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Quantities definition for the AdV control system readout

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Contents

Abstract

This document formulates quantities to be measured by the Advanced Virgo (AdV) read out system in order to design a suitable electric scheme. We first define the signal, with simulated amount of RF components, to obtain a realistic estimation of the power to be read out at each port of the interferometer. Then, several plots are given for several scenarii. Finally, we conclude that 2 different electronic designs for the whole interferometer might be the best solution to meet requirements. Furthermore, the scenario using a $f_2 \sim 50\,\mathrm{MHz}$ modulation frequency appears a lot more feasible than the one using a $f_2 \sim 80\,\mathrm{MHz}$ one.

1 Definition of the signal

As the AdV optical layout is still subject to modifications, following considerations may suffer actualization. Nevertheless, it seems that most of the relevant aspects should remain stable from the detection point of view.

1.1 Optical field

The optical beam should be phase modulated by 3 different frequencies, namely f_1, f_2 and f_3, leading to a field expression of the form:

$$E_{opt} = A \times e^{j\omega_p t} \times e^{j\delta_1 \sin \omega_1 t} \times e^{j\delta_2 \sin \omega_2 t} \times e^{j\delta_3 \sin \omega_3 t} \tag{1}$$

with ω_p the carrier pulsation at $\lambda = 1.064\,\mu$m, ω_1, ω_2 and ω_3, the modulation pulsations and δ_1, δ_2 and δ_3, the corresponding modulation indexes.

The phase modulation term can be expressed using Bessel functions as follow:

$$e^{j\delta \sin \omega t} = \sum_{k=-\infty}^{+\infty} j^k.J_k(\delta).e^{jk\omega t} \tag{2}$$

where $J_k(\delta)$ is given by:

$$J_k(\delta) = \left(\frac{\delta}{2}\right)^k . \sum_{p=0}^{\infty} \left(\frac{(-1)^p}{2^{2p} p!\,(k+p)!} . \delta^{2p} \right) \tag{3}$$

Considering foreseen values of modulation indexes δ_i, of the order of 0.1, the above development can fairly be limited to the first order of p. Using the identity $J_k(\delta) = (-1)^k \times J_{-k}(\delta)$, one finally approximates the phase modulation term as:

$$e^{j\delta \sin \omega t} = J_0(\delta) + j.J_1(\delta).e^{j\omega t} - j.J_1(\delta).e^{-j\omega t} \tag{4}$$

Inserting this expression in eq. 1 gives:

$$\begin{gathered} E_{opt} = K.e^{j\omega_p t} ... \\ +j.A.J_0(\delta_3).J_0(\delta_2).J_1(\delta_1) e^{j(\omega_p+\omega_1)t} - j.A.J_0(\delta_3).J_0(\delta_2).J_1(\delta_1).e^{j(\omega_p-\omega_1)t} ... \\ +j.A.J_0(\delta_3).J_0(\delta_1).J_1(\delta_2).e^{j(\omega_p+\omega_2)t} - j.A.J_0(\delta_2).J_0(\delta_1).J_1(\delta_2).e^{j(\omega_p-\omega_2)t} ... \\ +j.A.J_0(\delta_2).J_0(\delta_1).J_1(\delta_3).e^{j(\omega_p+\omega_3)t} - j.A.J_0(\delta_2).J_0(\delta_1).J_1(\delta_3).e^{j(\omega_p-\omega_3)t} \end{gathered} \tag{5}$$

with $K = A.J_0(\delta_1).J_0(\delta_2).J_0(\delta_3)$ and the terms in $J_1(\delta_x).J_1(\delta_y)$ or so beeing neglicted.

One recognizes in eq. 5 the expression of the phase modulated beam with the so-called upper and lower sideband components (respectively $\omega_p + \omega_i$ and $\omega_p - \omega_i$) associated to each modulation frequency.

1.2 Entering numbers

To estimate the beam measurable on each port of the interferometer, the convenient transfert function should be applied to eq. 5 using optical simulations. Those considerations are well beyond the scope of this note. Instead, we refer to the document [1] in which simulations based on possible scenarii of the optical layout for the AdV interferometer are detailled.

The first set of parameters is the modulation frequencies to be used. Those are summarized in table 1. From the detection point of view, only crude estimation of the modulation frequencies are needed.

The next parameter to be considered is the optical power in each sideband at each port of the interferometer. Again, the relevant numbers were extracted from [1] and summarized in table 2. It should be noted that the power components given in Watts in [1] are converted in $\sqrt{\text{W}}$ in this note. So it can be used directly with eq. 5. Futhermore, the author of [1] scaled the optical beam so that the total incident power impinging on each photodiodes does not exceed 100 mW. The latter number beeing a rough estimation of the maximum amount of power a photodiode can handle without permanent damage. The scaling is performed by applying the factor given in the last column of table 2 on the optical power.

	Option 1	Option 2
f_1	6 MHz	6 MHz
f_2	82 MHz	88 MHz
f_3	8 MHz	8 MHz

Table 1: Extracted from [1], possible set of modulation frequencies

Port	Carrier	LSB1	USB1	LSB2	USB2	LSB3	USB3	Coeff
ASYp	0.3046	0.0037	0.0036	0.3987	0.3975	0.0002	0.0002	1%
SYM	1.2845	0.5450	0.5450	0.3755	0.3808	0.5559	0.5559	3%
POBS	0.3479	0.0313	0.0313	0.0297	0.0134	0.0010	0.0010	75%
POX	0.3564	0.0313	0.0313	0.0297	0.0134	0.0010	0.0010	70%
POY	0.3564	0.0313	0.0313	0.0144	0.0284	0.0010	0.0010	70%
XP	1.8815	0.0003	0.0003	0.0003	0.0001	0.1e-4	0.1e-4	2%
YP	1.8762	0.0003	0.0003	0.0001	0.0003	0.1e-4	0.1e-4	2%

Port	Carrier	LSB1	USB1	LSB2	USB2	LSB3	USB3	Coeff
ASYp	0.3046	0.0040	0.0040	0.0111	0.0105	0.0002	0.0002	1%
SYM	1.2845	0.5450	0.5450	0.5550	0.5550	0.5559	0.5559	3%
POBS	0.3479	0.0313	0.0313	0.0007	0.0003	0.0010	0.0010	75%
POX	0.3564	0.0313	0.0313	0.0007	0.0003	0.0010	0.0010	70%
POY	0.3564	0.0313	0.0313	0.0003	0.0007	0.0010	0.0010	70%
XP	1.8815	0.0003	0.0003	0.0001	0.2e-4	0.1e-4	0.1e-4	2%
YP	1.8762	0.0003	0.0003	0.2e-4	0.0001	0.1e-4	0.1e-4	2%

Table 2: Extracted from [1], RF components at each port in $\sqrt{\mathrm{W}}$ for option 1 (upper table) and option 2 (lower table) scenario. LSBi and USBi standing for lower and upper sideband, respectively, of modulation frequency i ($\omega_p \pm \omega_i$).The last column gives the percentage of the optical power that is effectively picked up by photodiodes.

Depending on the particular photodiode to be used in the detection system, the maximum allowable power on photodiode, 100 mW, is subject to variation and could reach values greater than 200 mW. This parameter will remain unchanged in this note but it should be kept in mind that a greater amount of power could be possible when entering SNR considerations.

The amount of power affordable on the ASYp is set by the OMC thus it does not offer any tuning possibility like others. This makes this port a signal of a special interest as it is the weakest one. On the other side, the SYM port receives the strongest RF components which makes it of prime importance for the electronic design.

1.3 Optical power

The numbers from table 2 are transposed in the optical field expression from eq. 5 which can be reformulated as follow:

$$
\begin{aligned}
E_{opt} = K.e^{j\omega_p t} &+ j.A_1.e^{j(\omega_p+\omega_1)t} - j.B_1.e^{j(\omega_p-\omega_1)t} ... \\
&+ j.A_2.e^{j(\omega_p+\omega_2)t} - j.B_2.e^{j(\omega_p-\omega_2)t} ... \\
&+ j.A_3.e^{j(\omega_p+\omega_3)t} - j.B_3.e^{j(\omega_p-\omega_3)t}
\end{aligned} \tag{6}
$$

with K the carrier, A_i and B_i, the USB and LSB coefficient, respectively, of the i$^{\text{th}}$ modulation frequency from table 2.

The photodiode is sensitive to the optical power defined as:

$$P_{opt}(t) = E_{opt} \times E^*_{opt} \tag{7}$$

By developping and using the *ad'hoc* identities, one obtains the expression for P_{opt}:

$$
\begin{gathered}
P_{opt} = K^2 + A_1^2 + A_2^2 + B_1^2 + B_2^2 + C_1^2 + C_2^2 + ... \\
-2.K.(A_1+B_1).\sin(\omega_1 t) - 2.K.(A_2+B_2).\sin(\omega_2 t) - 2.K.(A_3+B_3).\sin(\omega_3 t) ... \\
-2.A_1.B_1.\sin(2\omega_1 t) - 2.A_2.B_2.\sin(2\omega_2 t) - 2.A_3.B_3.\sin(2\omega_3 t) ... \\
+2.(A_1A_2 + B_1B_2).\cos((\omega_2-\omega_1)t) - 2.(A_1B_2 + B_1A_2).\cos((\omega_2+\omega_1)t) ... \\
+2.(A_1A_3 + B_1B_3).\cos((\omega_3-\omega_1)t) - 2.(A_1B_3 + B_1A_3).\cos((\omega_3+\omega_1)t) ... \\
+2.(A_2A_3 + B_2B_3).\cos((\omega_2-\omega_3)t) - 2.(A_2B_3 + B_2A_3).\cos((\omega_2+\omega_3)t)
\end{gathered} \tag{8}
$$

The incident optical power is made up of a DC component and optical beatings at ω_i, $2\omega_i$, the sum and the difference of the 3 modulation frequencies. The most demanding requirement, in term of bandwidth of the system, is given by the $2.f_2$ component that stands at 176 MHz in the option 2 scenario. As an illustration of the difficulty, the common bandwidth limit with large area photodetectors like those to be used on AdV stands below 10 MHz.

