Estimation of Radii of Curvature from mirror maps

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Commissioning meeting

OUTLINE:

- **-which Radius of Curvature are we interested in?**
- **-fitting strategy:**
  - **-choosing the function**
  - **-choosing the weighting**
- **-application to VIRGO+ mirror maps**
- **-validation of RoC results by means of FFT simulation of focusing**
- **-check of low spatial frequency parameters by means of Finesse simulations**
  - **- piston and tilt**
  - **- RoC**
- **-comparison with experimental data**
- **-conclusions**

- which Radius of Curvature are we interested in?

The one that can reproduce focusing effects of the mirror:

$$\begin{pmatrix} A & B \\ -2/R & D \end{pmatrix}$$

-For an ideal spherical mirror this is the same than the geometrical RoC

-For a real life mirror (to be placed in a cavity) it is important to analyze also the "effective" RoC as seen by the modes.

Looking for an effective RoC

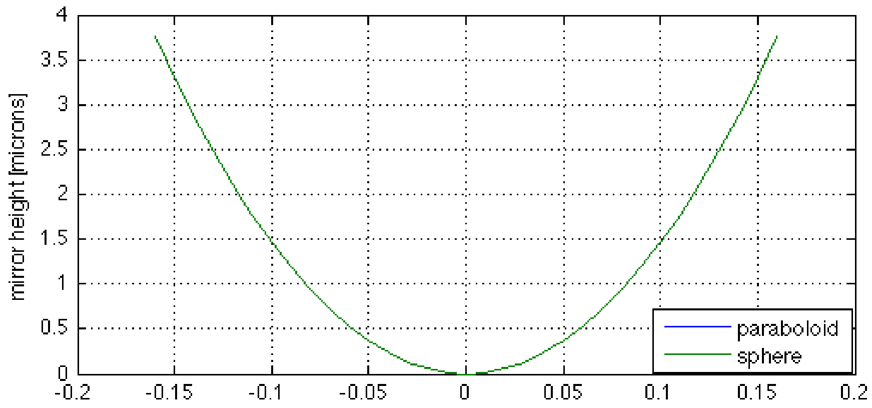
-fitting strategy: choosing the function

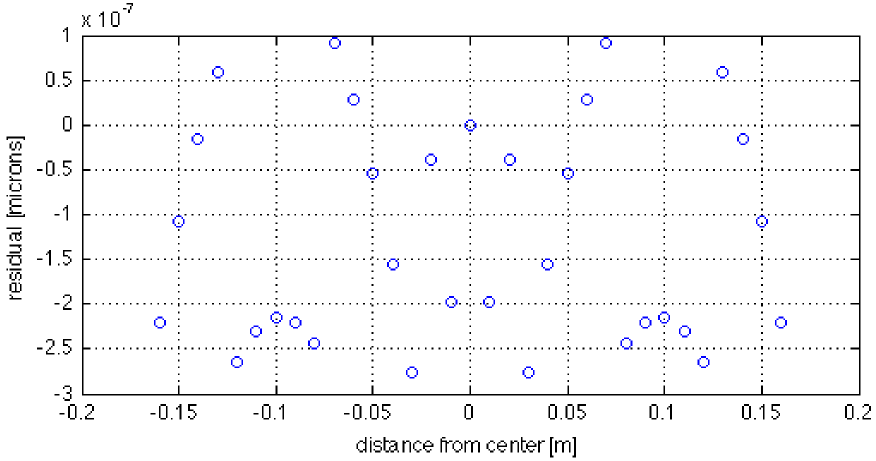
EGO

Two features to take into account:

-*accuracy*:
The function should describe well enough the surface to fit

-*reliability*:
"good" results while keeping numerical computation easy enough





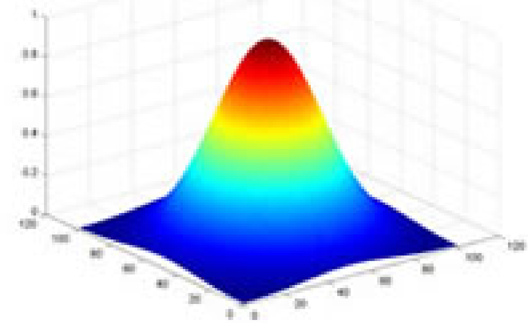
- our choice: a paraboloid

$$z(x,y) = c1\ x^2 + c2\ y^2 + c3\ xy + c4\ x + c5\ y + z0$$

it allows astigmatic curvatures, and gives piston and tilt in one shot.

-fitting strategy: choosing the weighting

- what is at the far edge of the mirror should be less important than what is at the center, for the sake of effective RoC estimation.



- RoC as seen from the beam point of view leads to consider the use of beam intensity as a weight for the fitting.

- **our choice: TEM00 intensity with suitable waist**

w(r,z) = $$I(r,z) = \frac{|E(r,z)|^2}{2\eta} = I_0 \left(\frac{w_0}{w(z)} \right)^2 \exp\left(\frac{-2r^2}{w^2(z)} \right) ,$$

(z= 0; w0 = ??)

Iterative procedure:
1) use expected beam size on the mirror → find RoC
2) compute new beam size on the mirror using previously found RoC
3) if difference is not below a set accuracy, repeat procedure

End when w0 used for the fitting is consistent with the RoC value

Let's apply this procedure to Virgo+ mirror maps...

