Nonlinear waveform reconstruction study

Higher-Order Spectral Analysis (HOSA)

Andrea Cesarini

European Gravitational Observatory (EGO), Cascina, Pisa, Italy

Study

Aim:

Searching for a nonlinear approach for waveform transient signal reconstruction

Supervisor:

Dr. Elena Cuoco

Internship:

Data and Noise Analysis Group

Generalizing spectral analysis

"Cumulants are combination of moments"

1st-order

mean

$$c_1(\tau) = \mathrm{E}[x(t)]$$

2nd-order

correlation spectrum

$$c_2(\tau) = \mathrm{E}\left[x^*(t)x(t+\tau)\right]$$

$$P(f) = \int_{-\infty}^{\infty} c_2(\tau)\; e^{-j2\pi f \tau}\, d\tau$$

Fourier transform of the autocorrelation

The statistics of a Gaussian signal is completely described by using its 1st- and its 2nd-order statistics

3rd-order

triple correlation

bispectrum

$$c_3(\tau_1, \tau_2) = \mathrm{E}\left[x^*(t)x(t+\tau_1)x(t+\tau_2)\right]$$

$$B(f_1, f_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c_3(\tau_1, \tau_2)\; e^{-j2\pi(f_1\tau_1 + f_2\tau_2)}\, d\tau$$

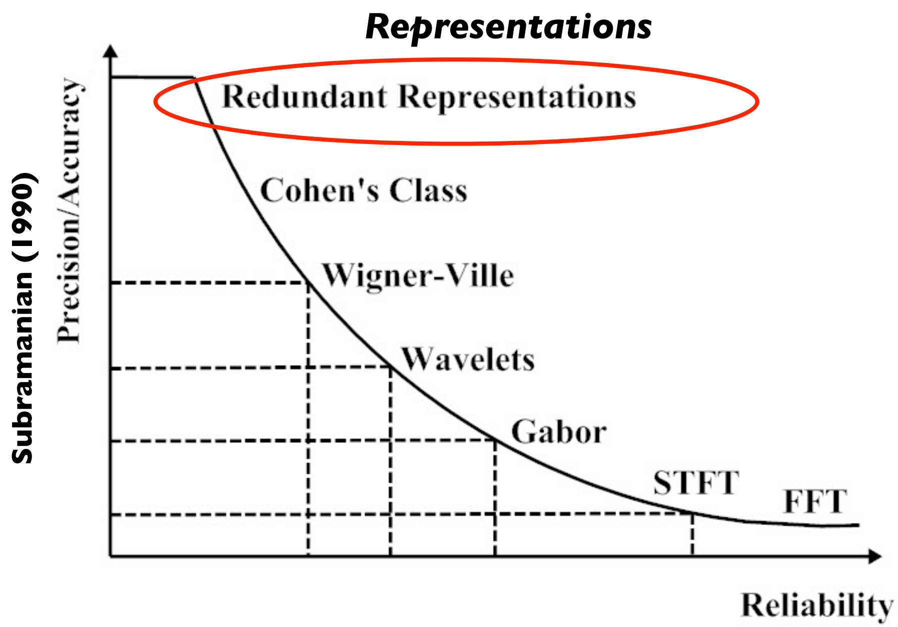
2nd-order Fourier transform

Higher-Order Spectral Analysis (HOSA)

Higher-order statistics do not maintain memory of gaussianity (filtering).

Higher-order spectral analysis (HOSA)

Generalized (non Gaussian) signals



The spectral density is a function of frequency and not a function of time. However, the spectral density of small "windows" of a longer signal may be calculated and plotted versus the time associated with the window. The figure shows the variations in the accuracy and reliability of these techniques (Subramanian, 1990).

- Gaussian noise suppression
- Study of dataset quasi-Gaussianity
- Nonlinear frequency coupling

Higher-order cumulants (>2) of pure Gaussian signals are zero and contain redundant information about the deterministic portion of the signals.

In real cases when signals are heavily corrupted, averaging the redundant information is possible to treat the noise.

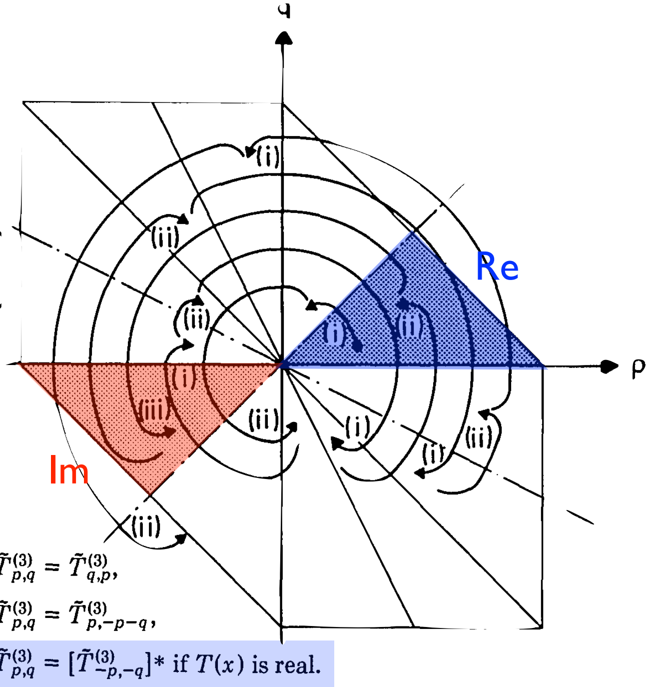
1D Spectrum

Phase coupling associated with nonlinearities cannot be correctly identified

Bispectrum Analysis

Discrete time treatment

Bispectrum redundant information ("Polyspectra")



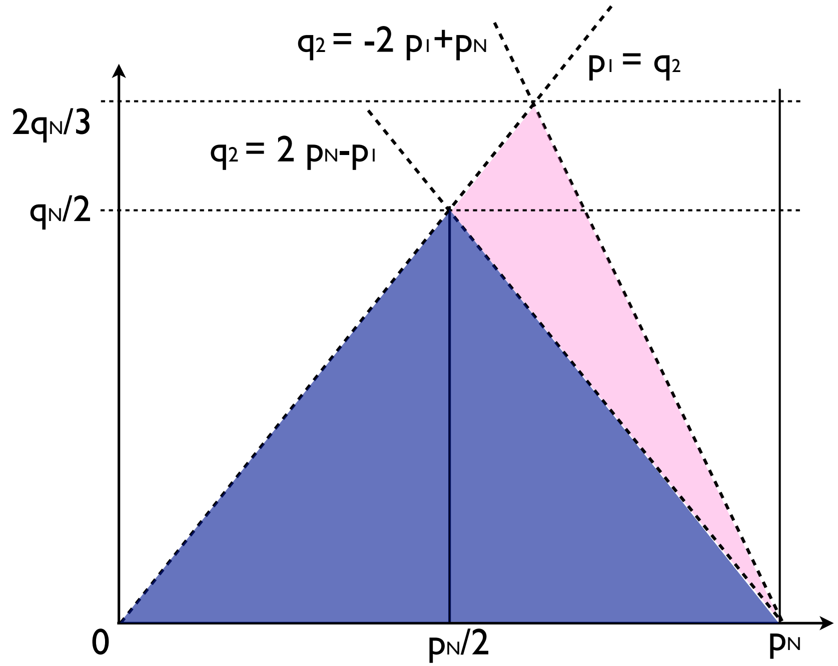
Adapted from Bartelt et al., (1984)

- Bispectrum symmetries of a band-limited signal.