1.4 Final photocurrent and associated shot noise

From an electric point of view, the photodiode receiving a beam of power P_{opt} will generate a current I_{ph} defined as follow:

$$I_{ph} = \eta \times \frac{q}{\mathrm{h}\nu} \times P_{opt} \tag{9}$$

Or, more commonly:

$$I_{ph} = \eta \times \frac{\lambda[\mu\mathrm{m}]}{1.2375} \times P_{opt} \tag{10}$$

where η is the quantum efficiency of the photodiode. The expected value of $\eta = 90\,\%$ will be used for the quantum efficiency.

From the expression of photocurrent in eq. 10, one estimates the sensitivity floor settle by the shot noise which is defined as:

$$\sigma_{SN} = \sqrt{2.q. < I_{ph} > .B} \tag{11}$$

q beeing the electric charge and B the system bandwidth.

This σ_{SN} has to remain the main noise contribution of the system.

2 Some plots of the expected signals

2.1 Spectral cares

Prior to any plot, we would like to remind the reader some considerations when dealing with discret signals. The following rules plus additionnal usefull highlights concerning signal windowing and equivalent noise bandwidth are summarized in comprehensive and easily available note[2]. Although those are basic stuff, frequent errors and misunderstandings can be avoid if those quantities are handled with care.

1. Signals are simulated using Matlab software, code listings are joined in the annex of this note. A time vector is generated with the signal vector itself. The pitch δt of the time vector sets the value of the sampling frequency $F_e = \frac{1}{\delta t}$ whereas its length $T = N \times \delta t$ defines the frequency resolution of the Fourier transform, or width of the frequency bin, $\Delta f = \frac{1}{T} = \frac{F_e}{N}$.

2. In order to avoid Gibbs phenomena (leakage artefact) when computing the Fourier transform, the modulation frequencies have to be rounded so that they are an exact multiple of the frequency resolution Δf.

3. The last consideration deals with the representation to be used. Indeed the simulated waveforms contain the contribution of 2 process of a different nature:

    - the signal itself, made up of a linear combination of pure sinewaves, is a stationnary process. It should be plotted using power spectrum (PS) expressed either in V^2, A^2, W^2 or using the squareroot of those units. On a PS, the value of a spectral line is relyable as it does not depend on the value of Δf, whereas the floor value does. (This means that the floor value will vary with the vector length!)
    - the additive noise, in our case the shot noise, is a second order stationnary process. A power spectral density (PSD) has to be used to represent such a quantity. It is expressed either in V^2/Hz, A^2/Hz, W^2/Hz or, again, using the squareroot of those units (*i.e.* Wattever/$\sqrt{\text{Hz}}$). In this frame, only the floor value should be considered as the spectral line value depends on Δf. (Here, it is the spectral line that will vary with the length of the simulated signal!)

Given the fact that consideration #2 above is fulfilled, one can fairly avoid the windowing step (Hanning, Hamming, Welch, *etc.* ...). Thus, one passes from one representation to the other in a straightforward way:

$$PS = \Delta f \times PSD \tag{12}$$

or

$$PS = \sqrt{\Delta f} \times PSD \tag{13}$$

depending on the ordinate axis unit.

As we are considering the stationnary aspect of the signal, only the PS representation is used in this note. Noise densities are listed as numbers in table 3. Nevertheless, as the shot noise was included when simulating signals, its value can be retrieve from the plot using $\Delta f = 1\,\text{KHz}$ ($N = 1 \times 10^6$ and $F_e = 1\,\text{GHz}$).

Calculations can be summarized as follow:

- the optical power is computed as a function of time at the considered port using eq. 8,
- then, the optical power is converted in a photocurrent using eq. 10,
- the shot noise current is calculated using eq. 11 and added to the photocurrent in the time domain,
- finally, the PS of the photocurrent is computed in A and the shoit noise value can be extracted using the relation given in eq. 13 in A/$\sqrt{\text{Hz}}$.