- application to VIRGO+ mirror maps

analysis with make_flat_map - Rich_curv_tilt_hess
Lcav=2999.9 m;   ITM RoC= -1.5e5 m;

$z(x,y) = c1\ x^2 + c2\ y^2 + c3\ xy + c4\ x + c5\ y + z0$

Hessian matrix:

$$H = \begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix}$$

$$(H - k\,\mathbb{1})\,\underline{v} = 0$$

$$\downarrow$$

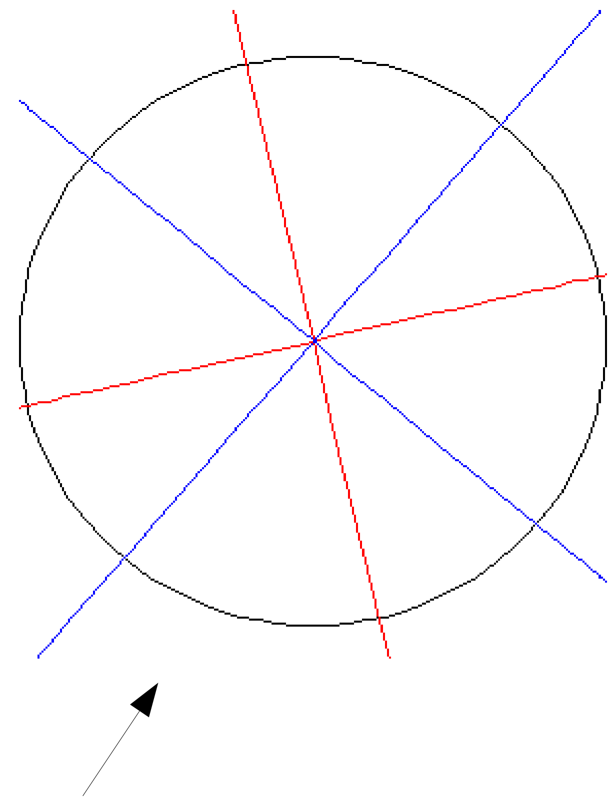
$k_{1,2} \rightarrow$ principal curvatures
$\underline{v}_{1,2} \rightarrow$ principal planes
(with angle ***alpha*** $= tg^{-1}(v_y / v_x)$)

	North End (C09061 on VEM10)	**West End (C09059 on VEM09)**
RoC_avg (m)	3273	3403
RoC1 (m)	3285.2	3424.5
RoC2 (m)	3260.2	3382.0
alpha (deg)	-76.9	49.8
Gouy_avg (deg)	73.2	69.9
Gouy_1 (deg)	72.9	69.4
Gouy_2 (deg)	73.6	70.4
xtilt (rad)	2.36e-7	1.79e-7
ytilt (rad)	7.80e-8	7.24e-8
piston (um)	-0.28	-0.23
Beam size mirr (mm)	61.0	56.4

- application to VIRGO+ mirror maps

analysis with make_flat_map - Rich_curv_tilt_hess
Lcav=2999.9 m;   ITM RoC= -1.5e5 m;

	North End (C09061 on VEM10)	West End (C09059 on VEM09)
RoC_avg (m)	3273	3403
RoC1 (m)	3285.2	3424.5
RoC2 (m)	3260.2	3382.0
astigmatism (m)	25.2	42.5
Gouy diff. (deg)	0.7	1.0
Beam parameters with IM RoC = -150km		
w0 (mm)	17.4	19.2
w0_1 (mm)	17.6	19.5
w0_2 (mm)	17.2	19.0
w_end (mm)	61.0	56.4
w1_end (mm)	60.4	55.8
w2_end (mm)	61.6	57.0



NE principal curvature axes (alpha= -76.9)
WE principal curvature axes (alpha= 49.8)

(as seen by TEM00)

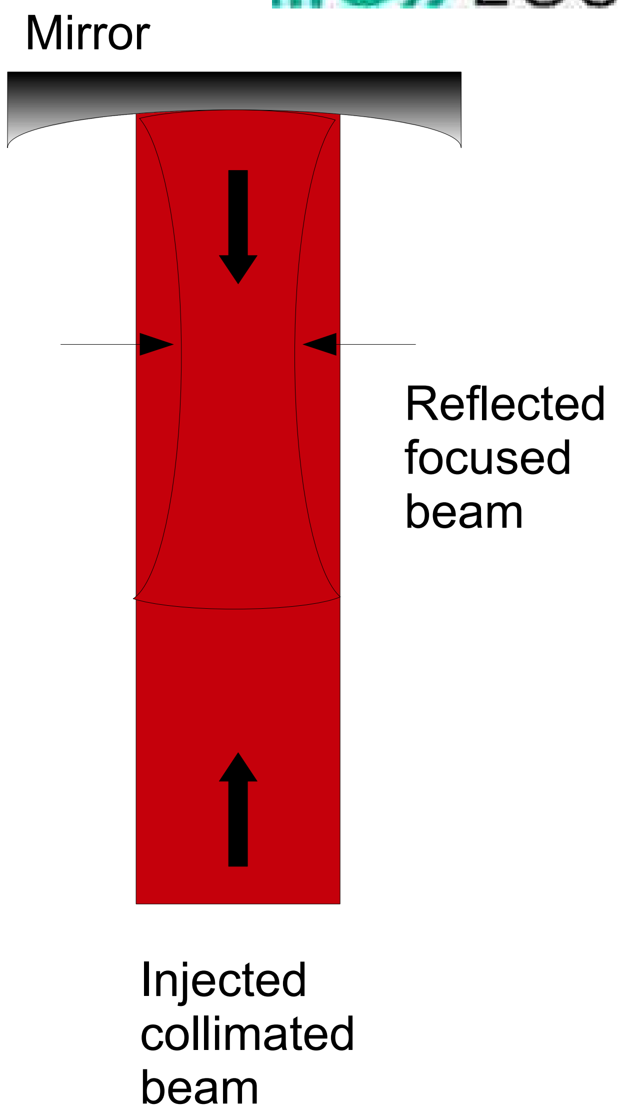
How to validate this procedure?