- For a real signal the bispectrum is completely determined by a single octant (blue).

- For a complex signal the symmetry relation (iii) is not applicable and a second octant is necessary in order to determine the bispectrum (red).

Stationarity vs. Nonstationarity



- The isosceles triangle (blue) is a consequence of the stationary and real signal and the symmetry properties.

- A nonzero extra triangle (pink) is a direct consequence of nonlinearity.

Bispectrum Analysis (2)

Indirect bispectral estimate

In practical cases (time-limited and band-unlimited signals), the bispectrum is computed by using the Fourier transform of the triple correlation (indirect estimate). The estimated cumulant is convolved by a 2D window function in order to obtain an improved estimation for the bispectrum of the signal.

Several windowing functions have been developed. Here we test the Parzen, the Sasaki and the Mean Squared Error (MSE) optimal Rao-Gabr windows.

Window	Index			
	J	**V**	**B**	**E**
Daniell	99468.5	0.1199	8990	1078.5
Parzen	8392.43	0.0409	1324.78	54.2
Hamming	60664.8	0.9067	6261.80	567.76
Priestley	288002	0.2032	10909.3	2216.91
Sasaki	1315.2	0.0486	2007.43	97.29
MSE optimal	2220.74	0.0691	458.69	31.68

Oroian et al., 2008

Rao-Gabr Optimal window

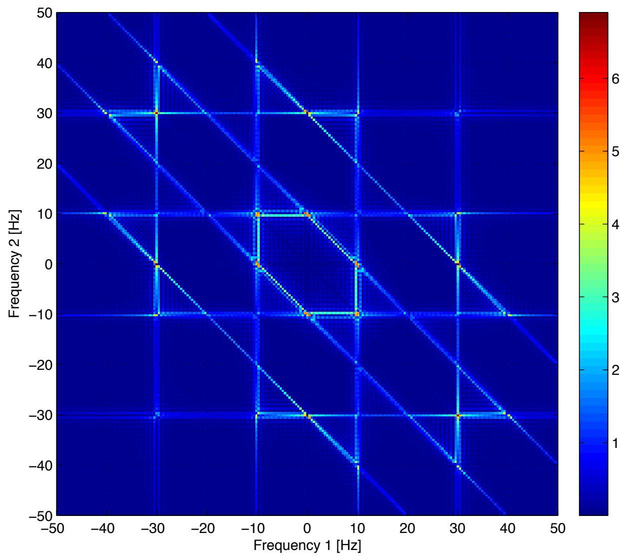
bias~J
variance~V

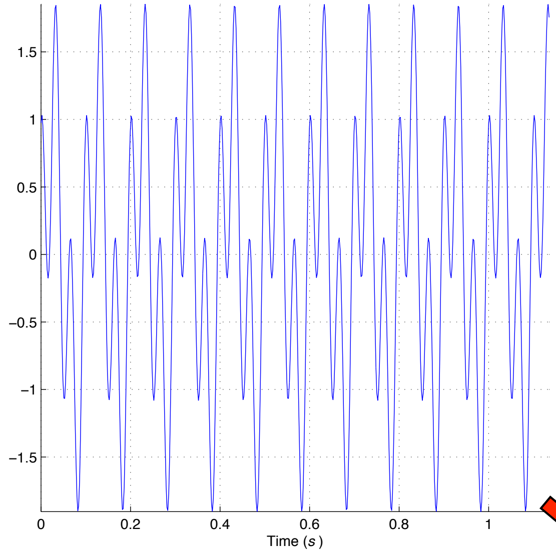
MSE~E=VxB

Bispectrum Analysis (2)