2.2 Power spectrums at each port in linear frequency scale

2.2.1 Option 1 scenario

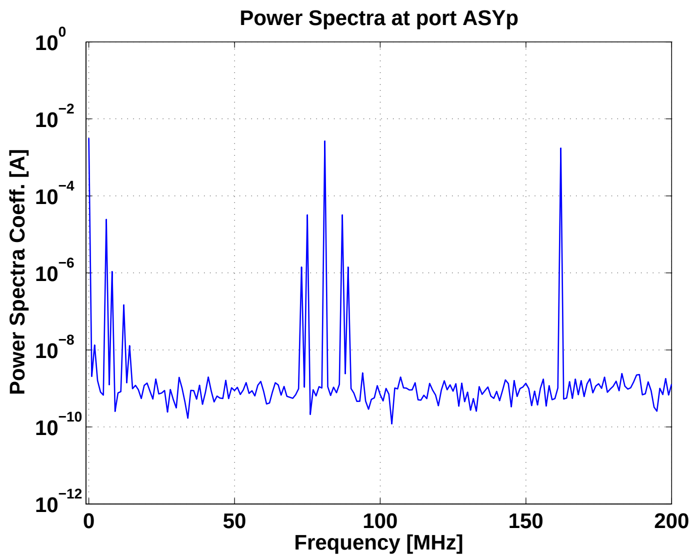

Figure 1: Power Spectra at ASYp

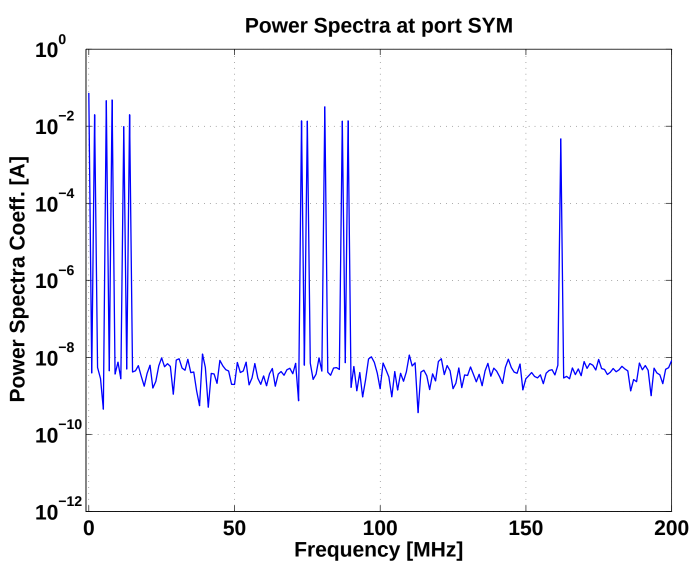

Figure 2: Power Spectra at SYM

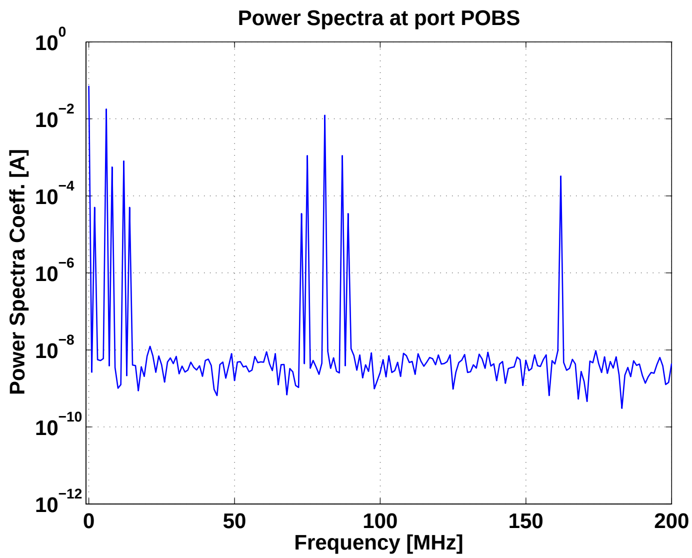

Figure 3: Power Spectra at POBS

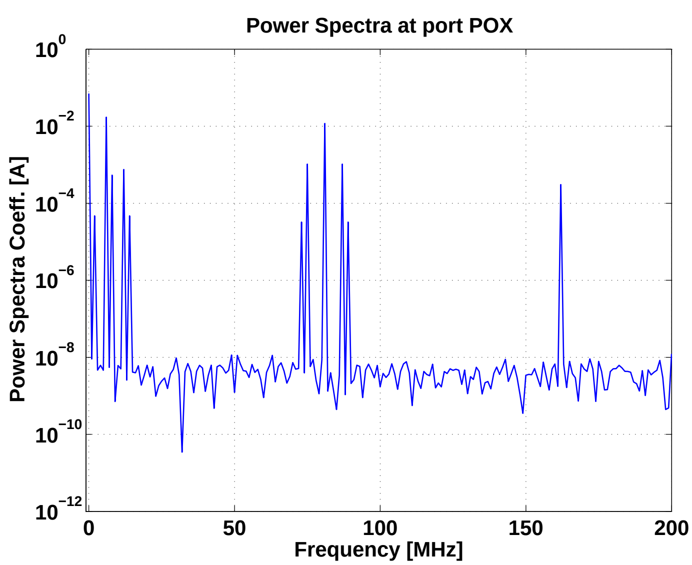

Figure 4: Power Spectra at POX

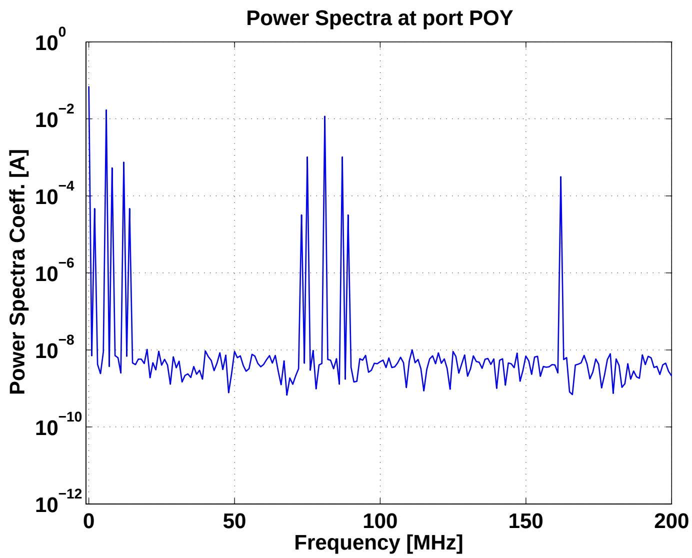

Figure 5: Power Spectra at POY

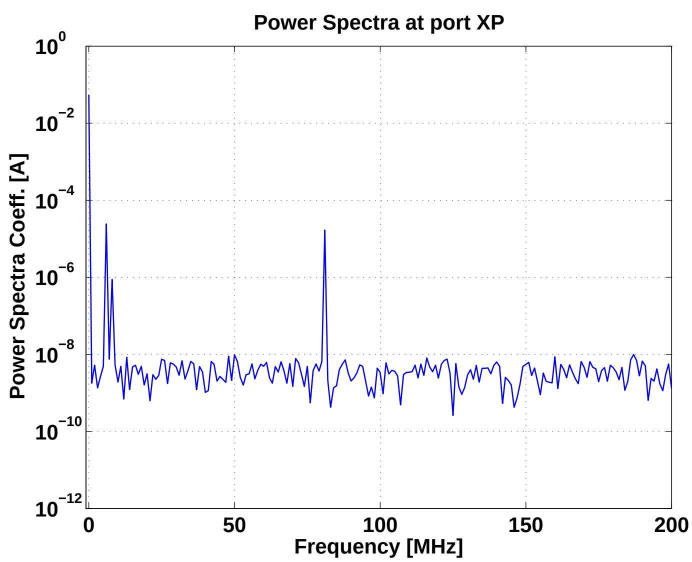

Figure 6: Power Spectra at XP

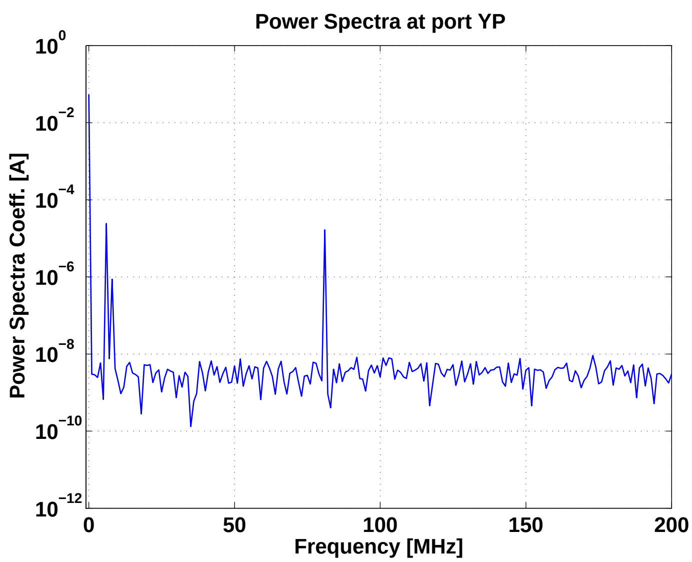

Figure 7: Power Spectra at YP

2.2.2 Option 2 scenario

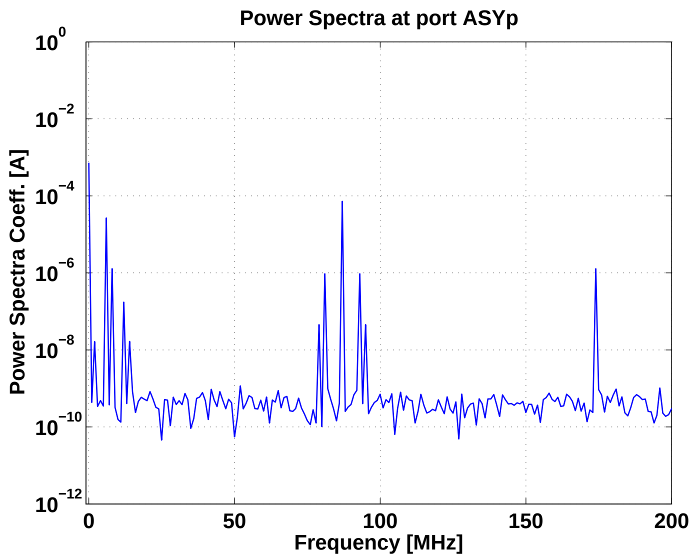

Figure 8: Power Spectra at ASYp

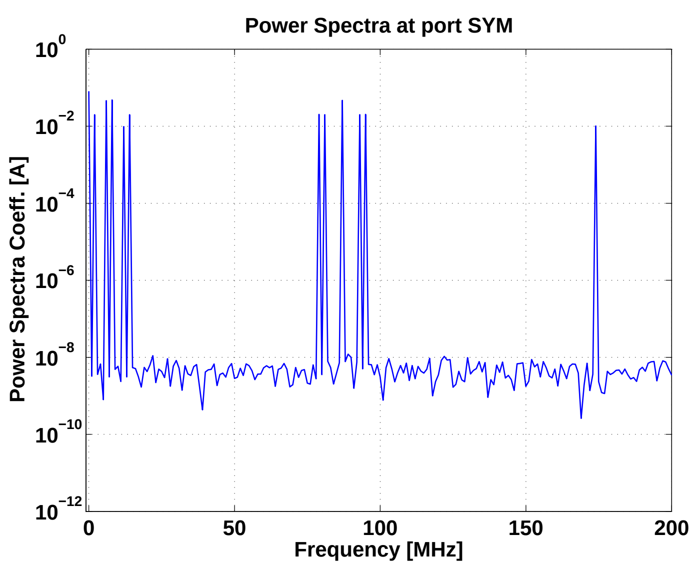

Figure 9: Power Spectra at SYM