-validation of RoC results by means of FFT simulation of focusing

Reminder: the effective RoC is the one that explains focusing effects of the mirror

FFT simulation:

- Reflect off mirror with a gaussian beam with plane wavefront.
- Find d_{min} from mirror giving minimum rms point spread.
- Find equivalent RoC (ideal mirror) which gives minimum rms point spread at *same* d_{min}.



-validation of RoC results by means of FFT simulation of focusing

- Results of FFT simulation
- Results of fitting with the same mode as a weight
- difference between fitting and simulation

	TEMmn	w0 (mm)	w mirr (mm)	RoC1 (m)	RoC2 (m)	RoCx (m)	RoCy (m)	alpha (deg)	**RoC avg (m)**	**RoC eff (m)**	delta (m)
NE	Unweighted over 150mm diam (1)									3300	
	00	17.5	60.6	3285.2	3260.2	3261.5	3283.9	-76.9	**3272.7**	**3271.4**	**1.3**
	01	17.8	59.7	3306.3	3282.2	3282.8	3305.6	-80.3	**3294.2**	**3299.8**	**-5.6**
	10	17.8	59.7	3306.6	3283.3	3284.3	3305.5	-77.7	**3294.9**	**3289.9**	**5**
	02	18.2	58.8	3324.9	3306.7	3307.1	3324.4	-80.6	**3315.8**	**3341.2**	**-25.4**
	20	18.1	58.9	3330.8	3299.5	3300.2	3330.1	-81.3	**3315.1**	**3326.9**	**-11.8**
WE	Unweighted over 150mm diam (1)									3410	
	00	19.3	56.1	3424.4	3381.9	3399.5	3406.6	49.8	**3403.0**	**3397.5**	**5.5**
	01	18.9	57.1	3388.1	3349.0	3361.7	3375.2	55.1	**3368.4**	**3370.7**	**-2.3**
	10	18.8	57.2	3381.5	3346.8	3359.0	3369.1	53.4	**3364.0**	**3360.0**	**4**
	02	19.1	56.6	3400.6	3366.2	3375.7	3391.0	58.1	**3383.3**	**3386.6**	**-3.3**
	20	19.0	56.9	3391.0	3358.9	3365.6	3384.2	62.7	**3374.9**	**3361.4**	**13.5**

(1) *see VIR-0381A-10*

Are results consistent with Finesse simulations?

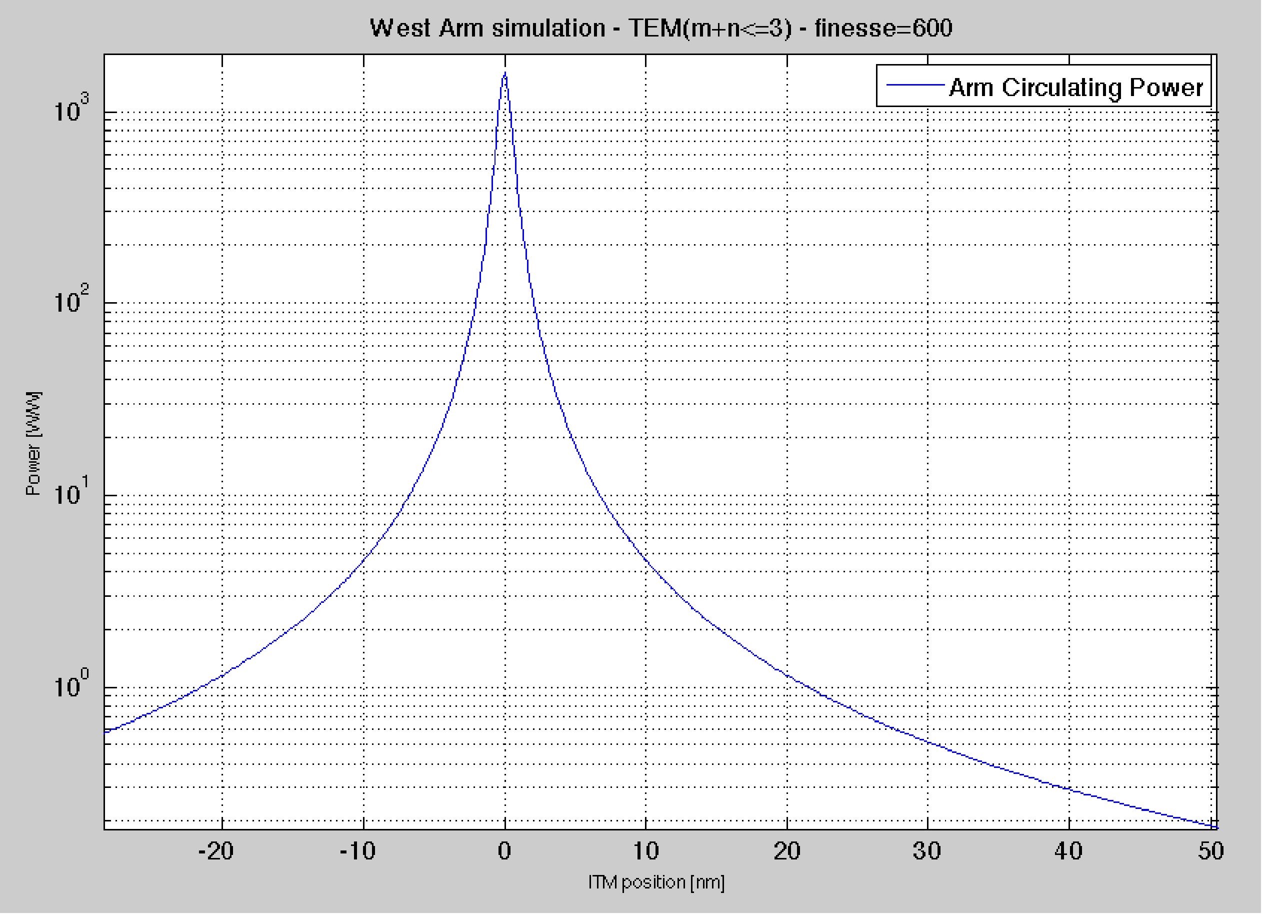
-check of parameters by means of Finesse simulations: *piston*

