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Cavity alignment sensitivity to shift and tilt

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1 Introduction

This note presents the work done for the alignment of the North Arm in order to verify experimentally the theoretical predictions. The tilt / shift coordinate basis was expected to be the most convenient for the drift control because they are eigenvectors of the cavity alignment. Moreover, the alignment is expected, due to the Advanced Virgo arm cavity design, to be much more sensitive to the tilt than to the shift. A Finesse simulation has been done evaluating the transmitted power behavior as a function of the misalignment of the cavity mirrors. Here we compare the results of the simulation with the equivalent measurement made on a real cavity (the Advanced Virgo North arm cavity). Finally the matrix that relates the individual mirror alignment coordinates with the cavity eigenvectors in real units is presented.

1.1 Cavity axis coordinates

A misalignment can occur by changing the input beam and/or the cavity axis direction. There are four angular degrees of freedom for each (beam and cavity axis): two tilts, pitch (θ_x) and yaw (θ_y), and two translations, x and y. Here only the dof regarding the cavity in the pitch direction will be considered.
The cavity axis direction is determined by the mirrors alignment which is subject to drift, whose longitudinal microscopical positions are actively controlled to maintain the cavity in resonance. A change on the orientation of one mirror will modify the cavity axis direction both in tilt and in translation (shift). However, from the control point of view it is more convenient to work in a coordinate system that decouples the two DOF to be controlled. For this reason we want to change from the mirror basis to the tilt / shift which are the eigenvectors of the optical cavity alignment. Due to the high finesse of the cavity the allowable mirror misalignments to maintain the cavity locked are in the micro-radians regime, thus in the following we will use the *small angle* regime.

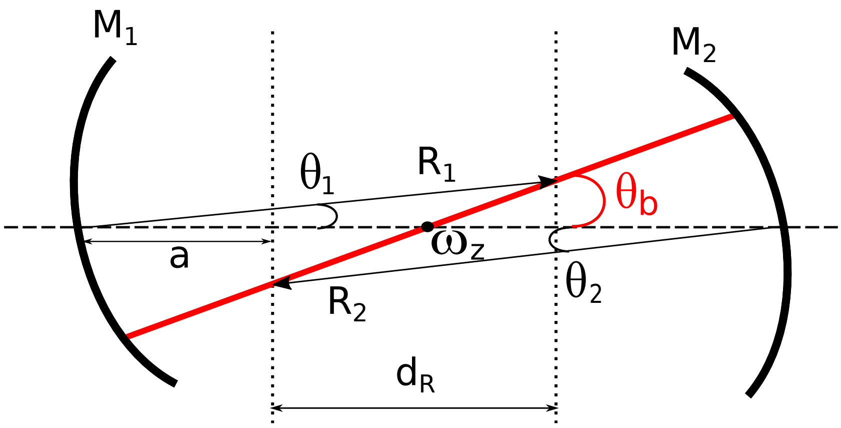

Figure 1: Scheme of a cavity with its optical axis tilted due to the cavity mirrors misalignment.

First we will study the tilt case. The equation of the optical axis can be written as a function of the optical parameters of the cavity. We are going to consider as zero-coordinate the position of the input mirror, as seen in Figure 1. We will write some useful parameters:

- Distance between mirrors center of curvature (d_R):

$$d_R = R_1 - a = R_1 - (L - R_2) \tag{1.1}$$

- Beam waist position (ω_z):

$$\omega_z = d_R/2 + a = \frac{R_1 - R_2 + L}{2} \tag{1.2}$$

- Optical axis slope (m_{oa}):

$$m_{oa} = \frac{R_1\theta_1}{d_R/2} \tag{1.3}$$

- Y-intercept of the optical axis (b_{oa}):

$$b_{oa} \approx -\omega_z \cdot m_{oa} \tag{1.4}$$

At this point we can write the optical axis expression:

$$y = m_{oa}x + b_{oa} = m_{oa}x - m_{oa}\omega_z = m_{oa}\left(x - \omega_z\right) \tag{1.5}$$

In order to know the combination of mirror angular displacement needed to make a pure tilt we solve the optical axis equation for a given point ($x = L - R_2$, $y = \theta_2 R_2$) we obtain the following relationship:

$$\theta_2 = -\frac{R_1}{R_2} \cdot \theta_1 \tag{1.6}$$

This equation shows the relationship between the angular displacement of both mirrors in order to produce a pure tilt of the optical axis.

