential O but with slight offsets (see fig. 5b). The angles for each case considered is shown in table 4.

Axis considered	North ϕ	East ϕ	South ϕ
Setup axis	$34.490°$	$34.500°$	$34.512°$
Difference from mean	$-0.007°$	$0°$	$0.012°$

Table 4: Angles ϕ of each setup to the beam axis. The setup axis angles consider the axis drawn between the two reference holes of a setup.



(a) Top view of each reference hole (blue points) and their axis direction (represented as blue lines), the angle ϕ of each setup to the beam axis is shown.



(b) Magnified top view at the center to show where the setups are pointing on the beam axis.

Figure 5: General top view (a) and magnified view (b) of the three setups in the reference frame of the NE tower centered in O. The blueprint axis direction of the setup is shown as dotted lines (at $\phi = 34.7°$).

3.2 Computing the optimal twist ψ

Using the results of the previous sections we fixed the angle ϕ for the O4 NCals to be $34.49°$. As explained at the end of section 2 a reference twist ψ will be input on the NCals. This reference twist is computed using FROMAGE at $\phi = 34.5°$ and $d = 1.7$ m giving a value of $\psi = 12.01°$ (see fig. 6).

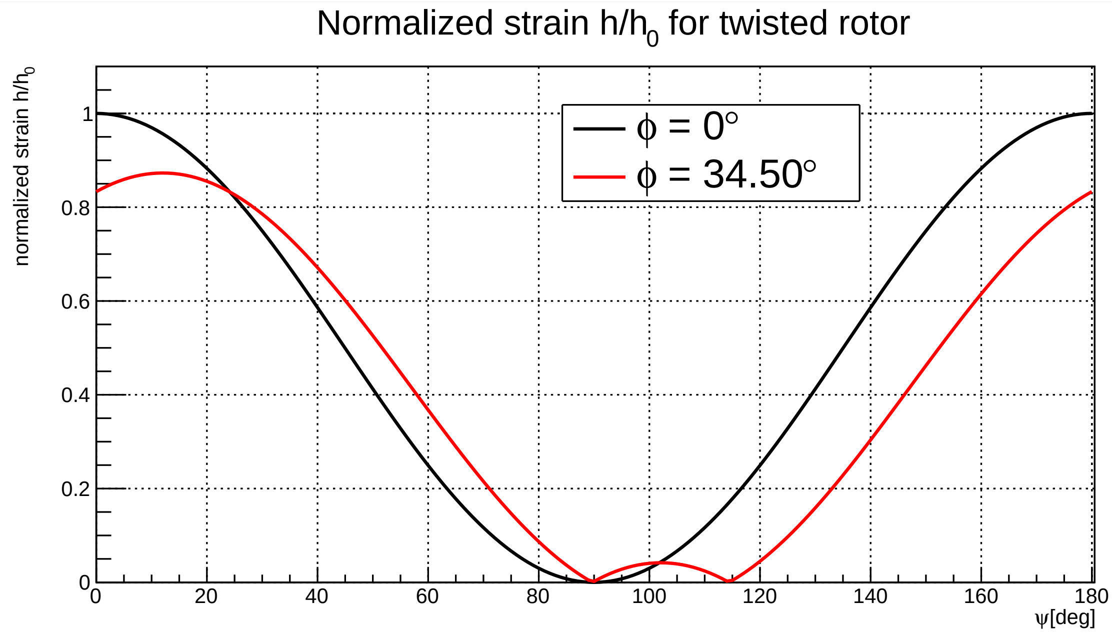


Figure 6: Normalized strain of a rotor at 1.7 m from the mirror for different twists ψ. The black curve shows a rotor at $\phi = 0°$, the red curve shows a rotor at $\phi = 34.5°$.

4 Uncertainties

The distance offset between the setup axis and the center of the reference frame O (see fig. 5) can be computed for each setup. Dividing this offset by the distance between the center O and the closest reference hole distance of each setup gives an uncertainty on ψ associated to the VRS survey. For the North setup the result is $0.156/1600 = 9.75 \times 10^{-2}$ mrad. For the East setup the result is $0.005/1600 = 3.13 \times 10^{-3}$ mrad. And for the South setup the result is $0.680/1200 = 0.57$ mrad. The NCal vertical suspension uncertainty relative to the reference plate is expected to be $0.02/800 = 2.5 \times 10^{-2}$ mrad. The machining uncertainty of the NCal support is $0.01/40 = 0.25$ mrad and its mechanical clearance when installed on the vertical plate is $0.05/40 = 1.25$ mrad. The rotor axes position in the NCal boxes have been machined with a precision of 0.05 mm over 154 mm giving a machining uncertainty of 0.32 mrad.

Using a quadratic sum of each source of uncertainty on ψ the result on the worst case scenario is $\delta\psi = \pm 1.43$ mrad ($\pm 0.08°$). This uncertainty is below the $0.1°$ taken for table 2 making the uncertainty of ψ on the NCal strain signal negligeable.

Using table 3 we can compute the uncertainty on ϕ. The worst case scenario being on the South setup with $1.14/1200 = 0.95$ mrad. As shown in eq. (1) at the first order, the ϕ error propagates as $\delta\phi sin(\phi)$ which makes 0.054% of uncertainty on the NCal strain signal.

5 O5 expectations

The NCal system for O5 will be different, four setups should be installed around the mirror at 1.3 m or slightly closer with an angle $\phi = 55.3°$. Based on the results from the previous sections we can estimate the twist ψ that will be used for O5 (see fig. 7).