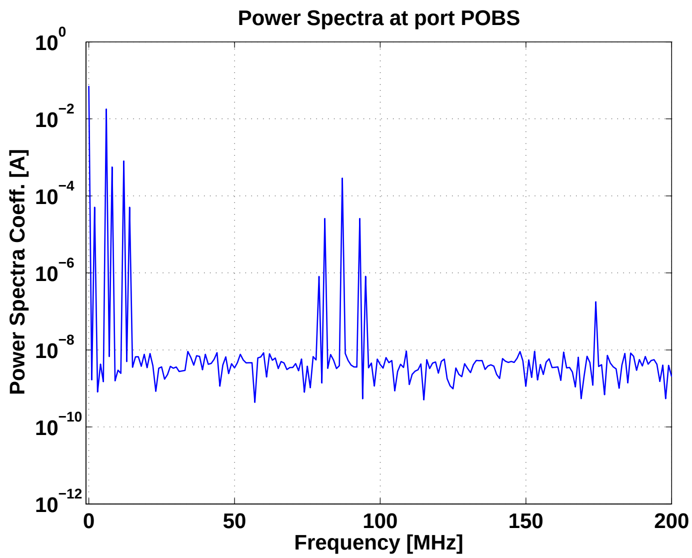

Figure 10: Power Spectra at POBS

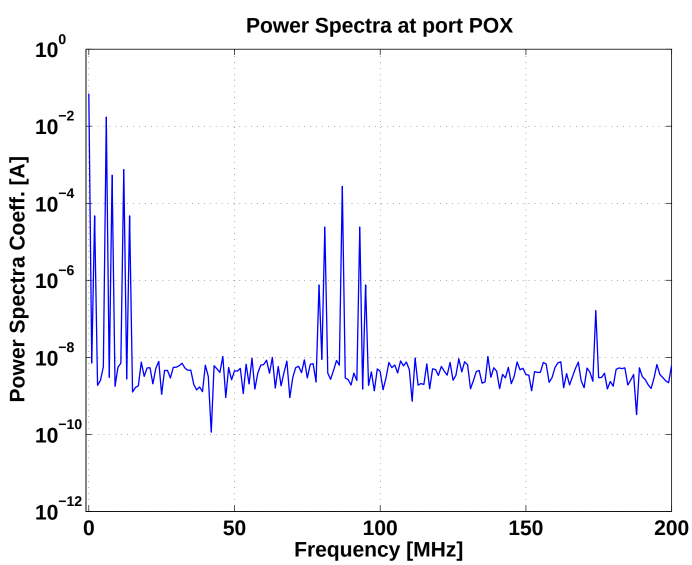

Figure 11: Power Spectra at POX

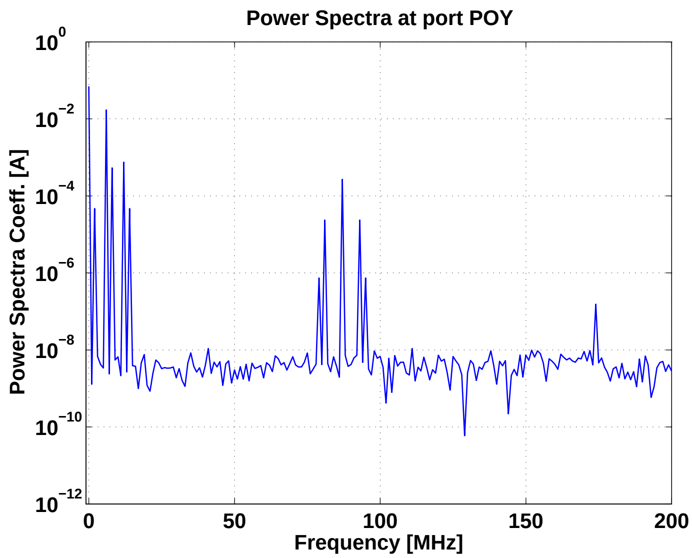

Figure 12: Power Spectra at POY

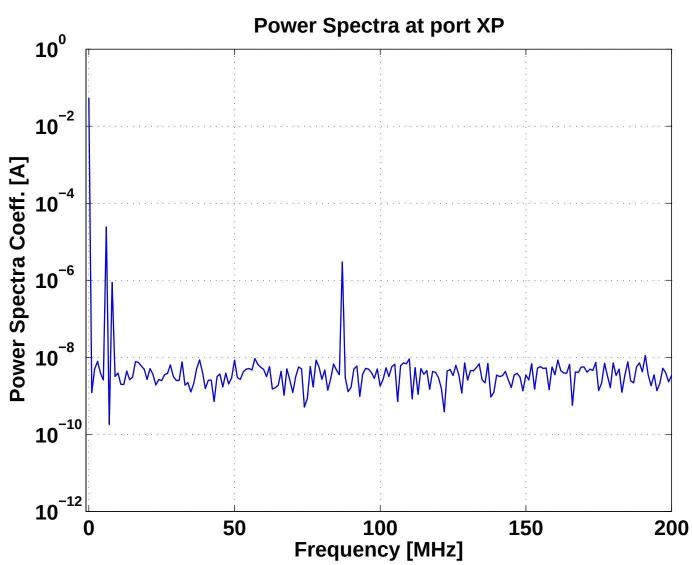

Figure 13: Power Spectra at XP

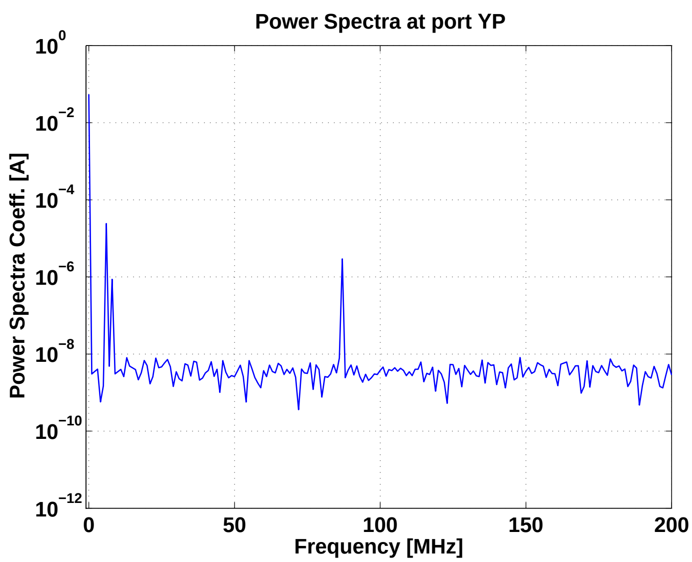

Figure 14: Power Spectra at YP

2.2.3 Modified modulation frequency on Option 1 scenario

Discussions concerning the modulation frequencies are beeing conducted at the time of writing. It seems that a lower value of $f_2 \sim 50\,\mathrm{MHz}$ would be prefered. Obviously, this kind of modification strongly impacts the read out electronic design (making it easier to realize!). Here are the corresponding plots using Option 1 power values.