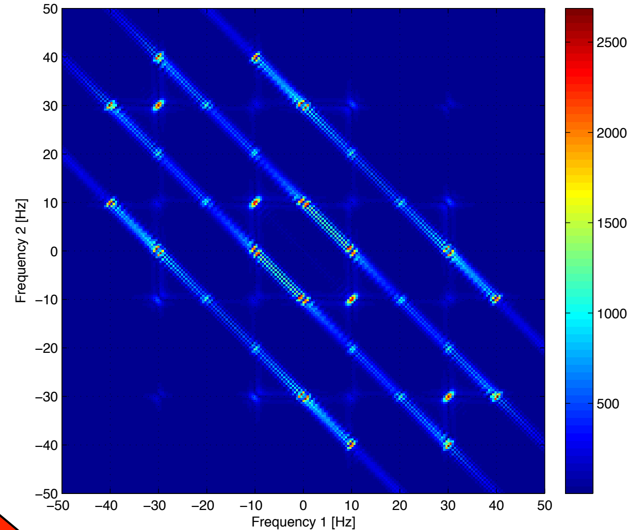
Windowing: sin10Hz-30Hz

None

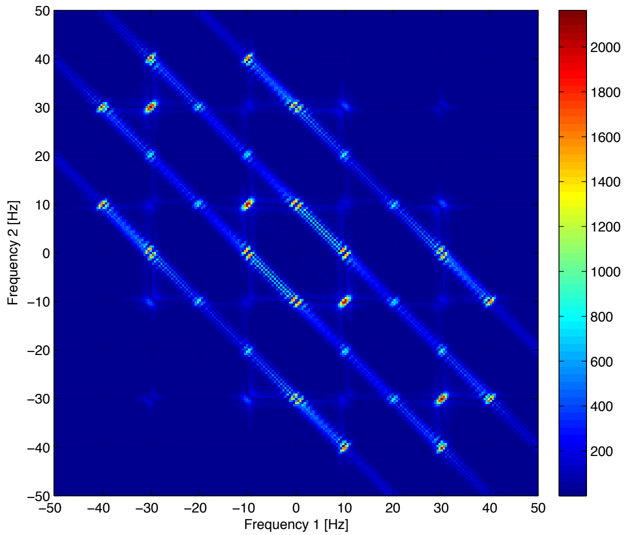


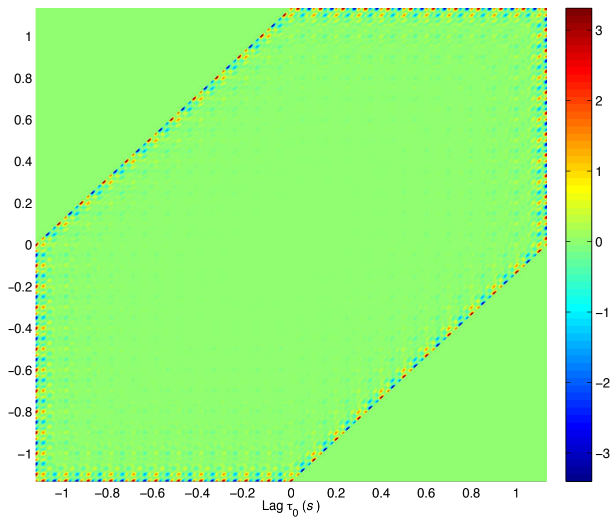


Sasaki

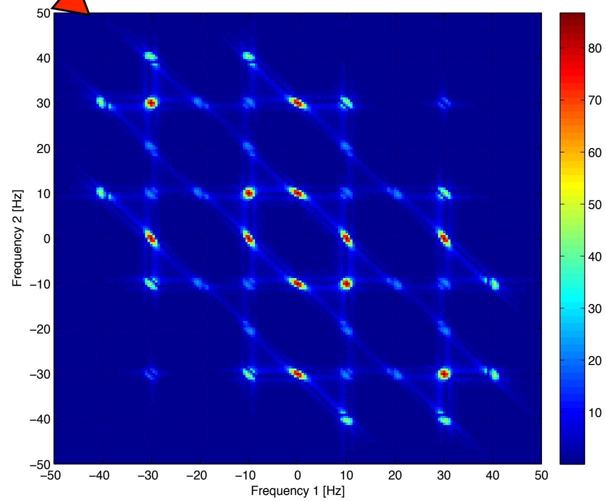


Parzen





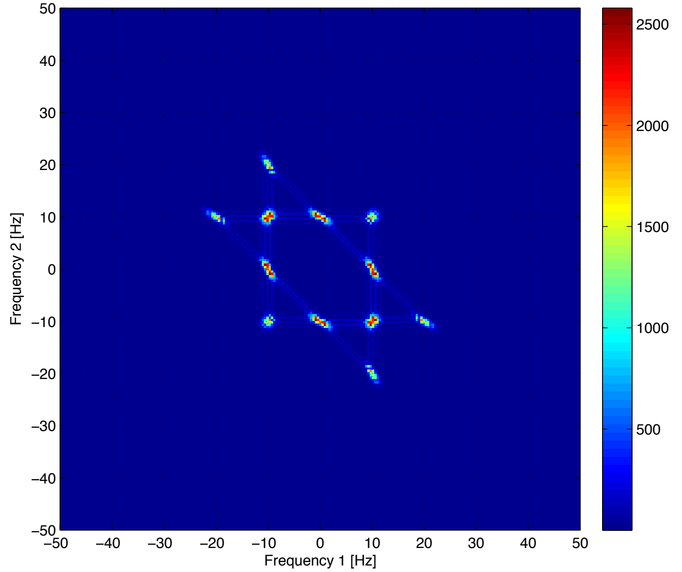
Rao & Gabr
Optimal window (MSE)

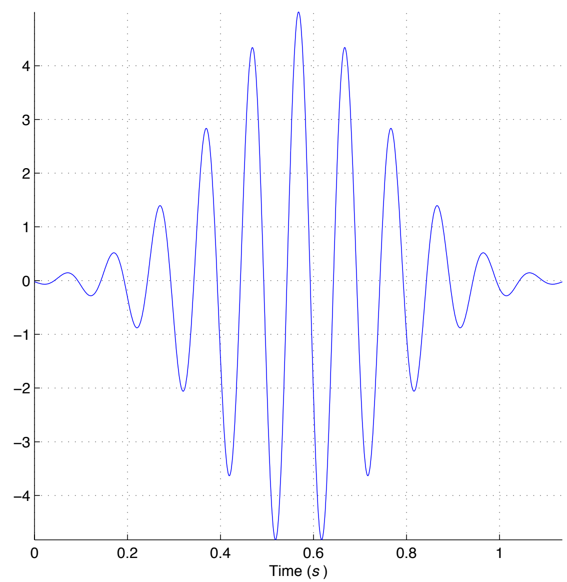


Bispectrum Analysis (3)