Finesse simulation: Arm cavity with one mirror map, high finesse.

-Measure total power in cavity
-scan TM position over half a wavelength
-find TEM00 resonance position
-use offset of TEM00 resonance to correct further the map
-repeat for the other TM
-check with both maps.

Sensitivity ≈0.5nm

(Fitting error ≈5nm)



-check of parameters by means of Finesse simulations: *tilt*

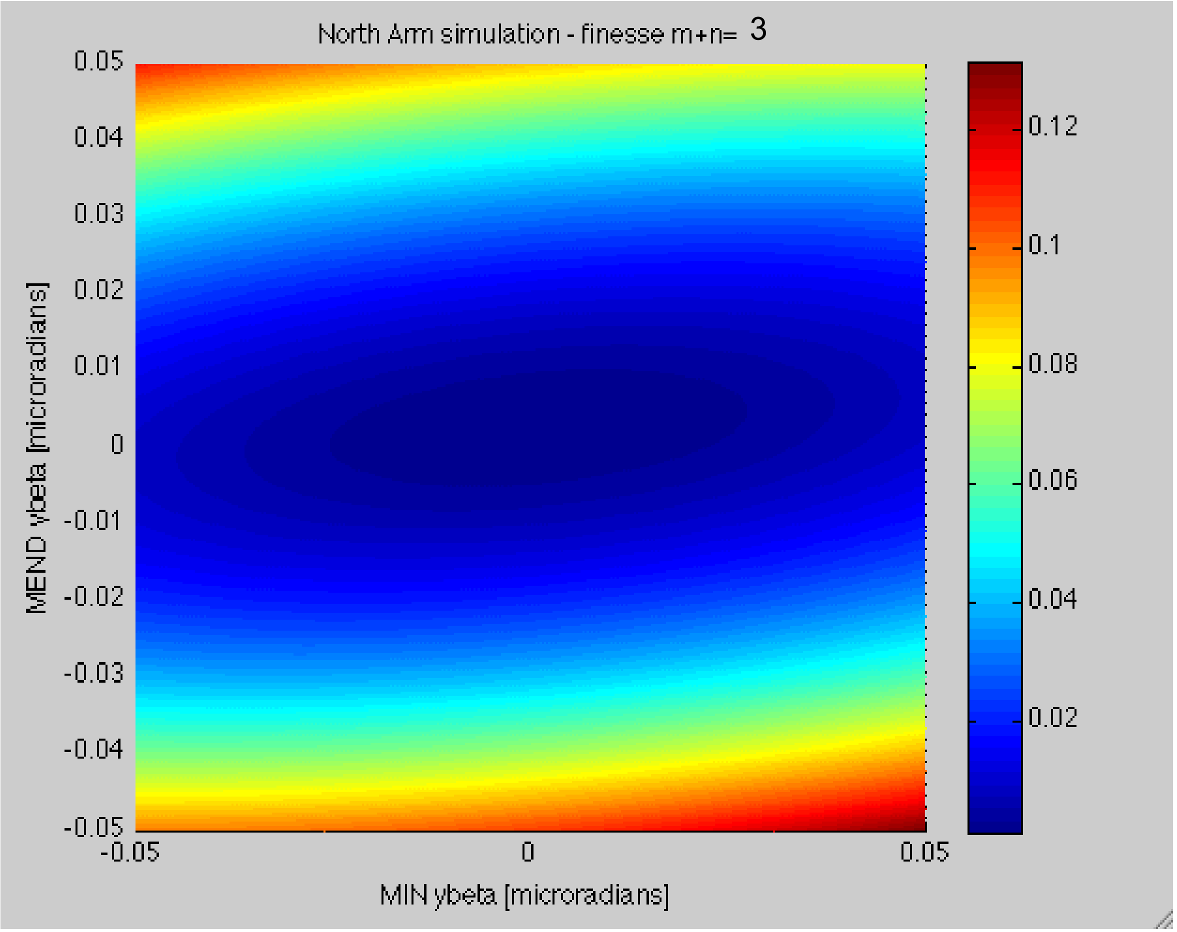
Finesse simulation:
Arm cavity with one mirror map, high finesse, locked on TEM00 resonance.