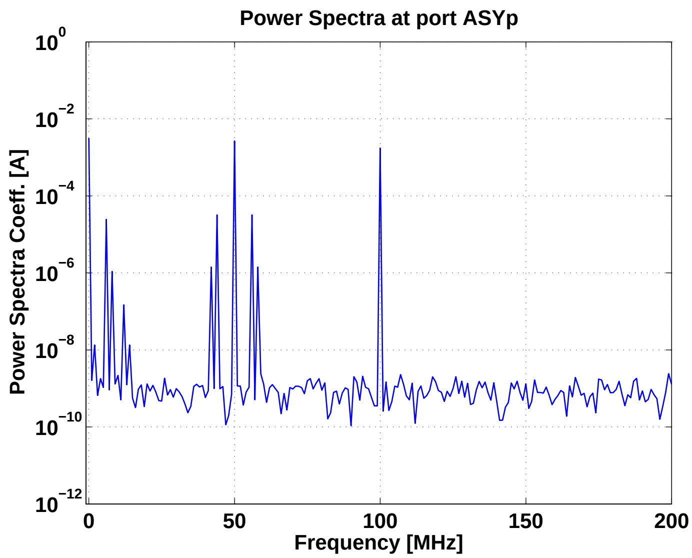

Figure 15: Power Spectra at ASYp

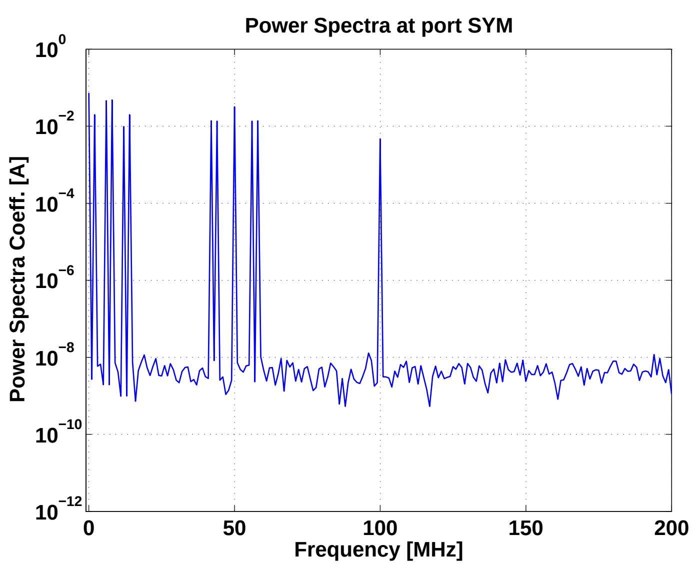

Figure 16: Power Spectra at SYM

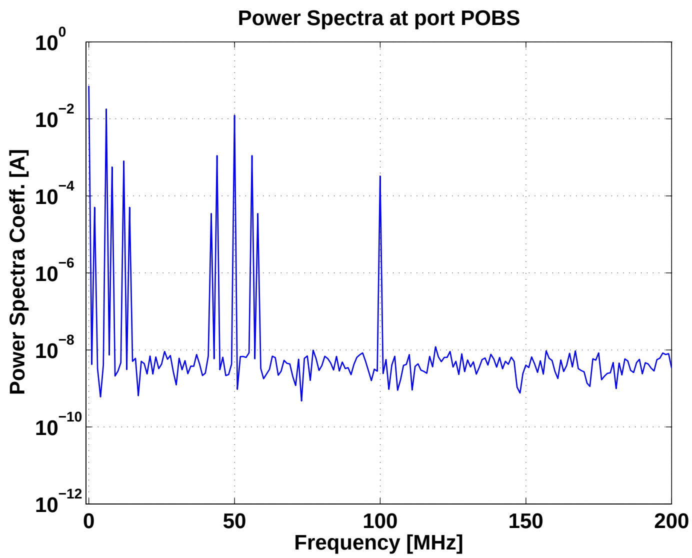

Figure 17: Power Spectra at POBS

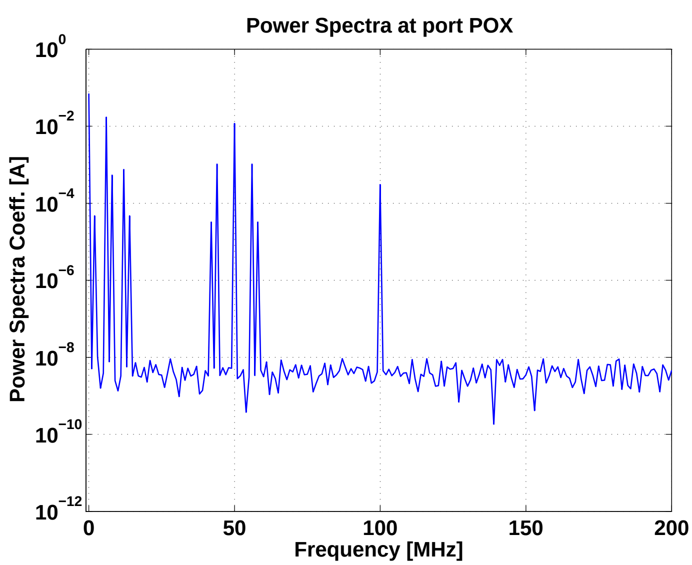

Figure 18: Power Spectra at POX

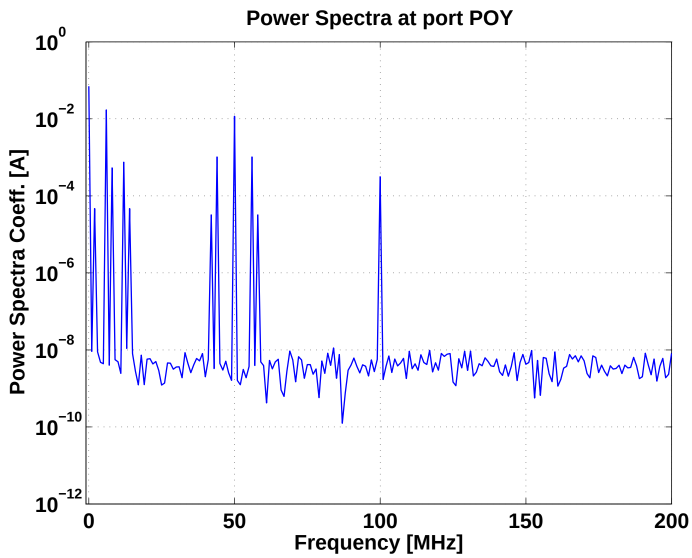

Figure 19: Power Spectra at POY

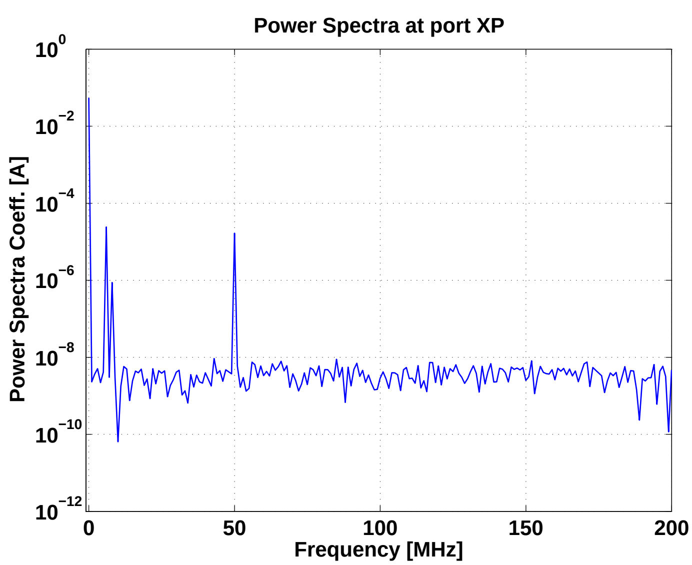

Figure 20: Power Spectra at XP

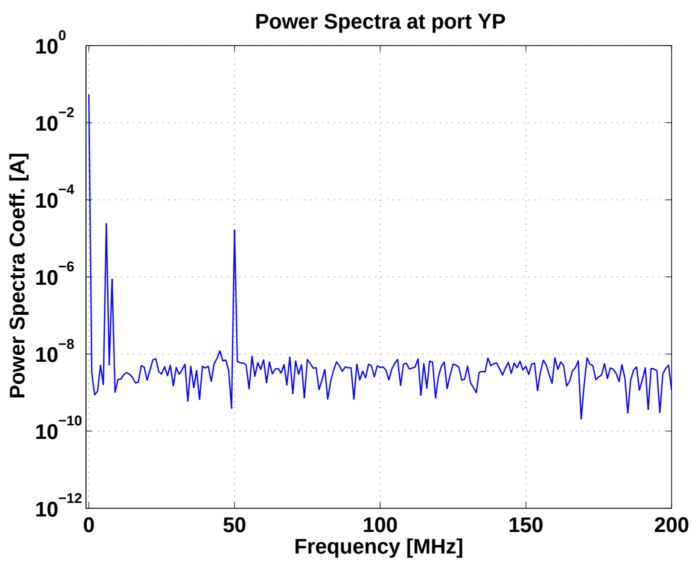

Figure 21: Power Spectra at YP

2.3 Power spectrums at each port in log frequency scale

2.3.1 Option 1 scenario

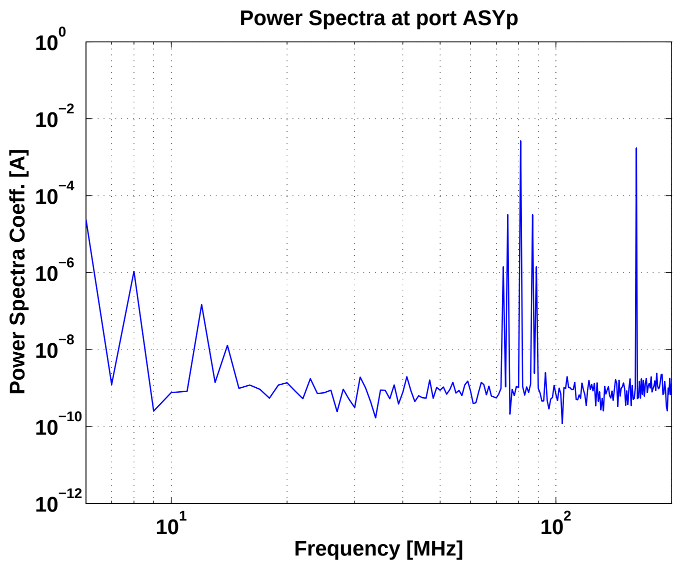

Figure 22: Power Spectra at ASYp

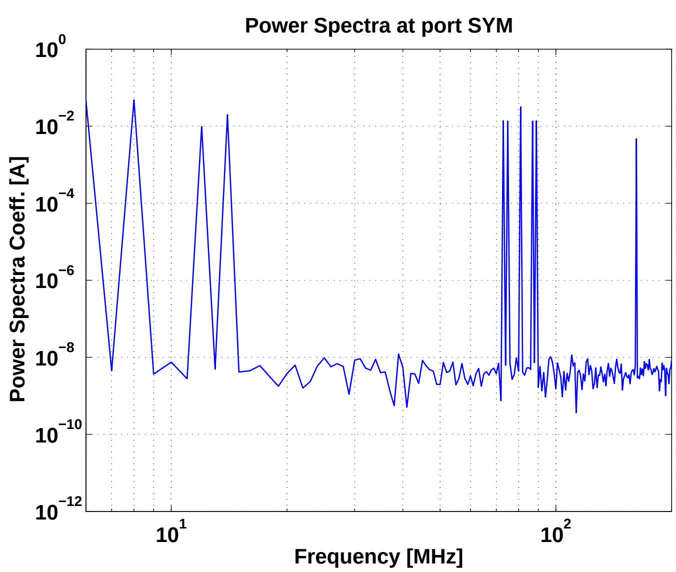

Figure 23: Power Spectra at SYM

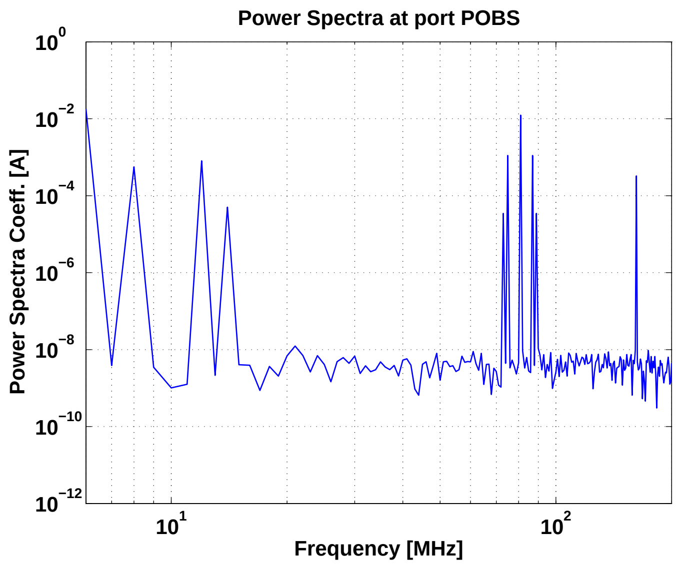

Figure 24: Power Spectra at POBS

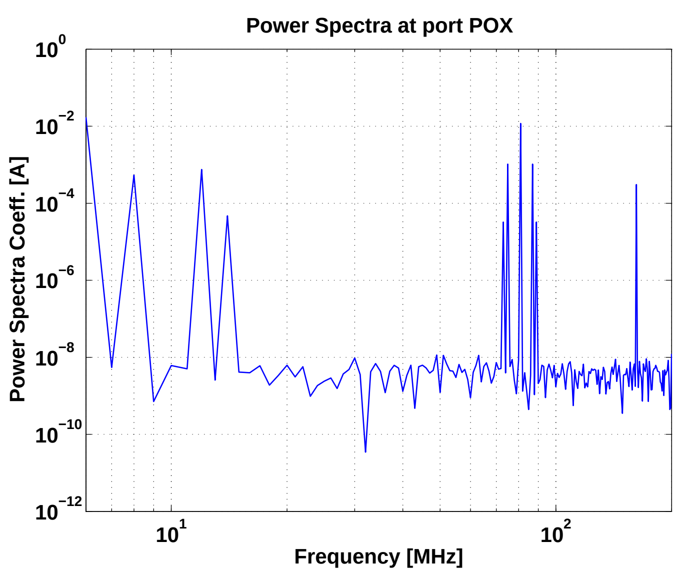

Figure 25: Power Spectra at POX