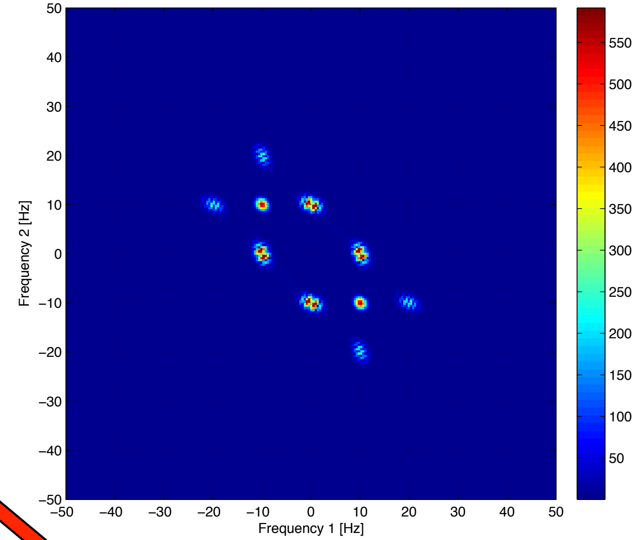
Windowing: SG10Hz

None

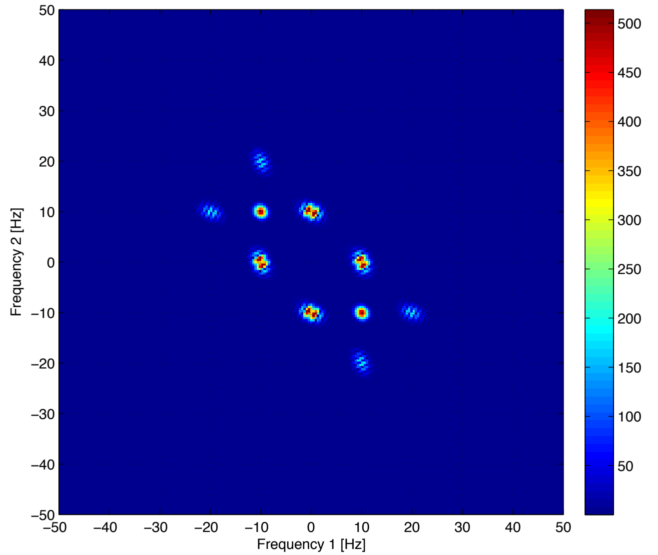


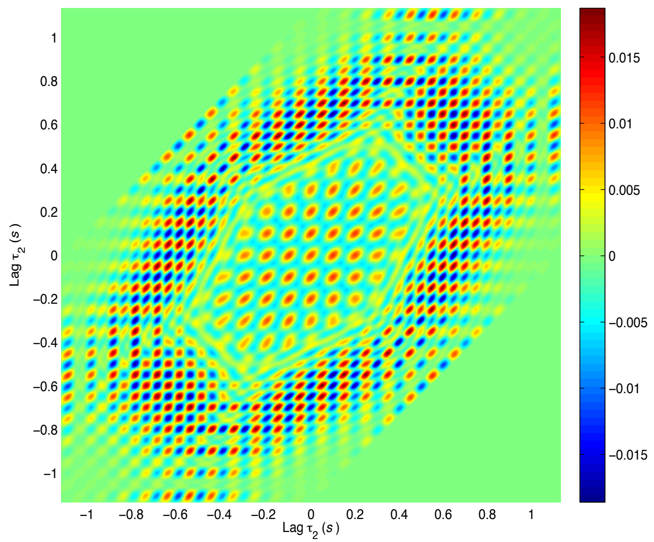


Sasaki



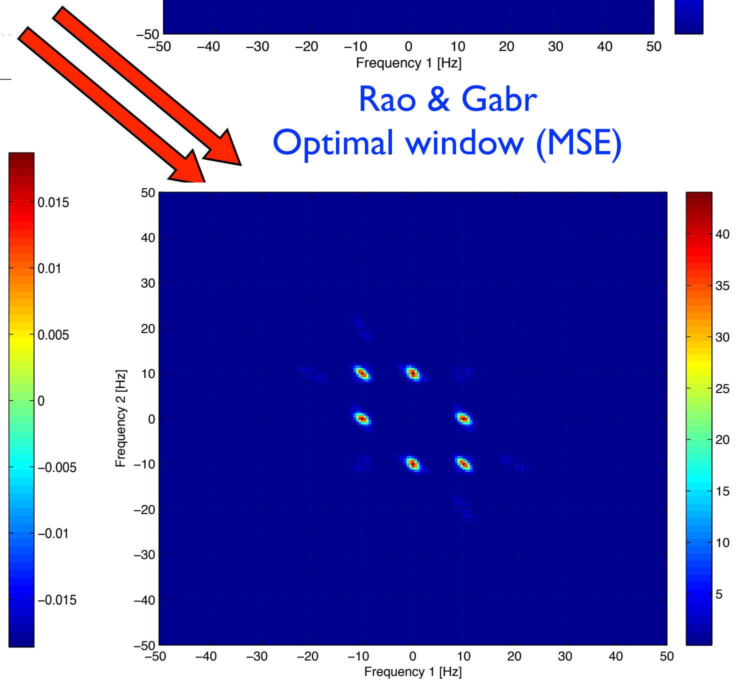
Parzen





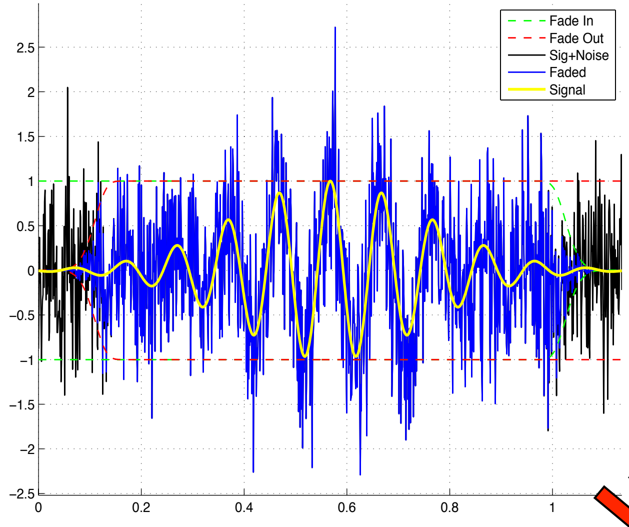
Rao & Gabr

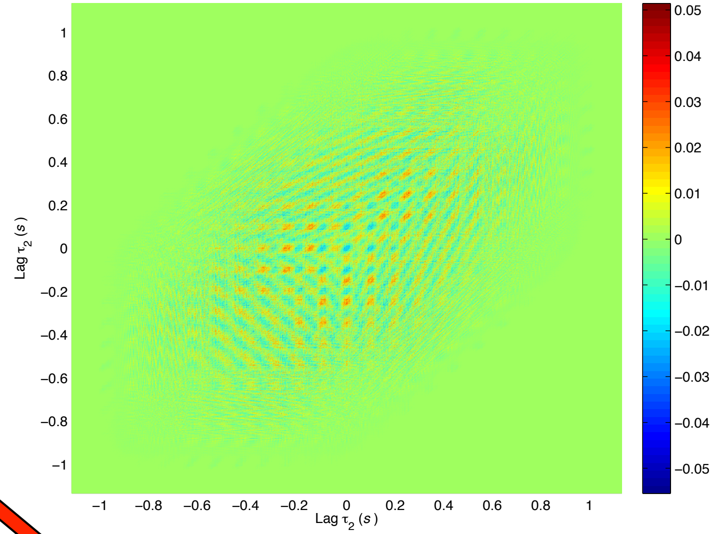
Optimal window (MSE)



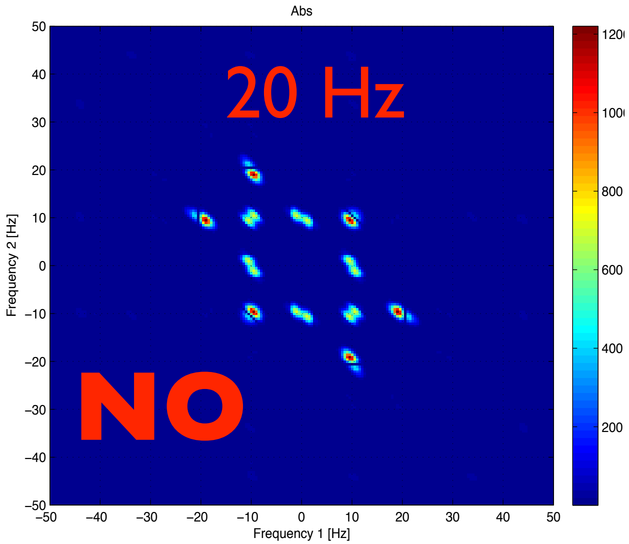
Bispectrum Analysis (4)

Windowing Rao-Gabr: SG10Hz+white at low SNR (<0.6)