-Measure power in TEM(m+n=1)
-scan *xtilt* angles both for ITM and ETM
-find minimum for power in TEM(m+n=1)
-use the tilt giving a minimum to correct further the maps
-repeat for *ytilt*
-check with the corrected maps
-repeat for the other map

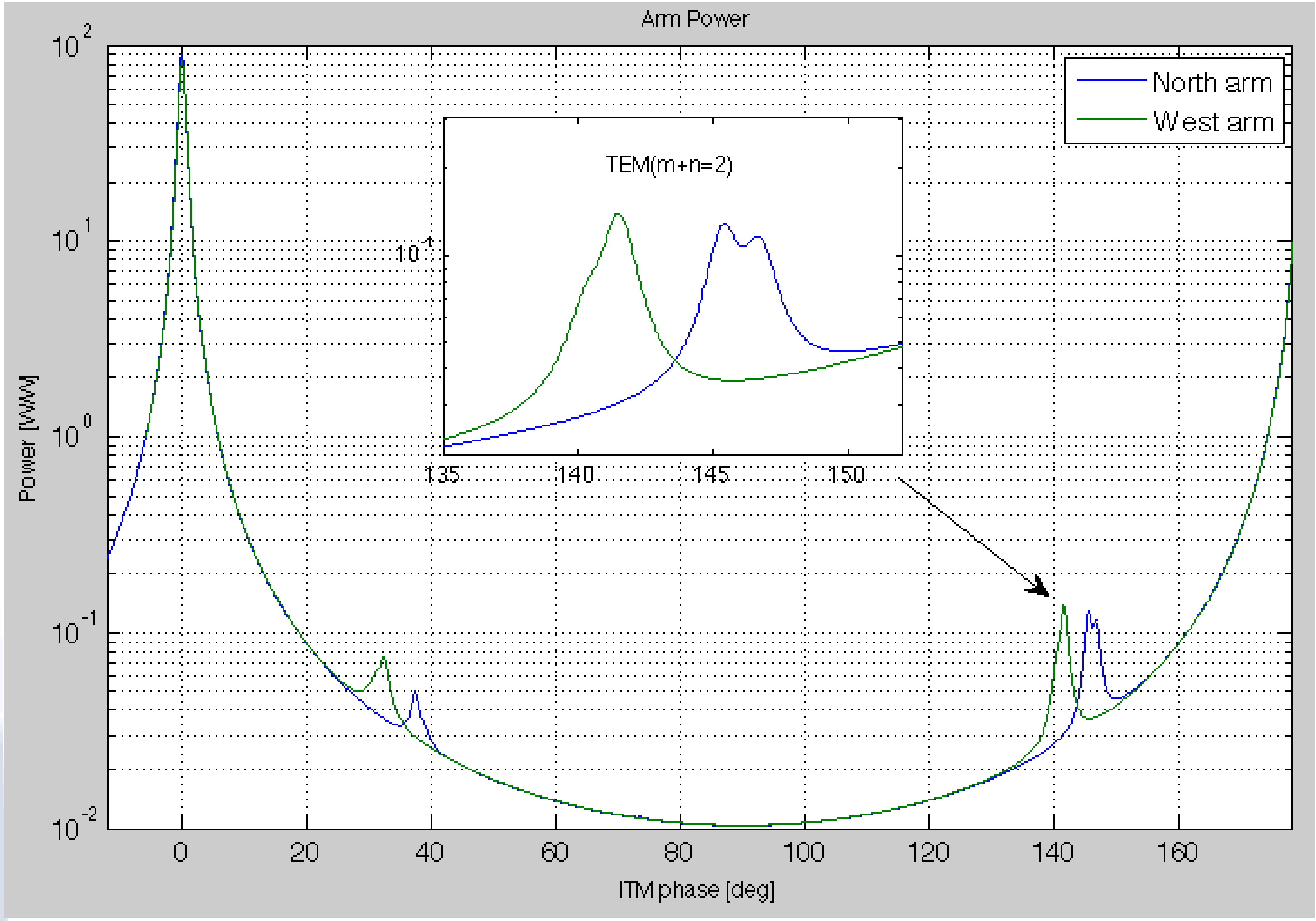
-check with both maps

Sensitivity ≈5nrad

(Fitting error ≈0.1urad)



-check of parameters by means of Finesse simulations: *RoC from Gouy phase*



RoC from Gouy phase.

Splitting of homs is due to astigmatism

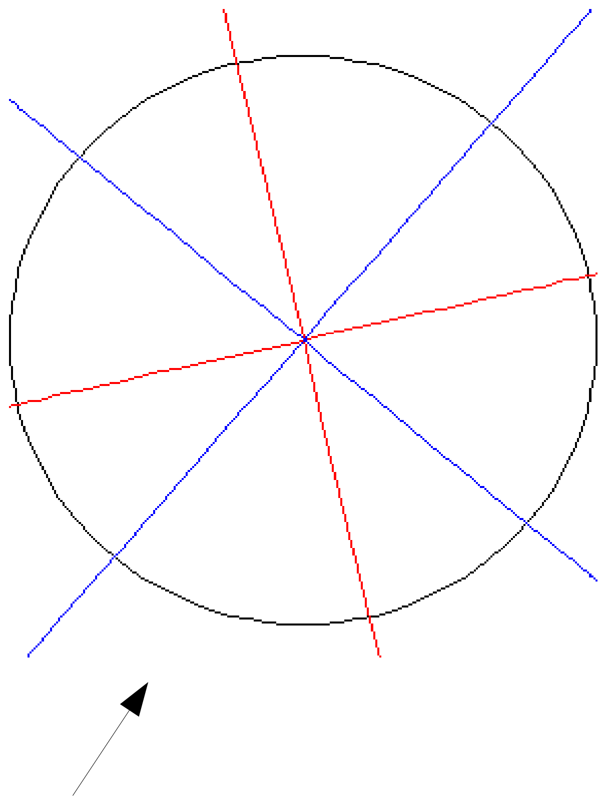
$$phi = \cos^{-1}(\sqrt{(g_{in} g_{end})})$$