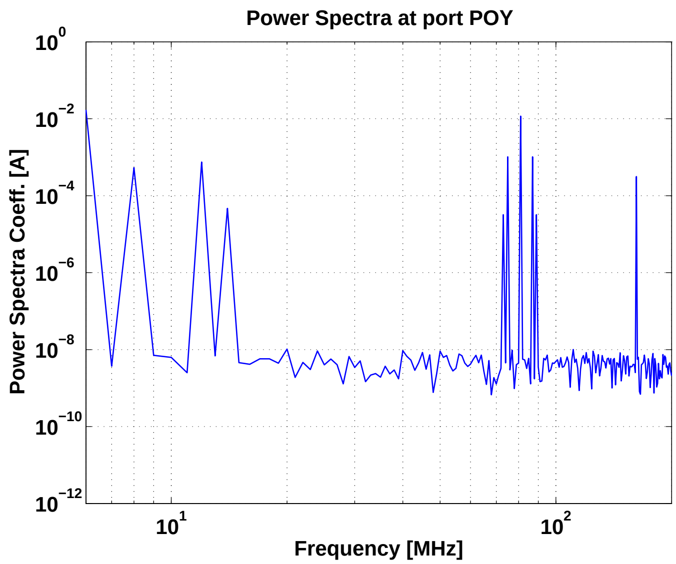

Figure 26: Power Spectra at POY

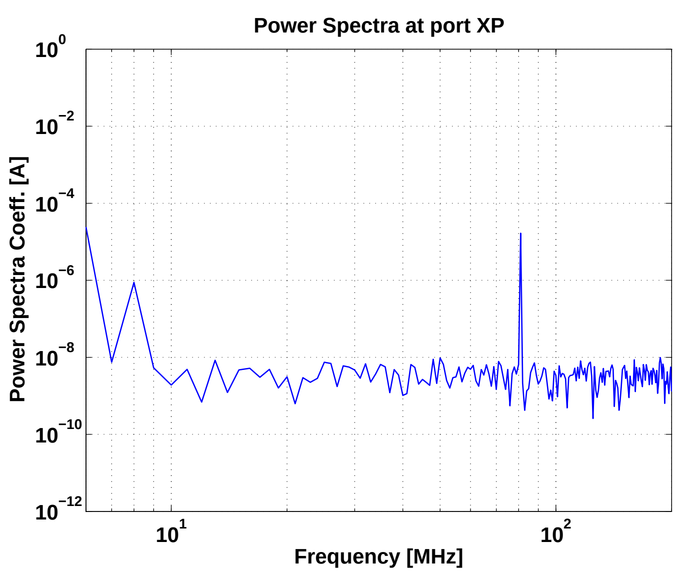

Figure 27: Power Spectra at XP

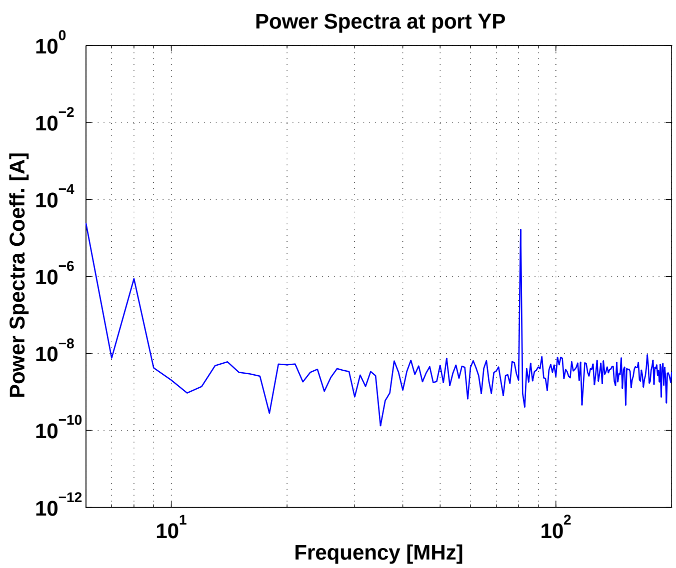

Figure 28: Power Spectra at YP

2.3.2 Option 2 scenario

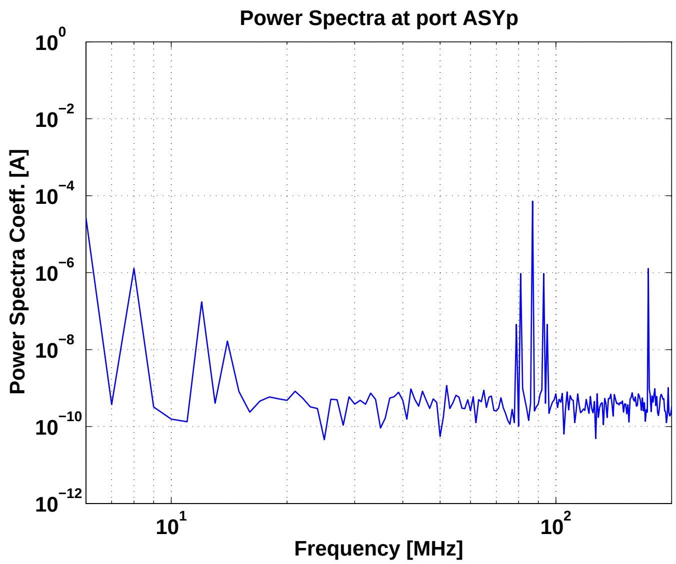

Figure 29: Power Spectra at ASYp

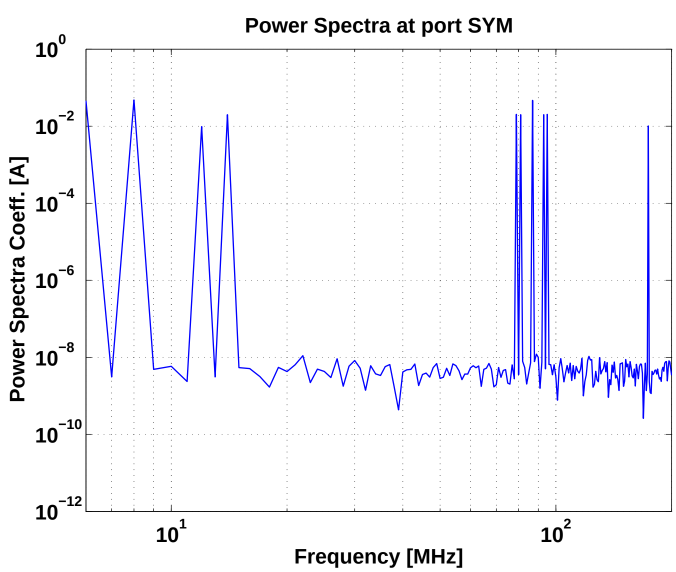

Figure 30: Power Spectra at SYM

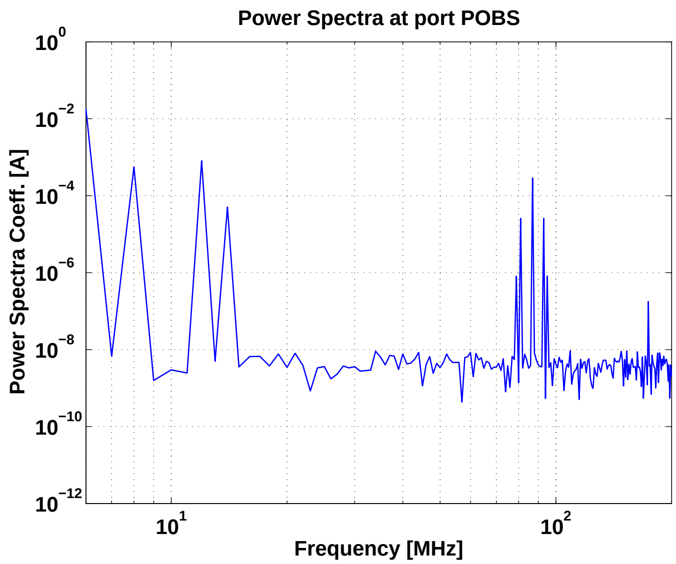

Figure 31: Power Spectra at POBS

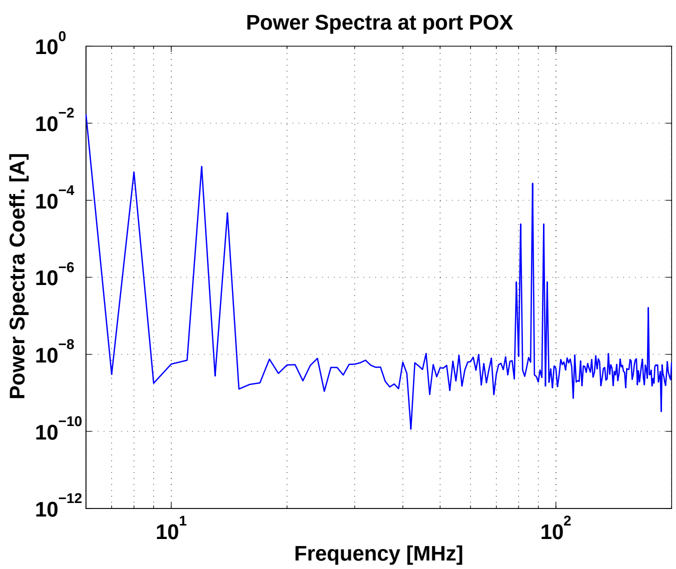

Figure 32: Power Spectra at POX

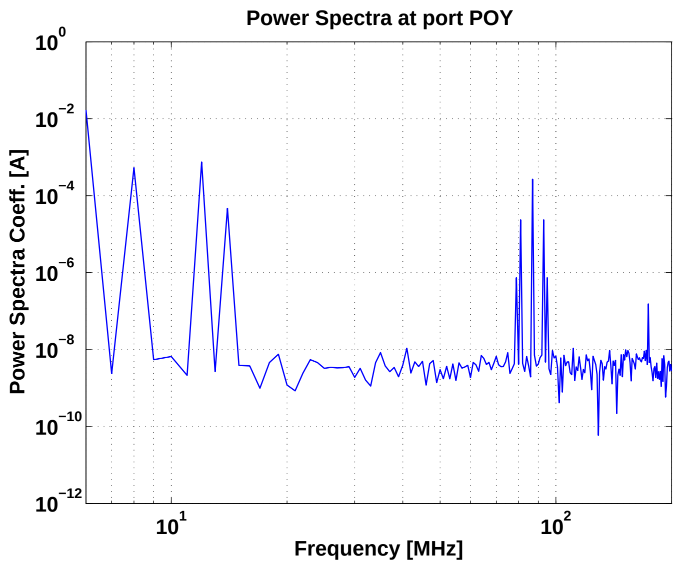

Figure 33: Power Spectra at POY

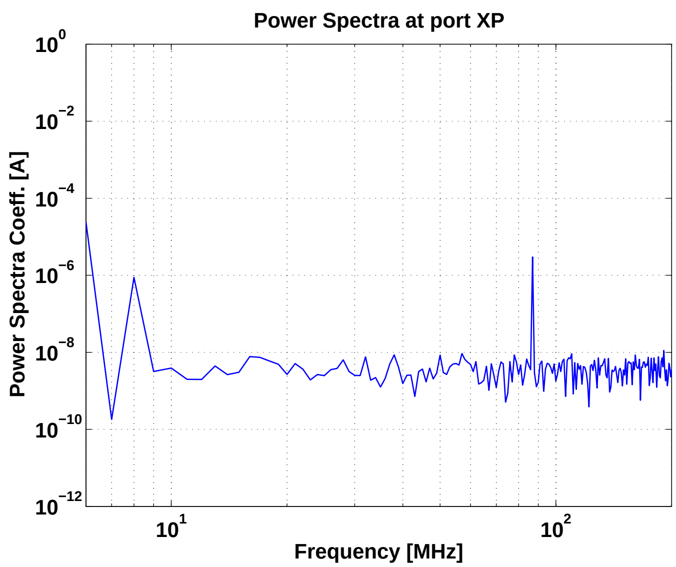

Figure 34: Power Spectra at XP

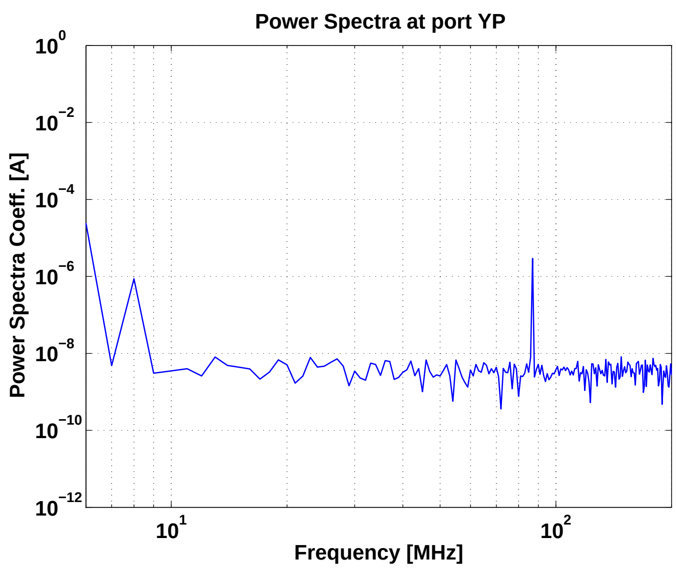

Figure 35: Power Spectra at YP

2.3.3 Modified modulation frequency on Option 1 scenario

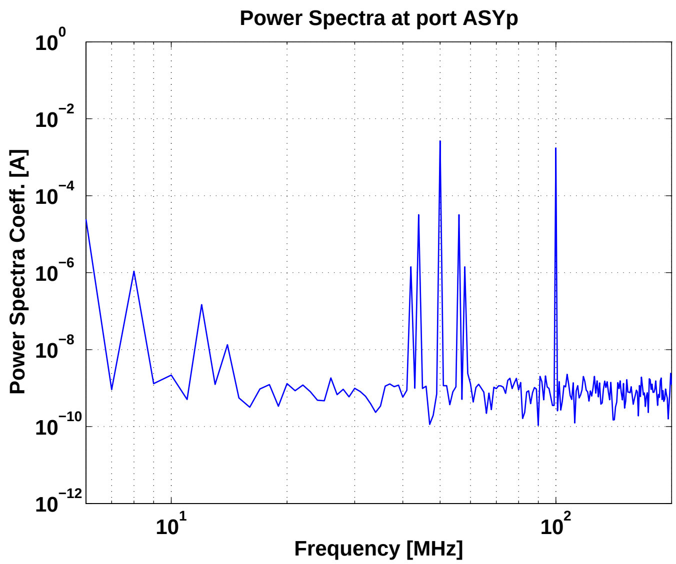

Figure 36: Power Spectra at ASYp