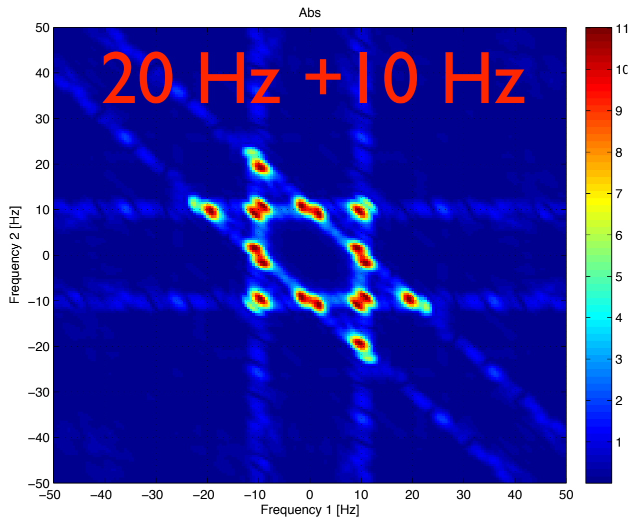




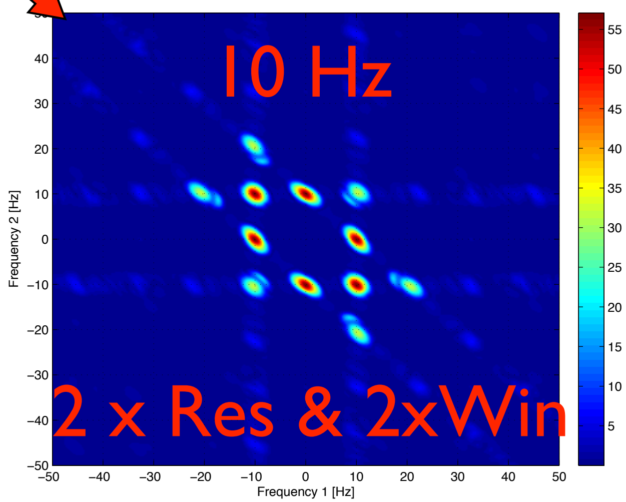
No fading



Fade



Fade



Signal extraction/filtering/reconstruction

Testing

Filtering Frequency-domain via DCT

D.V.Fevralev et al., 2006, "Combined bispectrum-filtering technique for signal shape estimation with DCT-based adaptive filter"

D.V.Fevralev et al., 2006, "Signal shape reconstruction by DCT-based filtering of Fourier spectrum recovered from bispectrum data"

Debug

Robust Amplitude and Phase reconstruction algorithm

G.Sundaramoorthy, 1990, "Bispectral Reconstruction of Signals in Noise Amplitude Reconstruction Issues"

M.Nakamura, 1993, "Waveform Estimation from Noisy Signals with Variable Signal Delay Using Bispectrum Averaging"

To be coded

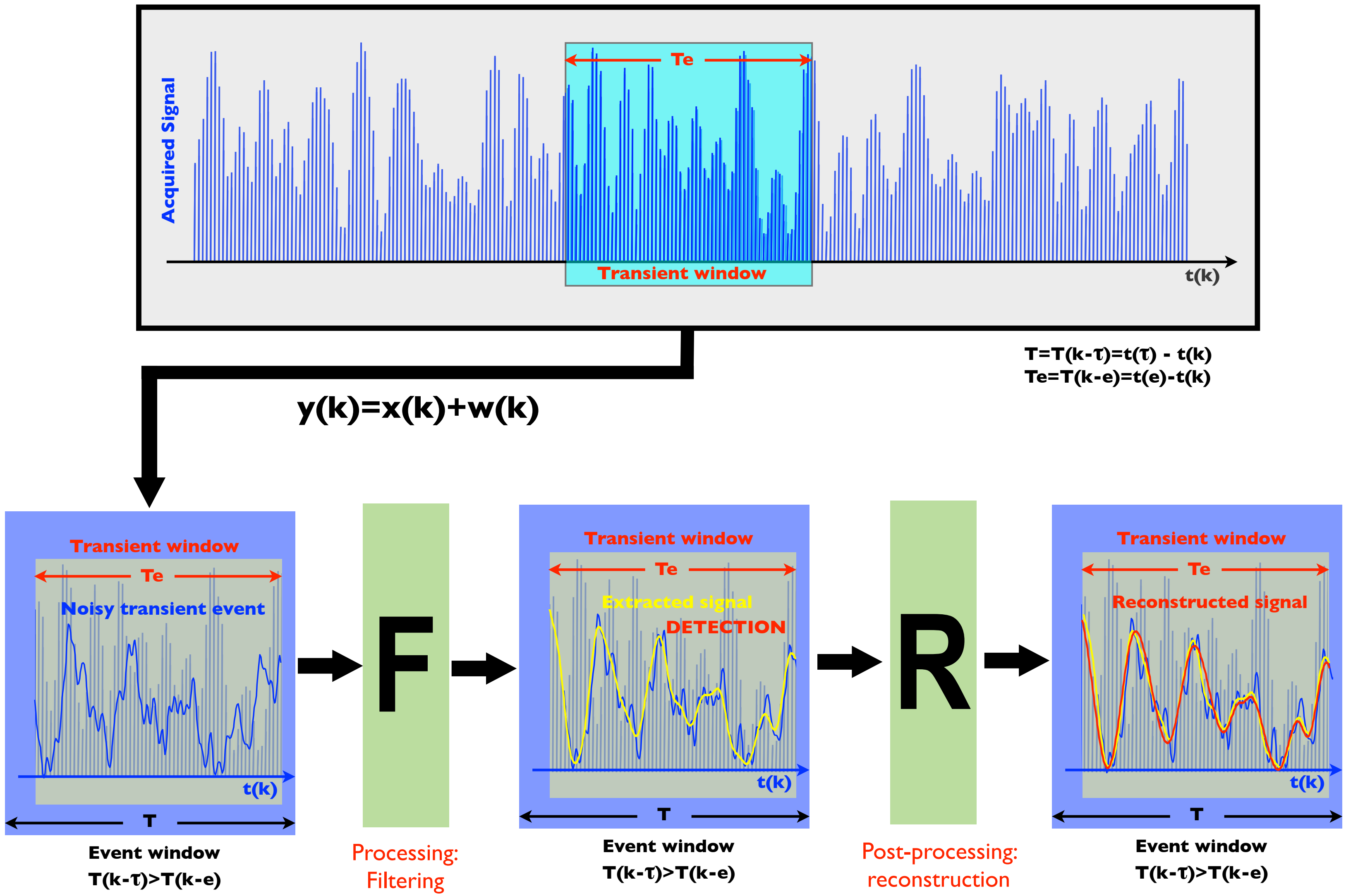
Inverse Filtering

M.Pulakka et al., 2005, "A Toolkit for Voice Filtering and Parametrisation"