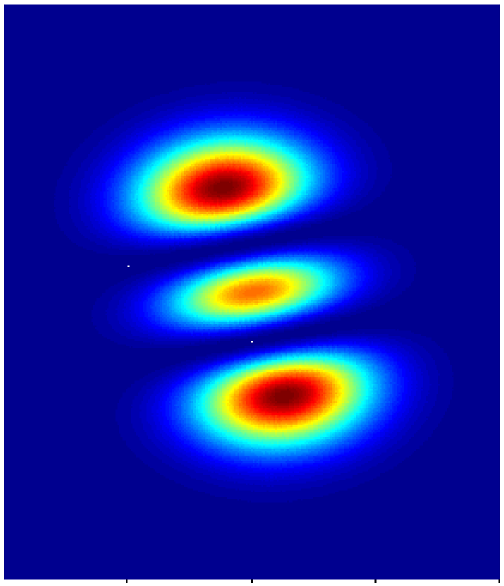
$$g = 1 - Lcav / RoC$$

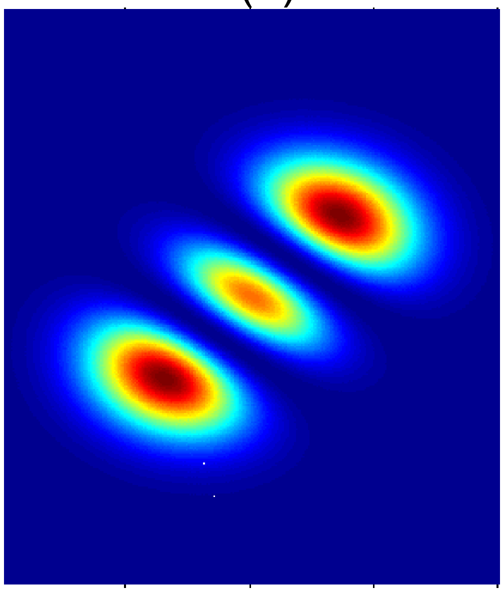
(See f.i. Siegman, Lasers)

	RoC02 (m)	RoC20 (m)	Avg RoC (m)	D_RoC (m)
North End M	3290	3269	3279 (*fitting:* 3273)	21 (*fitting:* 25)
West End M	3389	3366	3377 (*fitting:* 3403)	23 (*fitting:* 42.5)

Is it possible a mismatch between orientation of Finesse basis (likely along x,y) and the "natural" basis of the two arms?







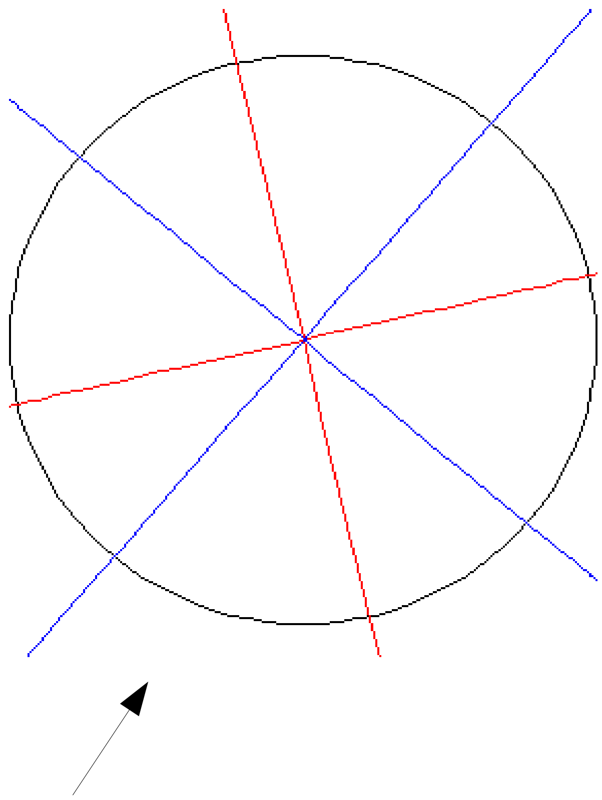
North ETM principal curvature axes
West ETM principal curvature axes

Consequences of orientation mismatch?
→ WORK IN PROGRESS

	RoC02 (m)	RoC20 (m)	Avg RoC (m)	D_RoC (m)
North End M	3290	3269	3279 (*fitting:* 3273)	21 (*fitting:* 25)
West End M	3389	3366	3377 (*fitting:* 3403)	23 (*fitting:* 42.5)

Is it possible a mismatch between orientation of Finesse basis (likely along x,y) and the “natural” basis of the two arms?