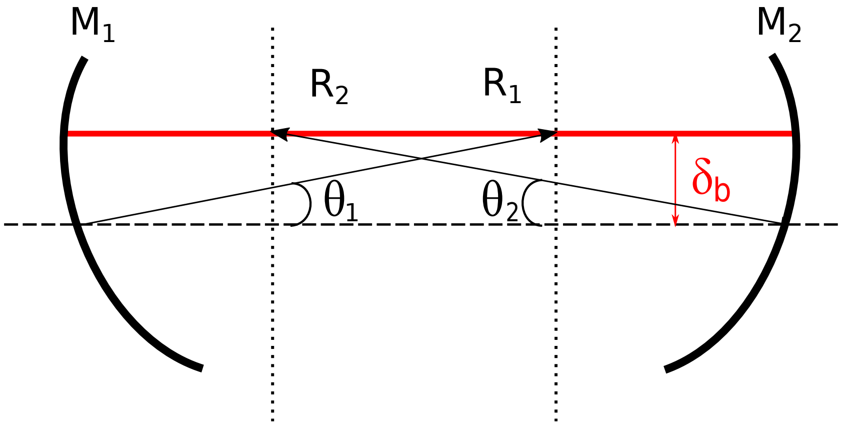

Figure 2: Scheme of a cavity with its optical axis shifted due to the cavity mirrors misalignment.

This reasoning can be repeated for a pure shift of the optical axis. In this case the equation of the optical axis becomes simpler because the slope is zero. A scheme of this configuration is shown in Figure 2. To obtain the relationship between the mirrors angular movement we only need two points of the line:

$$y = \theta_1 \cdot R_1 = \theta_2 \cdot R_2 \tag{1.7}$$

With this information we can build the matrix that changes from the mirror basis to the cavity eigenvectors:

$$\begin{pmatrix} \Theta \\ \Delta \end{pmatrix} = \begin{pmatrix} 1 & -R_1/R_2 \\ 1 & R_1/R_2 \end{pmatrix} \cdot \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

An additional interest on the tilt / shift coordinate system arises when working at high power. In this case the light beam exerts a torque upon the mirrors also known as *radiation pressure* [1]. This effect is equivalent to an optical spring connecting the two mirrors and modifying its mechanical transfer function. When taking in account radiation pressure the tilt coordinate corresponds to the (+)-mode or hard mode because it increases the frequency of the pendulum, making the system stiffer. On the other hand, the shift coordinate corresponds to the (-)-mode or soft mode because it decreases the frequency, making the system softer [2].

2 Finesse Simulation

A simulation of the North Arm alignment has been made using Finesse [3]. Starting from the ideal alignment, a scan of the misalignment on θ_x of both mirrors has been done, from 5μrad to -5μrad taking 25 points for each mirror. The objective was to observe the evolution of the transmitted power as a function of the misalignment which can be seen in Figure 3. Notice that only θ_x has been simulated, because we expect an equivalent behavior on θ_y. In this coordinate system both degrees of freedom are very coupled. The ellipse axis represent