J.Walker, 2003, "Application of the Bispectrum to Glottal Pulse Analysis"

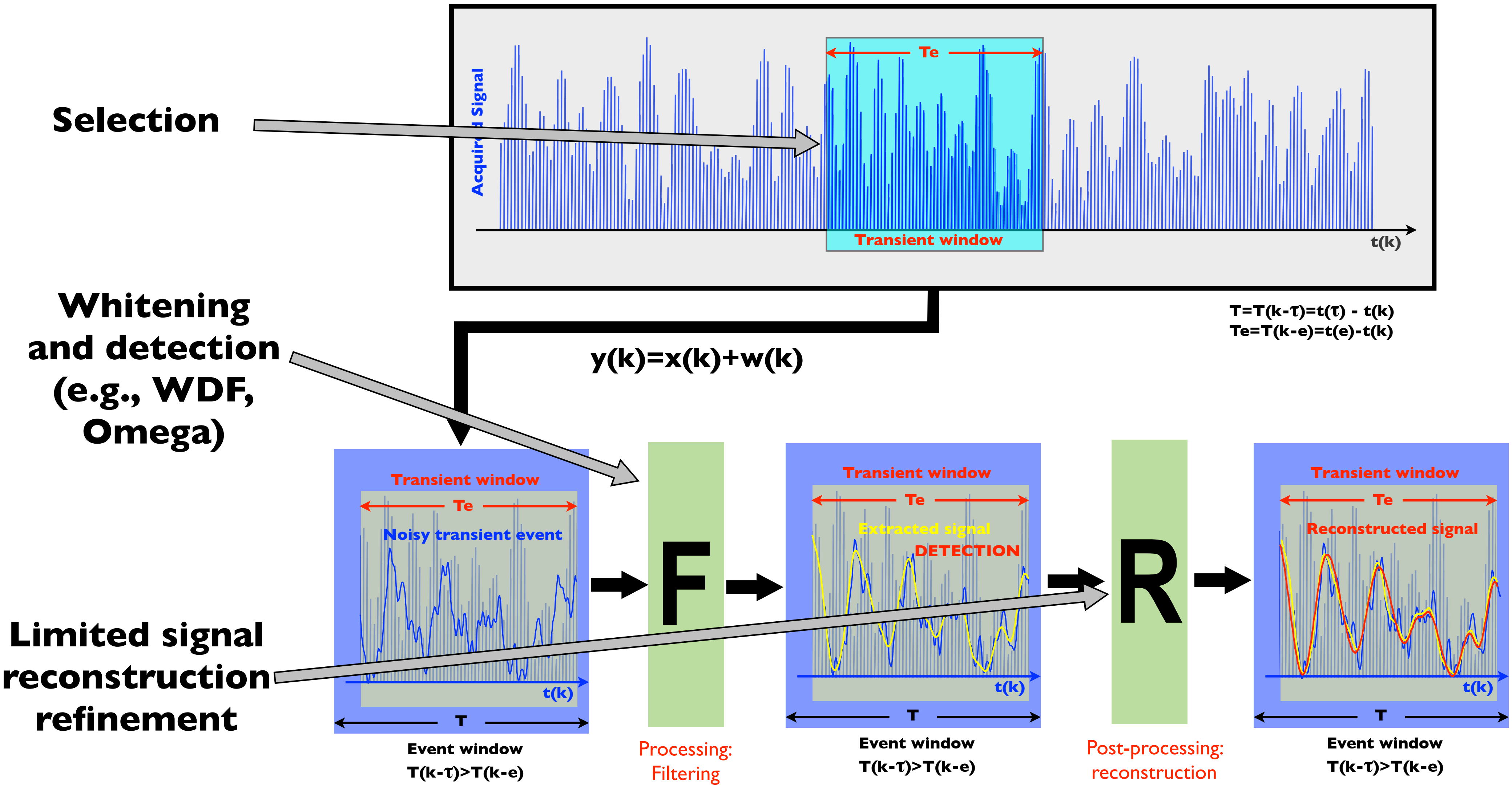
HOSA Integration

Traditional (stationary) approach



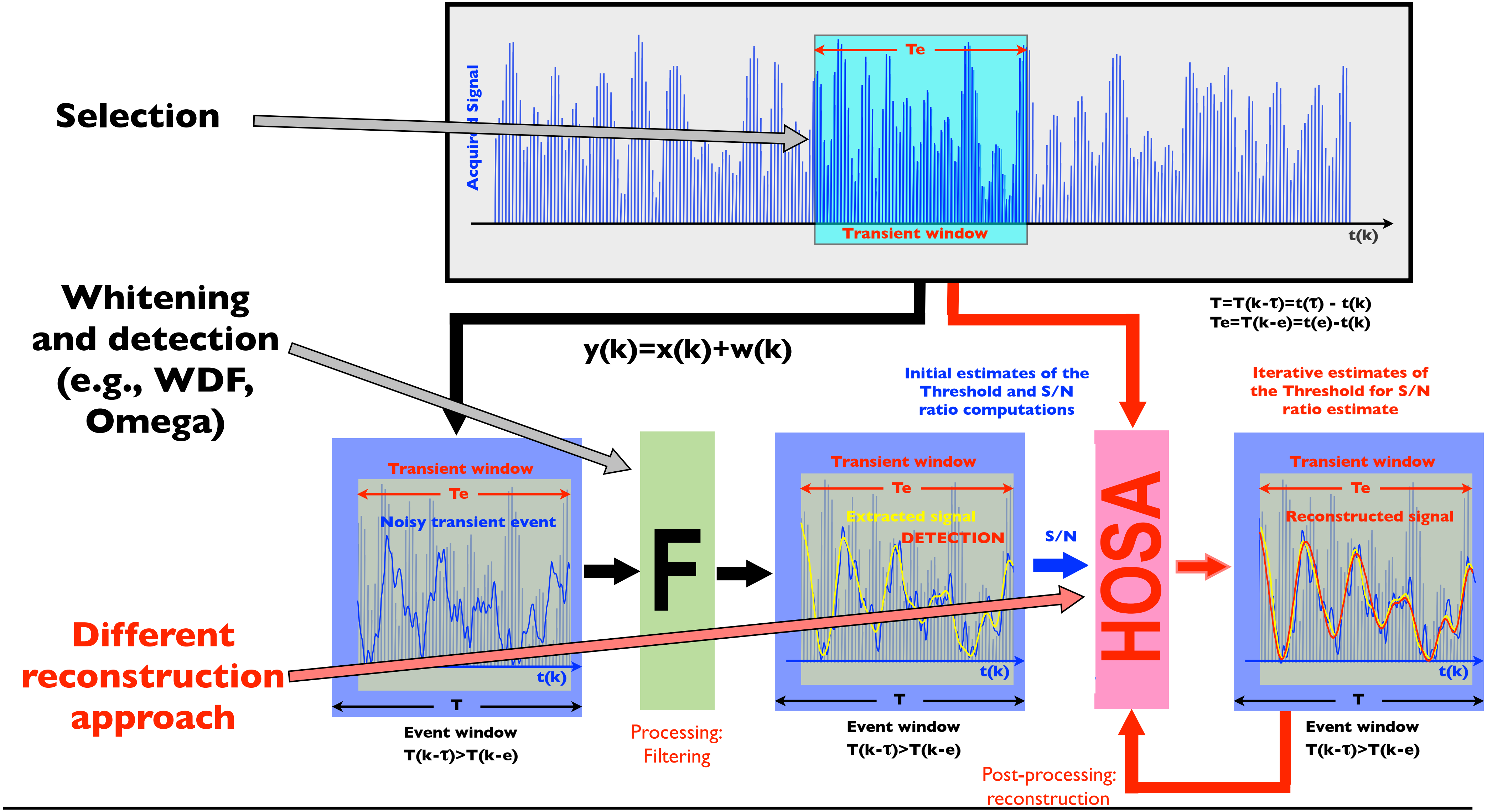
HOSA Integration (2)