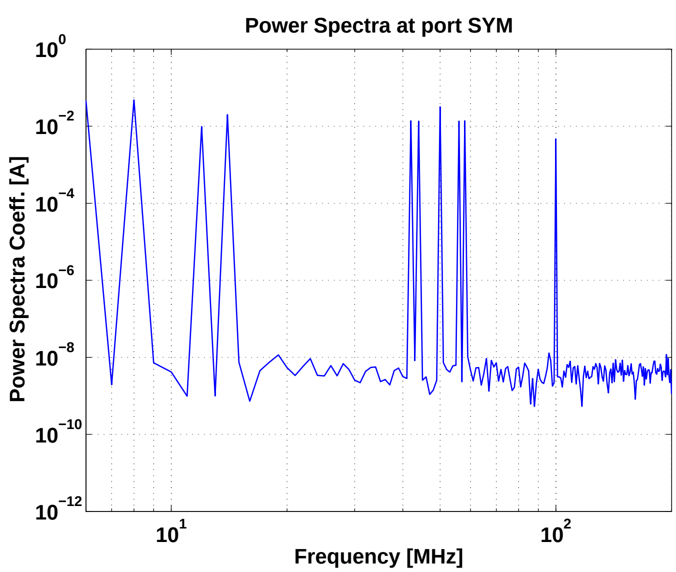

Figure 37: Power Spectra at SYM

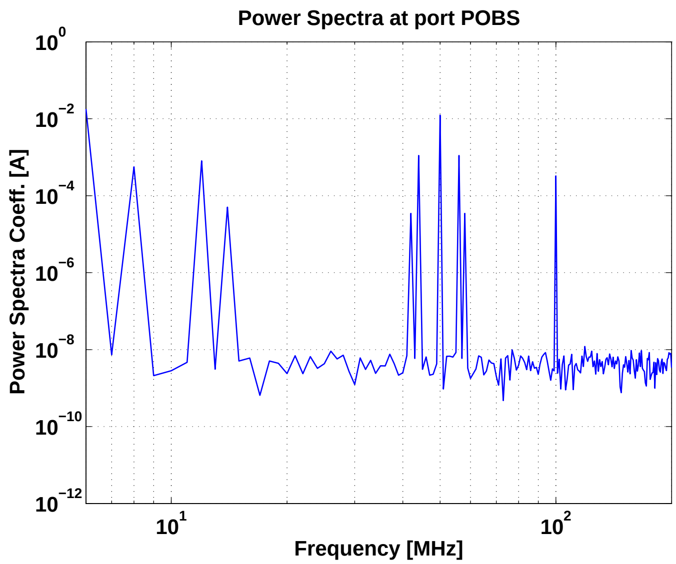

Figure 38: Power Spectra at POBS

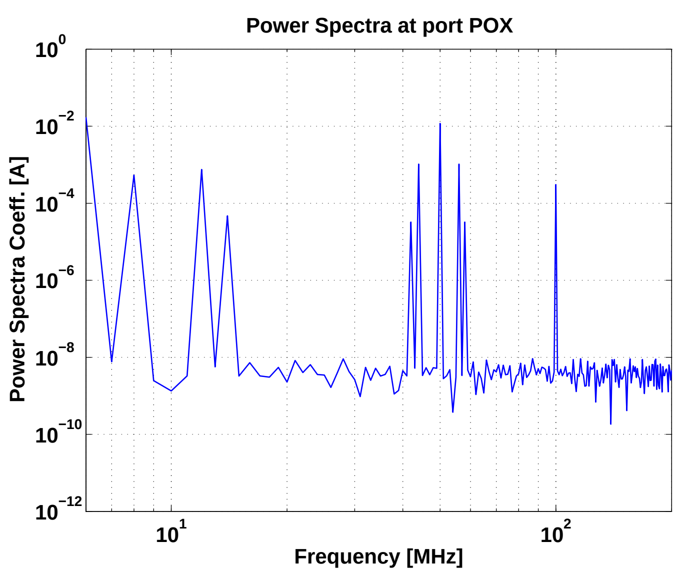

Figure 39: Power Spectra at POX

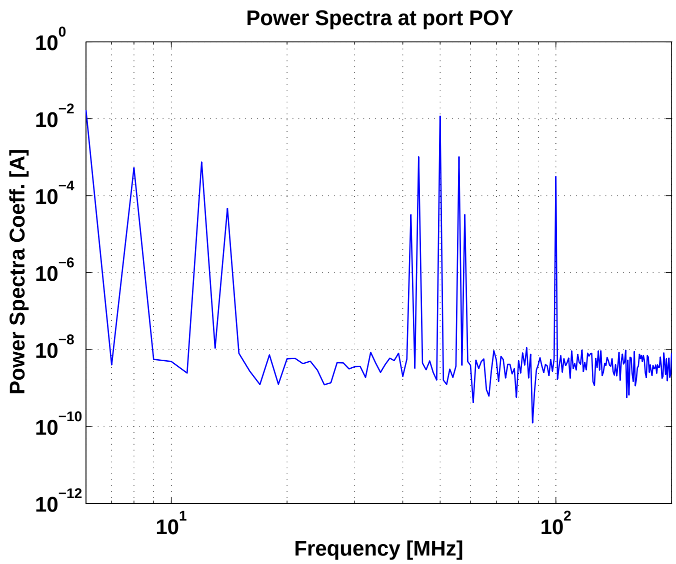

Figure 40: Power Spectra at POY

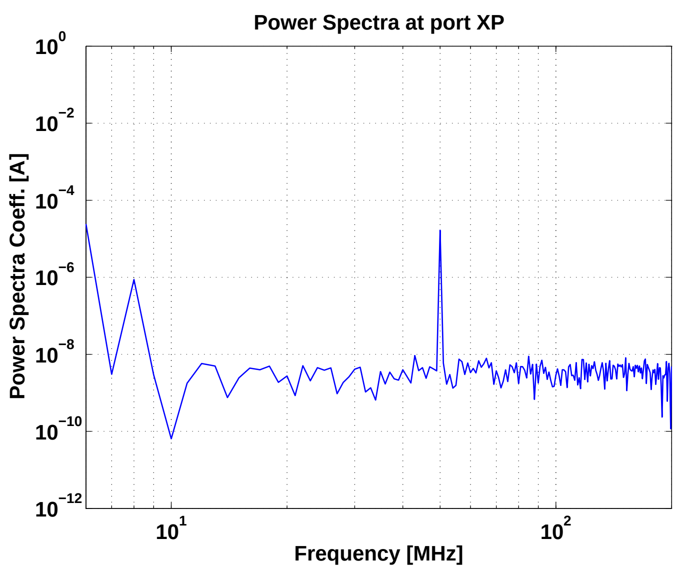

Figure 41: Power Spectra at XP

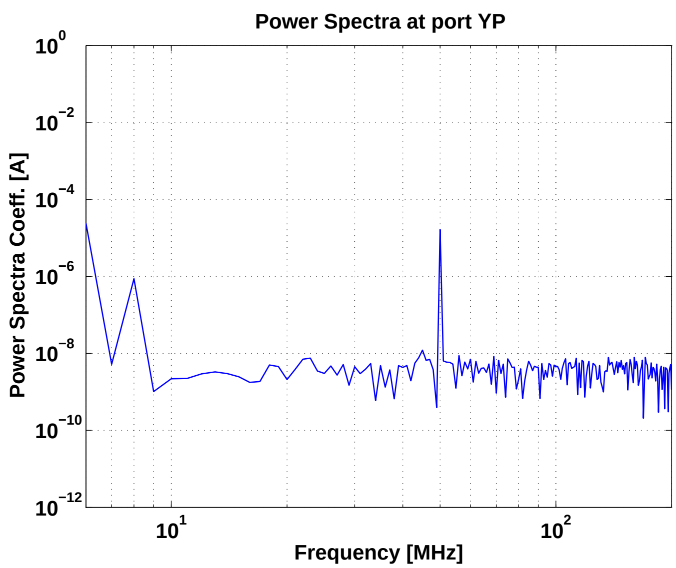

Figure 42: Power Spectra at YP

σ_{SN} [pA/$\sqrt{\text{Hz}}$]	ASYp	SYM	POBS	POX	POY	XP	YP
Option 1	31.9	152.9	151.9	150.2	149.8	132.4	132.0
Option 2	15.2	160.7	151.2	149.6	149.2	132.4	132.0

Table 3: Expected level of shot noise current in pA/$\sqrt{\text{Hz}}$ at each port of the interferometer for option scenario 1 & 2

2.4 Expected shot noise at each port

The shot noise current is easily calculated once the photocurrent has been generated. Values are summarized in table 3 for both scenarii.