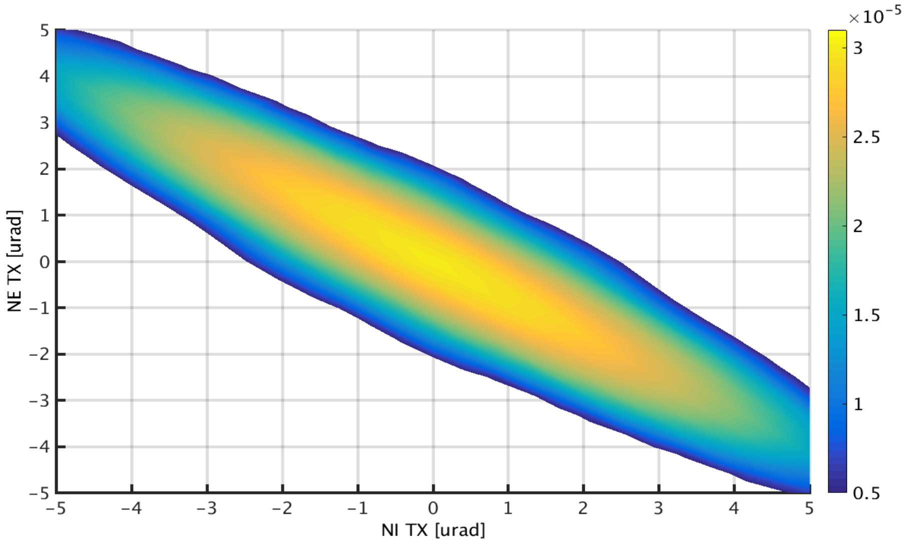

Figure 3: Finesse simulation result of the transmitted power of north arm as a function of the mirrors misalignment on θ_x.

the tilt and the shift coordinates, so it can be seen already that one is much more sensitive than the other. The ratio between them can be estimated by comparing the misalignment needed for one mirror to lose a given amount a power while doing a pure tilt or a pure shift. As seen on Figure 3 the ΔNI needed to decrease the power to $2.5 \cdot 10^{-5}$W with a pure tilt is $\sim$1.2μrad and with a pure shift is $\sim$6μrad. The total ratio Shift/Tilt is then $\sim$5. Notice that there is an uncertainty on this calculation due to the limited number of points simulated.

Now the same plot can be represented in the coordinate system tilt / shift mentioned in the previous section, see Figure 4.

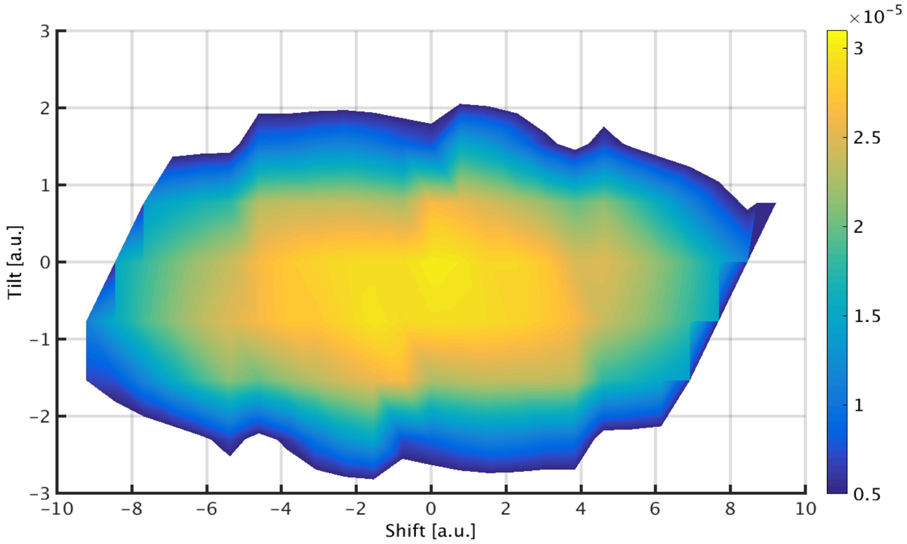

Figure 4: Finesse simulation result of the transmitted power of north arm on the tilt / shift basis.

It is clear now that the cavity is more sensitive to a tilt than to a shift. From the control point of view this means that the most critical degree of freedom is the tilt of the cavity, which should be controlled first. Notice as well that the tilt and the shift are in arbitrary units. This is because in order to obtain information on the coupling of the degrees of freedom and how to control them we are interested in comparable magnitudes. As the actuation is sent to the mirrors, the relevant magnitudes to compare are θ_1 and θ_2.