Traditional (stationary) approach



HOSA Integration (3)

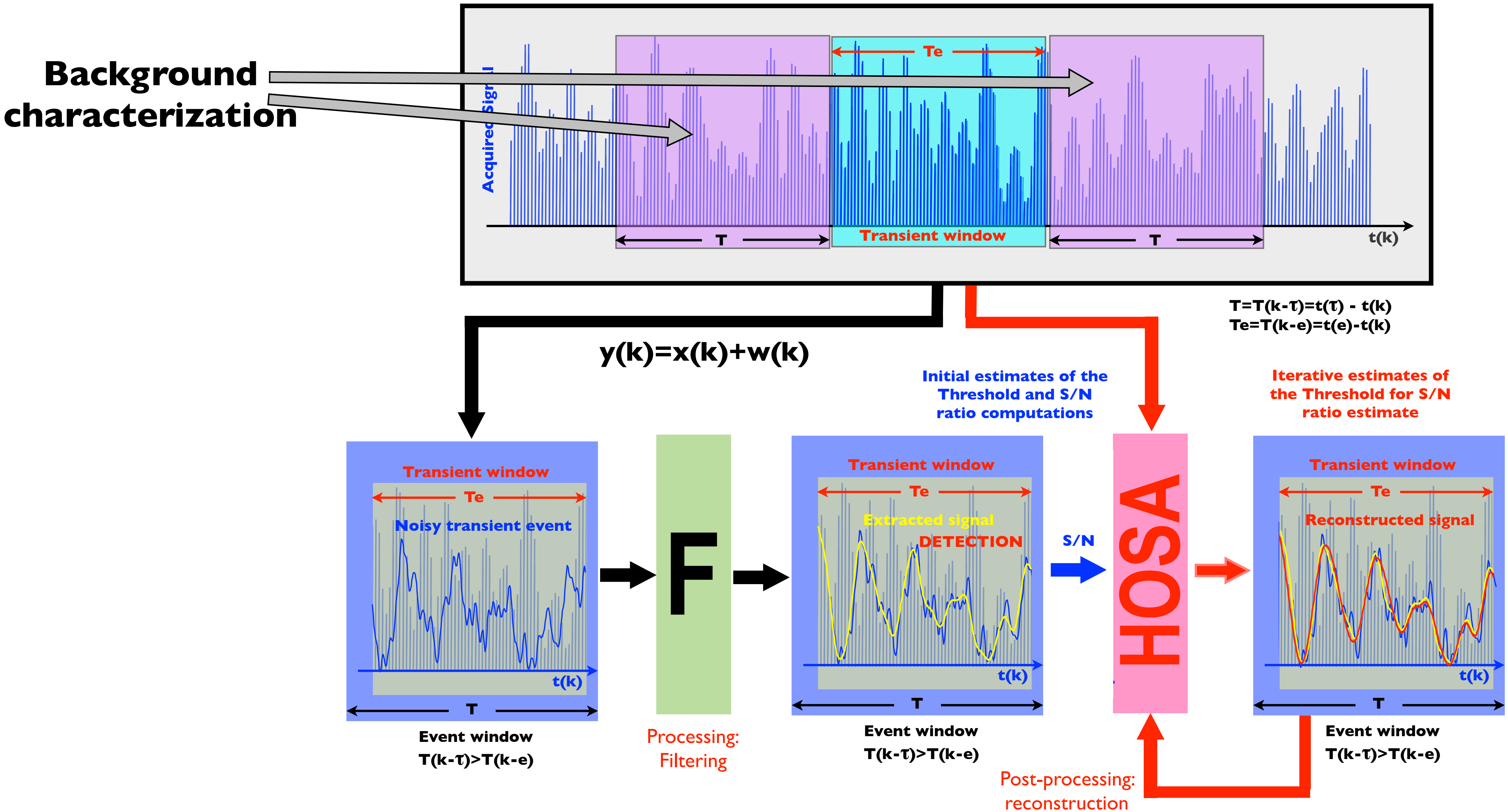
Advanced (stationary and nonstationary) approach