3 Final remarks

Given all those quantities, a couple of remarks should be adressed. First, noticeable differences exist between the amount of power expected on the ASYp port and others. The ASYp should receive a power of about 1 mW while other ports could expect 100 mW or more. Thus, it is not clear yet if a unique amplification solution could fit such different situations. A dedicated design might be needed in order to meet noise and SNR specifications. Then, the frequency bandwidth needed remains the main issue. The solution of a modulation frequency $f_2 \sim 50\,\text{MHz}$ does release a lot the constraint on the electronic design.

A Matlab function

A.1 Calling function

```
% This script calls the function simSignal with the ad'hoc parameters
%
% last modified : 06/07/2010
%
% Author : A. Belletoile

%%% Values simulated extracted from VIR-068A-08
% from top to bottom in coeff :ASY SYM POBS POX POY XP YP

%%% Amplitude coefficient to keep incident power on phd below 100mW
ampl = [0.010 0.030 0.751 0.701 0.697 0.020 0.020];

%%% Sampling Frequency
Fe =1e9;

optionSet = 1;

switch optionSet
    case 1 % Option 1 scenario

    f1 = 6e6;
    f2 = 81e6;
    f3 = 8e6;
    coeff(1,:)=[ 9.28e-2 1.37e-5 1.32e-5 1.59e-1 1.58e-1 2.50e-8 2.80e-8];
    coeff(2,:)=[ 1.65e-0 2.97e-1 2.97e-1 1.41e-1 1.45e-1 3.09e-1 3.09e-1];
    coeff(3,:)=[ 1.21e-1 9.77e-4 9.78e-4 8.80e-4 1.79e-4 9.59e-7 9.60e-7];
    coeff(4,:)=[ 1.27e-1 9.77e-4 9.78e-4 8.80e-4 1.79e-4 9.60e-7 9.61e-7];
    coeff(5,:)=[ 1.27e-1 9.79e-4 9.78e-4 2.08e-4 8.08e-4 9.56e-7 9.55e-7];
    coeff(6,:)=[ 3.54e-0 8.58e-8 8.59e-8 7.74e-8 1.57e-8 1.13e-10 1.13e
        -10];
    coeff(7,:)=[ 3.52e-0 8.60e-8 8.59e-8 1.83e-8 7.10e-8 1.12e-10 1.12e
        -10];

    case 2 % Option 2 scenario

    f1 = 6e6;
    f2 = 87e6;
    f3 = 8e6;
    coeff(1,:)=[ 9.28e-2 1.59e-5 1.59e-5 1.24e-4 1.10e-4 3.38e-8 4.01e-8];
    coeff(2,:)=[ 1.65e-0 2.97e-1 2.97e-1 3.08e-1 3.08e-1 3.09e-1 3.09e-1];
    coeff(3,:)=[ 1.21e-1 9.78e-4 9.78e-4 5.01e-7 8.79e-8 9.60e-7 9.62e-7];
    coeff(4,:)=[ 1.27e-1 9.78e-4 9.79e-4 5.01e-7 8.79e-8 9.61e-7 9.62e-7];
    coeff(5,:)=[ 1.27e-1 9.79e-4 9.78e-4 8.35e-8 4.84e-7 9.54e-7 9.53e-7];
    coeff(6,:)=[ 3.54e-0 8.59e-8 8.59e-8 2.60e-9 4.57e-10 1.13e-10 1.13e
        -10];
    coeff(7,:)=[ 3.52e-0 8.59e-8 8.59e-8 4.34e-10 2.51e-9 1.12e-10 1.12e
        -10];

    case 3 % Modified value of F2 with option 1 scenario

    f1 = 6e6;
    f2 = 50e6;
```

```
    f3 = 8e6;
    coeff(1,:)=[ 9.28e-2 1.37e-5 1.32e-5 1.59e-1 1.58e-1 2.50e-8 2.80e-8];
    coeff(2,:)=[ 1.65e-0 2.97e-1 2.97e-1 1.41e-1 1.45e-1 3.09e-1 3.09e-1];
    coeff(3,:)=[ 1.21e-1 9.77e-4 9.78e-4 8.80e-4 1.79e-4 9.59e-7 9.60e-7];
    coeff(4,:)=[ 1.27e-1 9.77e-4 9.78e-4 8.80e-4 1.79e-4 9.60e-7 9.61e-7];
    coeff(5,:)=[ 1.27e-1 9.79e-4 9.78e-4 2.08e-4 8.08e-4 9.56e-7 9.55e-7];
    coeff(6,:)=[ 3.54e-0 8.58e-8 8.59e-8 7.74e-8 1.57e-8 1.13e-10 1.13e
        -10];
    coeff(7,:)=[ 3.52e-0 8.60e-8 8.59e-8 1.83e-8 7.10e-8 1.12e-10 1.12e
        -10];
end


for i=1:7
    [Iop,DSPIop,fIop,sigmaSN] = simSignal(f1, f2, f3, sqrt(coeff(i,:)),
        ampl(i),i,Fe);
end
```

A.2 Computing function

```
function [Iph,DSPIph,fIph,sigmaSN] = simSignal(f1, f2, f3, coeff, ampl,
    name, Fe)
%SIMSIGNAL.M
%
% This function compute the triple phase modulated signal received by a
% photodiode given the RF component amplitude in coeff, the modulation
% frequencies f1, f2, f3,  an amplitude factor ampl, the name of the
% considered port name and the sampling frequency Fe.
%
% last modified : 06/07/2010
%
% Author : A. Belletoile

switch name
    case 1
        namefig = 'ASYp';
    case 2
        namefig = 'SYM';
    case 3
        namefig = 'POBS';
    case 4
        namefig = 'POX';
    case 5
        namefig = 'POY';
    case 6
        namefig = 'XP';
    case 7
        namefig = 'YP';
end

%%% RF components
carr = coeff(1);
lsb1 = coeff(2);
usb1 = coeff(3);
lsb2 = coeff(4);
usb2 = coeff(5);
lsb3 = coeff(6);
usb3 = coeff(7);

%%% time vector
N = 1e6;  % T = N / Fe
t = (1:N) / Fe;

%%% frequency set
w1 = 2*pi*f1;
w2 = 2*pi*f2;
w3 = 2*pi*f3;

%%% Compute optical power Popt
K = carr^2 +usb1^2 + lsb1^2 + usb2^2 + lsb2^2 + usb3^2 + lsb3^2;

Popt = K -2 * carr * (usb1 + lsb1) * sin(w1.*t) - 2 * carr * (usb2 + lsb2)
    * sin(w2.*t) - 2 * carr * (usb3 + lsb3) * sin(w3.*t) ...
```

```
    - 2 * usb1 * lsb1 *cos(2*w1.*t) - 2 * usb2 * lsb2 *cos(2*w2.*t) - 2 *
        lsb3 * usb3 *cos(w3.*t) ...
    + 2 * (usb1 * usb2 + lsb1 * lsb2) * cos((w2-w1).*t) - 2 * (usb1 *lsb2+
        lsb1*usb2) * cos((w1+w2).*t)...
    + 2 * (usb1 * usb3 + lsb1 * lsb3) * cos((w3-w1).*t) - 2 * (usb1 *lsb3+
        lsb1*usb3) * cos((w1+w3).*t)...
    + 2 * (usb3 * usb2 + lsb3 * lsb2) * cos((w2-w3).*t) - 2 * (usb3 *lsb2+
        lsb3*usb2) * cos((w3+w2).*t);

%%% Reduce optical power so less than 100mW inping on photodiodes
%%% given in VIR-068A-08
Popt = ampl * Popt;

%%% Convert optical power into current (W-->A)
eta = 0.9;
lambda = 1.064;

Iph = eta * lambda * Popt / 1.2375;

%%% Add shot noise
B = 0.5 * Fe;
sigmaSN = sqrt(2*1.6e-19*mean(Iph)*B);
noise = sigmaSN * randn(1,N);

Iph = Iph + noise;

[DSPIph,fIph]=periodogram(Iph,[],1000,Fe); %DSP in [IphUnit^2/Hz]
PSIph = DSPIph * Fe / N;

%%% Plot
figure
semilogy(fIph/1e6, sqrt(PSIph),'LineWidth',1)
set(gca, 'FontSize', 16, 'FontWeight', 'bold')
title(['Power Spectra at port ' namefig])
xlabel('Frequency [MHz]')
ylabel('Power Spectra Coeff. [A]')
axis([-1 200 1e-12 1])
grid

%%% Uncomment to save figure
%eval(['print -depsc2 -tiff PS_' namefig '_newFreq.eps']);
% eval(['print -dpng PS_' namefig '.png']);
close

% figure
% semilogy(fIph/1e6, sqrt(DSPIph),'LineWidth',1)
% set(gca, 'FontSize', 16, 'FontWeight', 'bold')
% title(['Power Spectral Density at port ' namefig])
% xlabel('Frequency (MHz)')
% ylabel('Power Spectral Density [A/sqrt(Hz)]')
% axis([-1 200 1e-15 1e-2])
% grid
% % eval(['print -depsc2 -tiff PSD_' namefig '.eps']);
% % eval(['print -dpng PSD_' namefig '.png']);
% close
```

References

[1] G. Vajente, "Simulation of Advanced Virgo Length Sensing and Control System", VIR-068A-08, 2008

[2] G. Heinzel, A Rüdiger and R. Schilling, "Spectrum and spectral density estimation by the Discrete Fourier Transform (DFT), including a comprehensive list of window functions and some new flat-top windows", Max Planck Institute für Gravitationsphysik, sciops.esa.int, 2002