2.1 Calibration

Once the ratio between dofs has been established it can be useful to calibrate in SI units the tilt / shift coordinates system. Starting with the tilt, the calibration can be derived from the slope of the optical axis equation for a pure tilt (Equation 1.3). For a θ_1 of 1 μrad of the North Input mirror, the optical axis of the North Arm cavity is tilted by:

$$\theta_b = \theta_1 \cdot \frac{2 \cdot R_1}{R_1 - L + R_2} = 27.6 \cdot \theta_1 \quad [\mu rad] \tag{2.1}$$

The calibration factor for a pure shift can be derived from Equation 1.7instead. For a θ_1 of 1 μrad of the North Input mirror the optical axis of the North Arm cavity is shifted by:

$$\delta_b = \theta_1 \cdot R_1 = 1.42 \cdot \theta_1 \quad [mm] \tag{2.2}$$

Now the change of basis matrix can be written in SI units:

$$\begin{pmatrix} \Theta_{cal}\ [\mu rad] \\ \Delta_{cal}\ [mm] \end{pmatrix} = \begin{pmatrix} 27.6 & -27.6 \cdot R_1/R_2 \\ 1.42 & 1.42 \cdot R_1/R_2 \end{pmatrix} \cdot \begin{pmatrix} \theta_1\ [\mu rad] \\ \theta_2\ [\mu rad] \end{pmatrix}$$

This allow us to calibrate Figure 4:

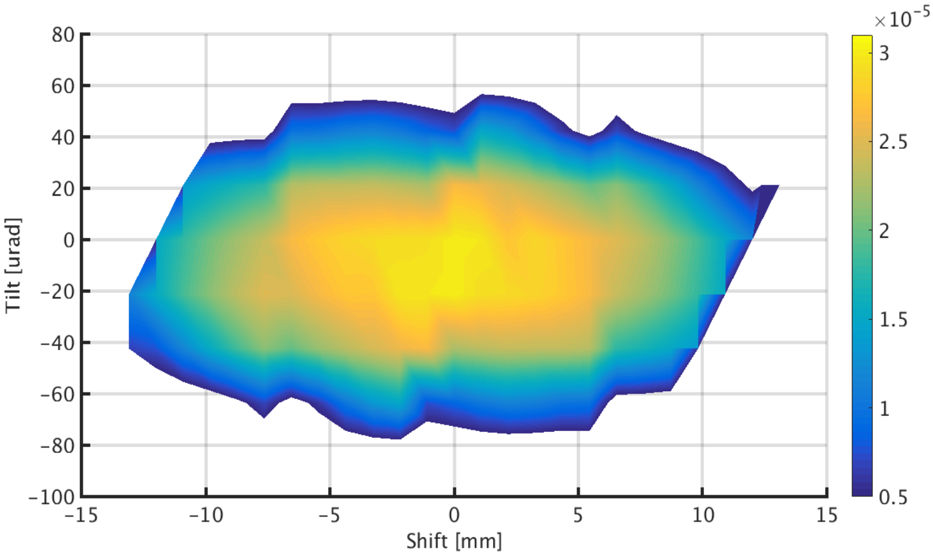

Figure 5: Finesse simulation result of the transmitted power of north arm on the tilt / shift basis having the tilt calibrated in micro-radians and the shift in millimeters.

3 Experimental measurements

This behavior has been reproduced in using the Advanced Virgo North arm cavity in order to check that it behaves as predicted by theory. Starting from an aligned state with the cavity un-locked, the evolution of the transmitted power resonances or "flashes" has been studied as a function of the misalignment of both mirrors. In order to do so, an angular line has been added to the θ_x of the North End mirror at the level of the marionetta, with an amplitude of $\pm 8\mu$rad and at a frequency of 20 mHz. The frequency was chosen under the resonances of the mirror pendulum (600 and 800 mHz) in order to avoid the attenuation due to the mechanical transfer function which decreases as $1/\mathrm{f}^2$ above the resonances. Otherwise, in order to make such an important angular excitation, the correction sent would be too high.
At the same time the θ_x of the North Input mirror has been changed by applying a slow ramp at the marionetta (several minutes to do $\pm 35\mu$rad). This action was repeated for different alignments of NE θ_x (always with the angular line on) until the misalignment was too important to see any flash on the B7 photodiode. When this happened, the mirrors were repositioned in order to find an alignment that allowed to have light resonating on the cavity. This process was iterated in order to make several complete scans of the cavity alignment. An example of this scan is shown on Figure 6.