		RoC02 (m)	RoC20 (m)
NE	Finesse sim.	3290	3269
	fitting	3324 (tem02 y)	3300 (tem20 x)
WE	Finesse sim.	3389	3366
	fitting	3391 (tem02 y)	3366 (tem20 x)



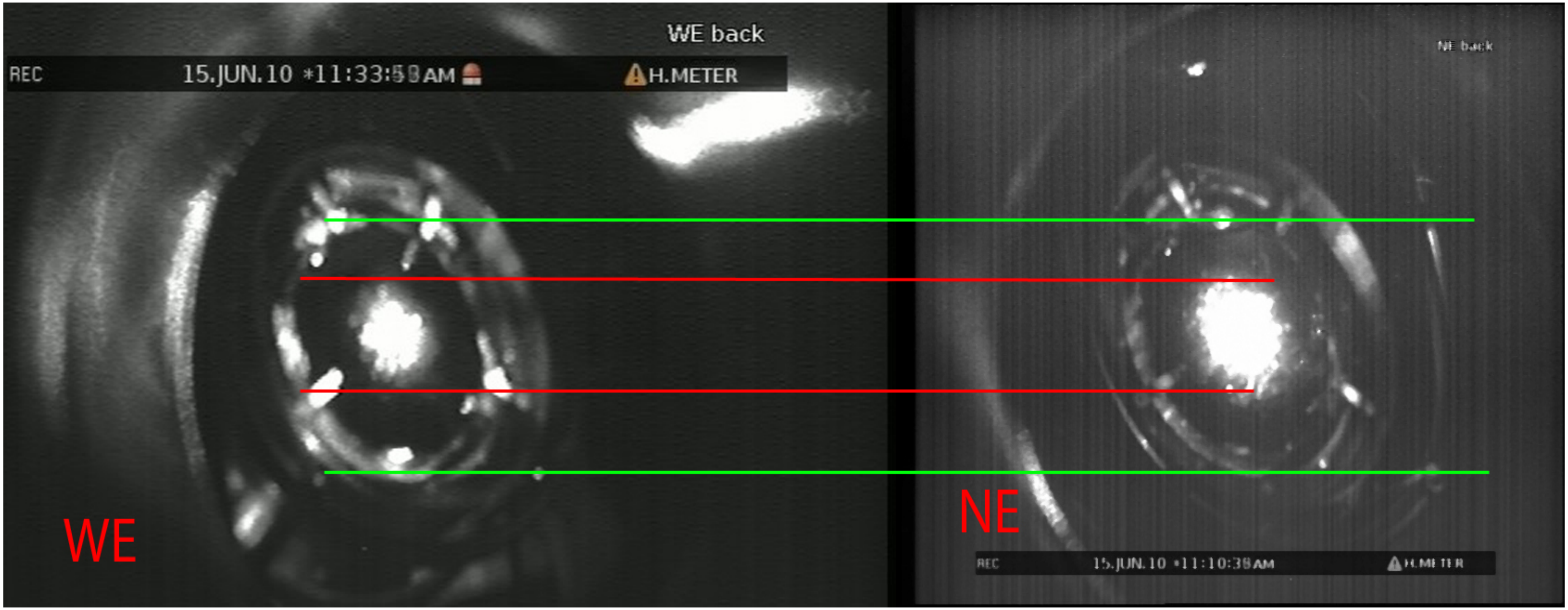
North ETM principal curvature axes
West ETM principal curvature axes

- better for WE, worse for NE:
further investigation needed

How does the fit compare with experimental data?

-comparison with experimental data

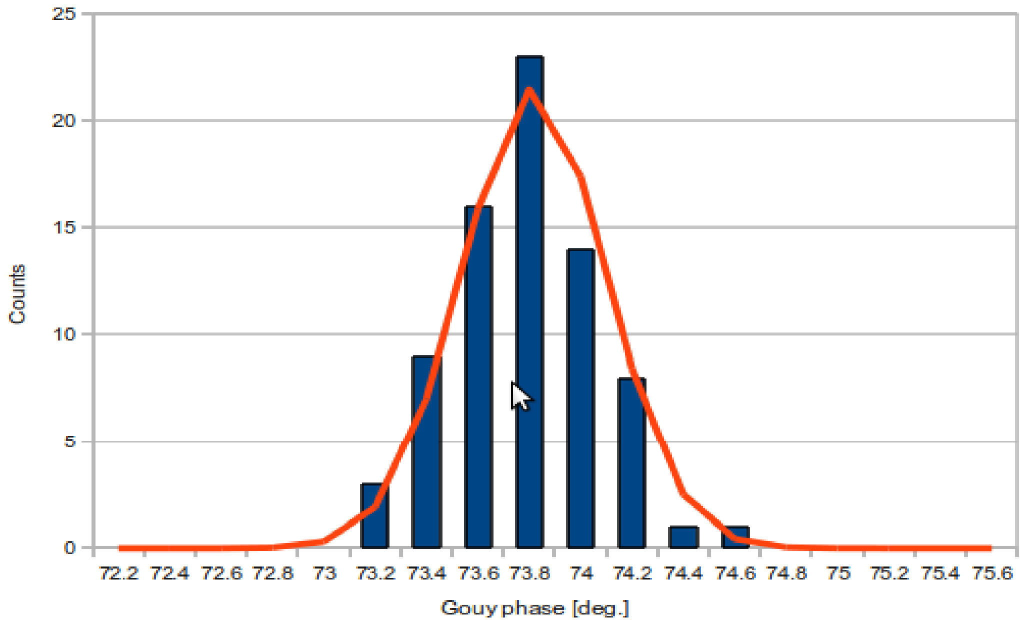
See logbook entry *26858* (Beams) by Vajente et al.