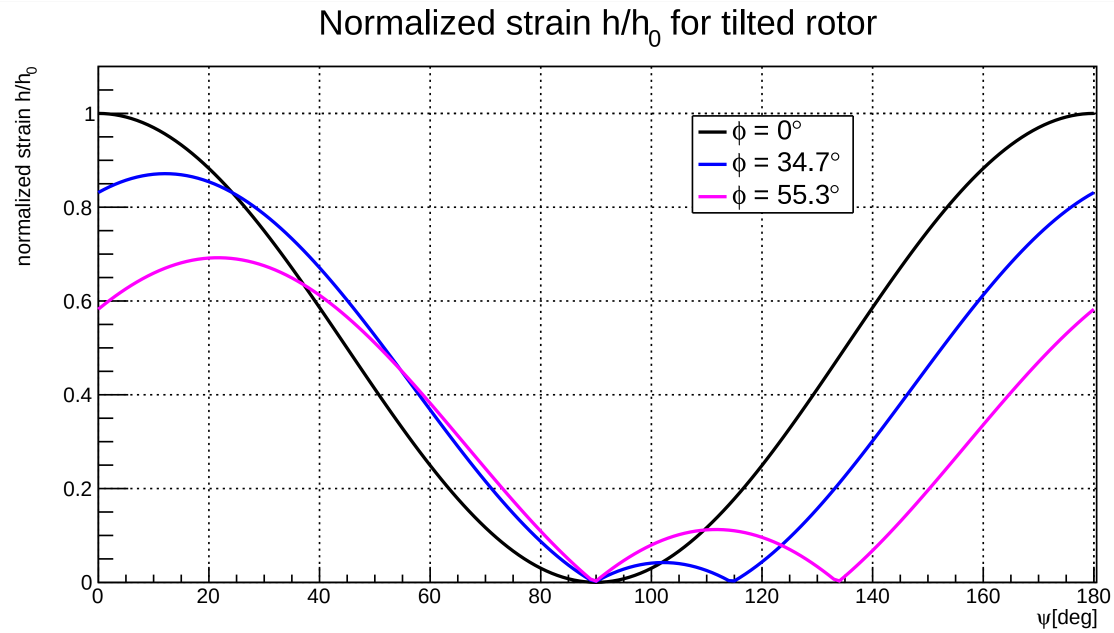


Figure 7: Normalized strain of a rotor at 1.7 m from the mirror for different twists ψ. The black curve shows a rotor at $\phi = 0°$, the blue curve shows a rotor at $\phi = 34.7°$ and the purple curve shows a rotor at $\phi = 55.3°$

The optimal twist at $d = 1.7$ m and $\phi = 55.3°$ is $\psi = 21.64°$. Using $\delta\psi$ lower than $0.1°$ as our uncertainty we compute a deviation for the NCal to mirror distance of 1.7 m around $\psi = 21.64°$. The results show that the relative amplitude deviation is of the order of $10^{-4}\%$ taking a fixed twist. The uncertainty on the amplitude from the twist is therefore negligeable for O5. One can notice in fig. 7 at $\psi = 0°$ the deviation of the O5 amplitude is greater than O4.

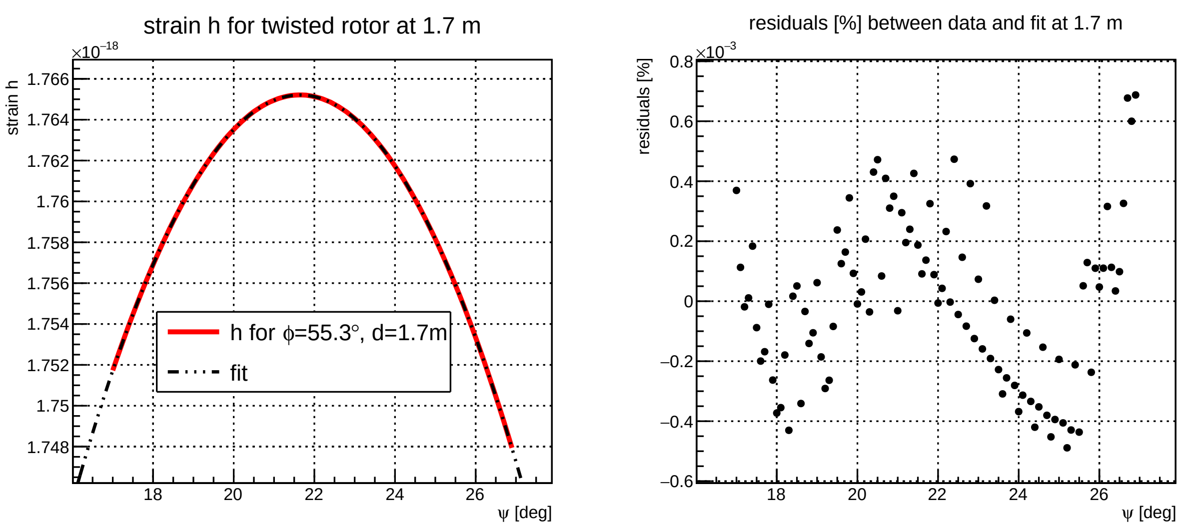


Figure 8: Left plot shows the maximum signal for a NCal to mirror distance of 1.7 m at $\phi = 55.3°$, the red curve shows the FROMAGE simulation and the black curve shows quadratic fit. Right plot shows the fit residuals which are usually smaller than $10^{-3}\%$.