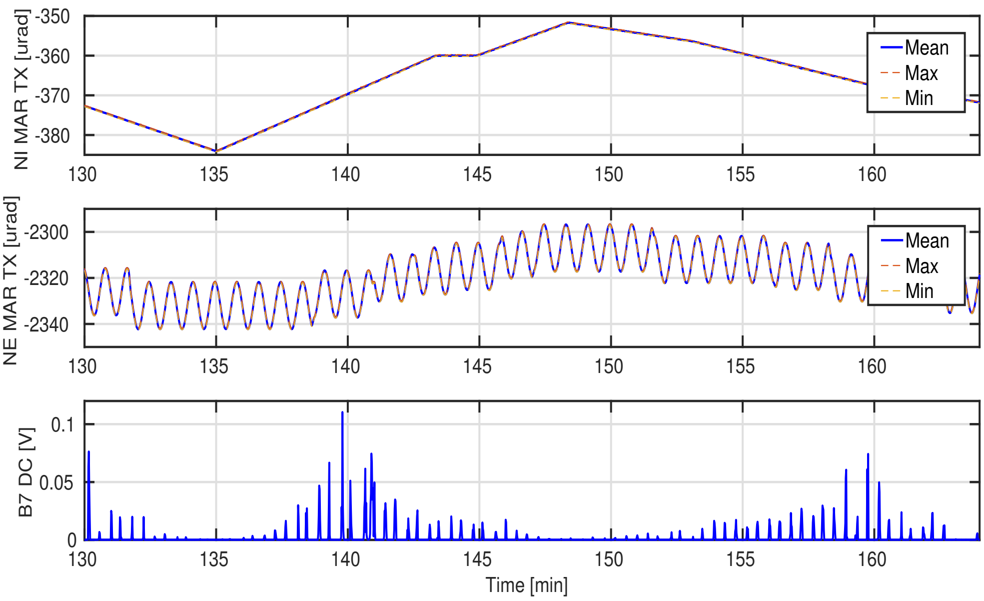

Figure 6: Scan of the transmitted power of north arm by misaligning the NE and NI mirrors on θ_x by acting on the marionetta. The data shown are sampled at 1Hz.

With the collected data, an analogue plot to Figure 3 can be done for the North Arm cavity of Advanced Virgo. The analysis has been made with the trend data (stored at 1Hz), and in order to have more statistics the mean, maximum and minimum values of the mirrors alignment positions have been used. For each alignment position, the maximum power of the transmission peaks of each scan has been taken in account. This means that for each position there were several power values. This way the impact that can have the cavity velocity on the power of the transmission peaks is minimized. The plot has been done by taking the maximum value of all of them and by taking the mean value obtaining an equivalent result for both configurations. Figure **??** shows the configuration where the maximum values are taken.