Qualitative agreement:

	West End	North End
Predicted beam size (mm)	56.4	61.0

-comparison with experimental data



North Arm swinging

See logbook entry *26776* (NE RoC) by G. Vajente

ETM RoC from hom resonances
Average separation between modes.
(ITM assumed to be perfectly flat)

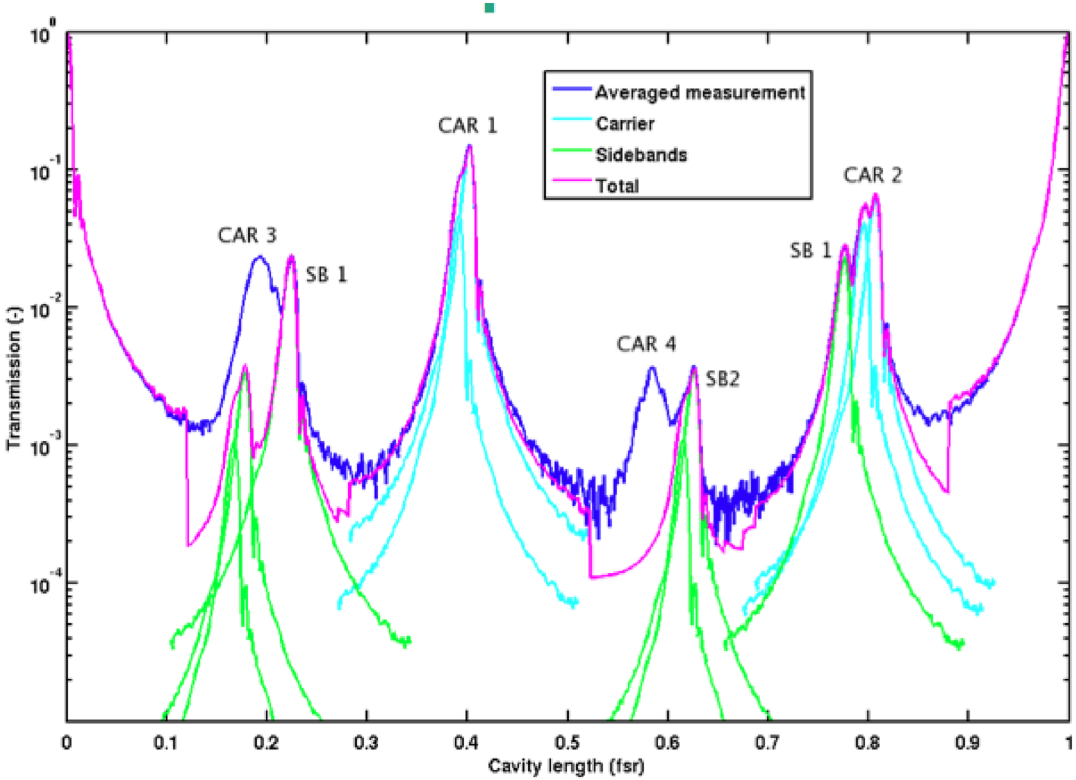
$$\text{phi} = \cos^{-1}(\sqrt{(g_{in} g_{end})})$$

$$g = 1 - \text{Lcav} / \text{RoC}$$

(See f.i. Siegman, Lasers)

	average
Gouy phase (deg)	73.7
RoC NE (m)	3257 *(fitting: 3273)*

-comparison with experimental data



West Arm swinging
(top stage actuation)

See VIR-0386A-10 (B. Swinkels)

ETM RoC from hom resonances
(ITM assumed to be perfectly flat)

	tem01	tem10	tem02	tem20	average
Gouy phase (deg)	70.4	72.4	143.1	145.3	
RoC WE (m)	3380	3302	3334	3293	**3327**
RoC WE *fitting* (m)	3388	3345	3401	3359	**3373**

	North End Mirror			West End Mirror		
	RoC (m)	RoC_1 (m)	RoC_2 (m)	RoC (m)	RoC_1 (m)	RoC_2 (m)
meas.	**3257**	n/a	n/a	**3327**	3357[a]	3297[a]
fit. gauss.	3273	3285[b]	3260[b]	3403	3424[b]	3382[b]
Finesse (1)	3279	3290[c]	3269[c]	3377	3389[c]	3366[c]
SIESTA (2)	3266	n/a	n/a	3412	n/a	n/a
LMA fit.(3)	3300	n/a	n/a	3410	n/a	n/a

(1) RoC inferred from gouy phase in Finesse simulation
(2) RoC seen by TEM00 beam in SIESTA simulation optimizing losses, *see VIR-0381A-10*
(3) spherical fitting over 150mm diam., no weight, *see VIR-0381A-10*

(a) average RoCs seen by TEM(m+n=1,2) modes, along cavity TEM basis axes
(b) average RoCs seen by TEM00 along map principal curvature axes
(c) average RoCs seen by TEM(m+n=2) along finesse TEM basis axes

Conclusions

- paraboloid fitting with gaussian weight (*pfgw*) shows excellent agreement with FFT simulation of focusing effects of a mirror map (around 10^{-3} discrepancy for TEM00)

- good agreement (<1% discrepancy) between fitting results and simulations (Finesse, SIESTA) *using the same mirror maps.*

- around 2-3% discrepancy between fitting results and measured data:
    - -orientation issues?
    - -influence of Input Mirror?
    - *further investigation is needed to compare with measured data*

- *pfgw* provides an estimation of effective RoC ***as seen by the beam*** better than usual unweighted spherical fitting → *we suggest to use this procedure to specify mirror requirements for AdV*