VIRGO

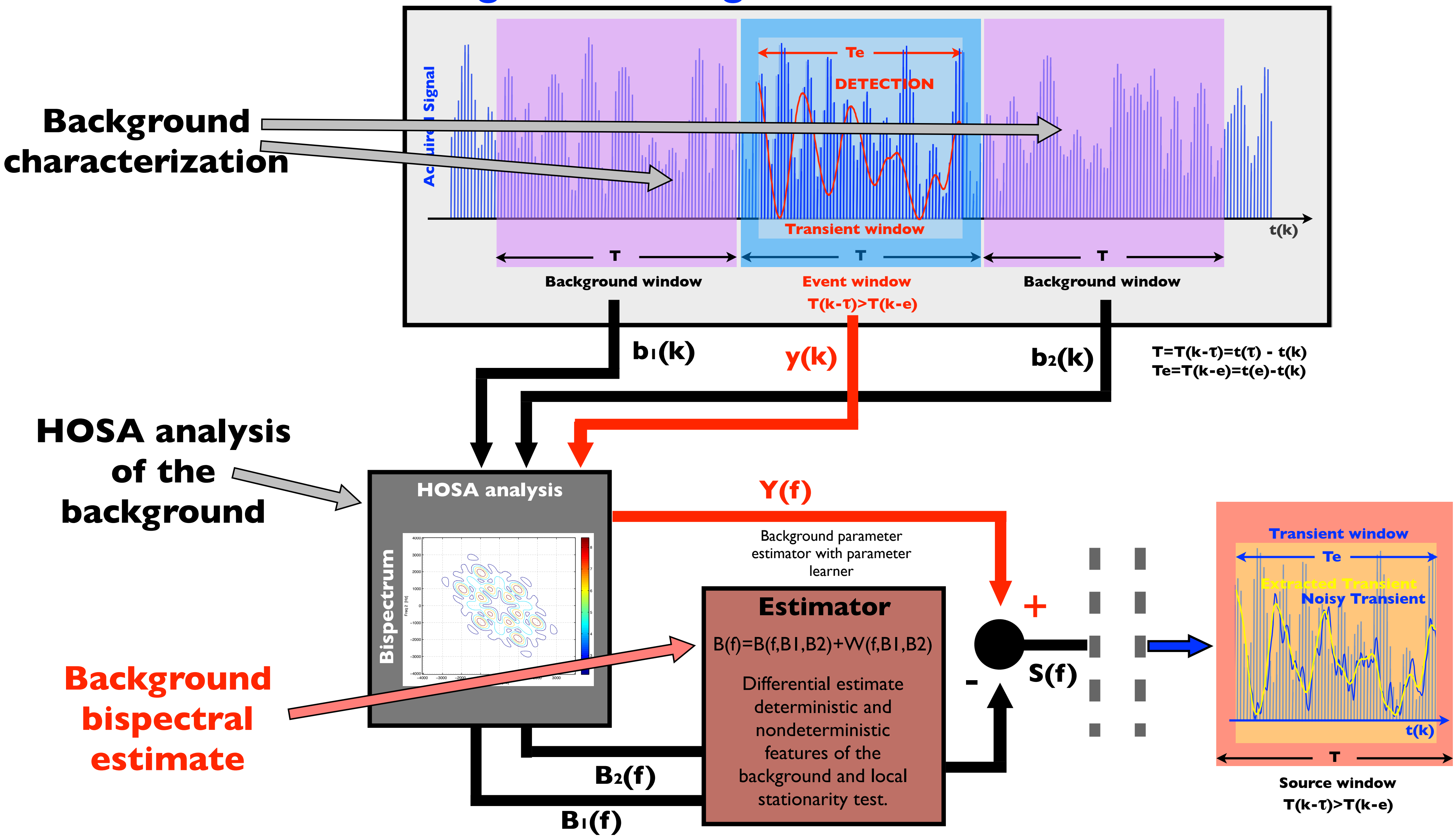
HOSA Integration (4)

Advanced (stationary and nonstationary) approach



HOSA Integration (5)

Signal-to-Background estimate



HOSA Integration (6)

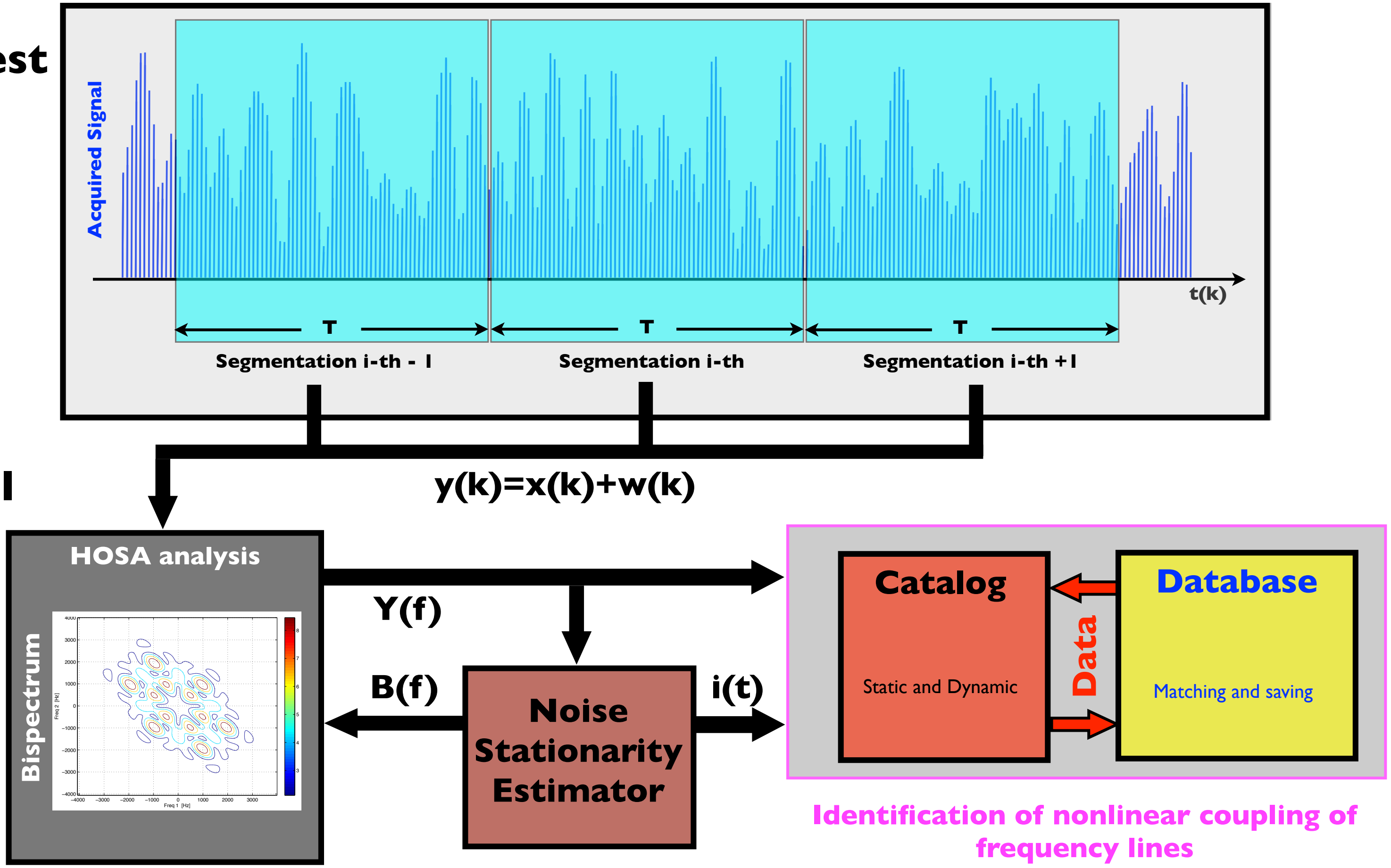
Nonstationary nonlinear correlations of frequency lines

- **Nonlinear correlation test**

- **Prompt data monitoring for nonstationary nonlinear coupling in the frequency-domain**

- **Search for multi-channel nonlinear coupling of events produced by:**

-> Gravitational wave
-> Laser fluctuation
-> Electromagnetics
-> Seism
-> Thermal noise



What next...

- Test on real data
- Modify for suiting performance of an in-time analysis
- Integrate it as tool on NMAPI framework

End