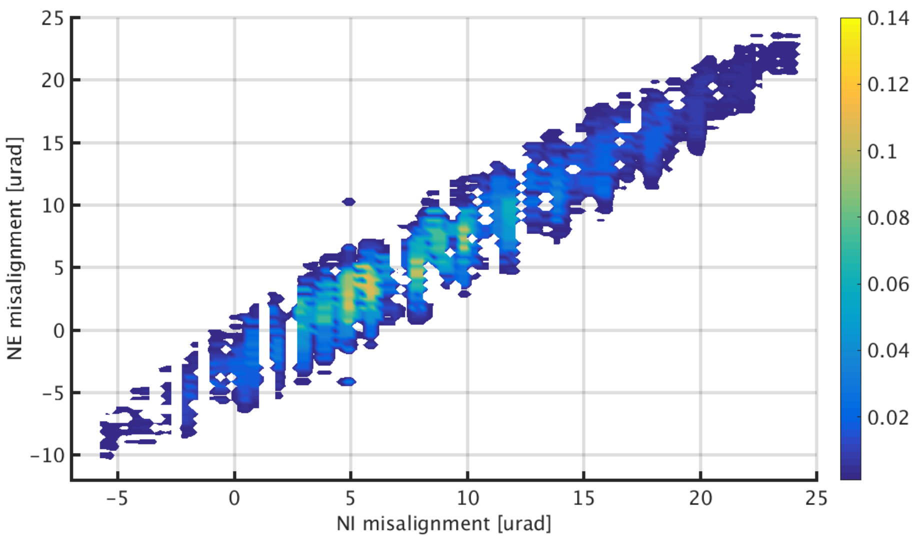

Figure 7: Measurements of the transmitted power of north arm as a function of the mirrors misalignment on θ_x.

It can be seen that the slope of Figures 3 and 7 has opposite sign. This is due to the fact that the direction of the marionetta actuation does not follow the same Finesse convention, although it does not interfere with our analysis because it does not affect the absolute value. As shown in Equations 1.6 and 1.7 the slope of these plots should give us the ratio between both radius of curvature that is used after in order to change basis. The plot on Figure 7 has been fitted to a line with a slope of 0.92 to be compared with the theoretical one which is 0.84. The discrepancy comes from the cross-calibration between the actuators of the NE mirror and the NI mirror which are expected to be identical within a 10% error [4]. Taking in account this uncertainty the experimental measurement is in well agreement with the values predicted by the theory.
Also in this case it can be seen that the cavity is much more sensitive to a tilt than to a shift. The ratio between shift and tilt coordinates can be calculated as done for the simulation: in this case for the power to decrease to 0.03 V it is needed a ΔNI of $\sim 2\mu$rad of pure tilt and $\sim 11\mu$rad of a pure shift. The ratio between them is in this case $\sim$5.5 which is in agreement with the result of the simulations taking into account the actuators calibration uncertainty of 10% previously mentioned.

Using the slope found experimentally the change of basis to tilt / shift has been made and it is shown on Figure 8.

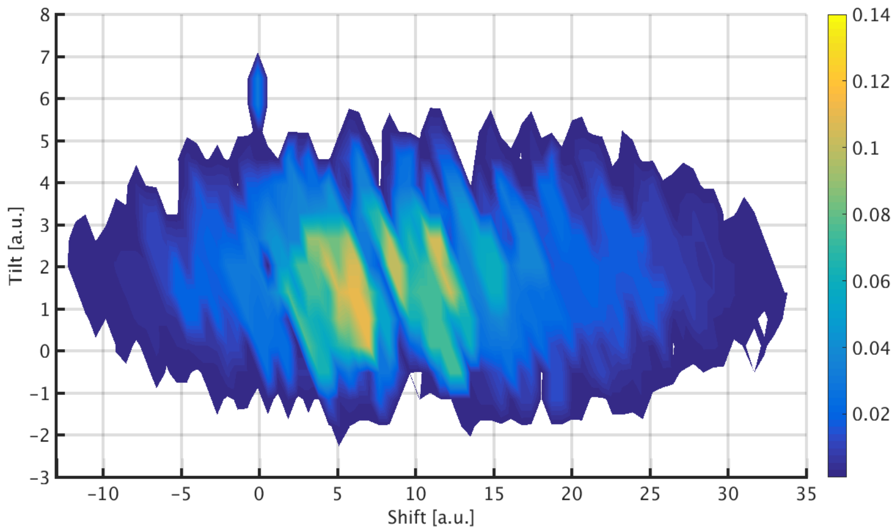

Figure 8: Measurements of the transmitted power of north arm on the tilt / shift basis.

4 Conclusion

The alignment of the North Arm cavity has been simulated, and the evolution of the transmitted power has been shown as a function of two coordinate systems: end / input mirror and tilt / shift of the optical axis. These results have been compared with real measurements and it has been proved that they are well in agreement.
This analysis shows that the most convenient basis for the angular control is the shift/tilt basis because it allows to implement easily the hierarchical control having the tilt controlled with larger frequency bandwidth with respect to the shift since it requires higher control accuracy.

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