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# Advanced Virgo Design: Differential Arm Length Noise from Vertical Movement of Wedged Input Mirrors

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## 1 Introduction

The use of input mirrors featuring a wedge is currently under discussion for the Advanced Virgo design [1].

As indicated by Figure 1 mirrors with wedge can cause differential arm length noise, originating from mirror motion perpendicular to beam axis. In case the mirror moves up in vertical direction, the beam path inside the mirror substrate increases, while the distance between the beam splitter and mirror surfaces decreases. The overall geometrical arm length of the small Michelson interferometer stays constant, but due to the fact that the mirror substrate has an index of refraction different from vacuum the optical path length changes.

The wegde can be chosen to be vertically or horizontally orientated, depending on which solution turns out to be most feasible to implement in the Advanced Virgo system constraints (vacuum vessels, optics etc). Since usually suspensions provide better isolation for horizontal than for vertical degrees of freedom, its seems saver to go for horizontal wedges. However, constraints from the vacuum and suspension systems might only allow for the implementation of vertical wedges in Advanced Virgo.

The scope of this document is to investigate whether the vertical isolation of a Virgo super attenuator is good enough to allow the use of wedged input mirrors, without spoiling the sensitivity of Advanced Virgo.<sup>1</sup>



Figure 1: Vertical movement of a wedged input mirror causes an optical path length change,  $\Delta L$ , originating from the different index of refraction of the mirror substrate and the vacuum.

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 $<sup>^{1}</sup>$ The authors believe that in case the super attenuator provides enough vertical isolation to allow vertical wedges, this should also be valid for horizontally orientated wedges.



# 2 Projection of the differential arm length noise from vertical movement of wedged input mirrors

The transfer function from vertical motion of the mirror itself,  $\tilde{V}_{mot}$ , to MICH<sup>2</sup> length noise is given by the following equation:

$$TF_{V \to MICH} = \tilde{V}_{mot} \cdot \tan(\alpha) \cdot (n_{sub} - 1),$$
 (1)

where  $\alpha$  is the angle of the wedge and  $n_{\text{sub}}$  the index of refraction of the mirror substrate. The coupling from MICH length noise to differential arm length (DARM) noise is according to [4] described by:

$$TF_{\text{MICH}\to\text{DARM}} = \frac{\pi}{2F},$$
 (2)

where F is the averaged Finesse of the two arm cavities.

Alltogether the projection of length noise from vertical motion of the wedged input mirrors can be described by the following equation:

$$h_{\text{wedge}} = 2 \cdot \tilde{V}_{\text{sei}} \cdot TF_{\text{SA}} \cdot \tan(\alpha) \cdot (n_{\text{sub}} - 1) \cdot \frac{\pi}{2F} \cdot \frac{1}{3000 \,\text{m}}$$
(3)

 $V_{\rm sei}$  describes the seismic in the Virgo central lab and  $TF_{\rm SA}$  the vertical isolation transfer function of full Virgo super attenuator. The  $\frac{1}{3000 \,\mathrm{m}}$  accounts for the calibration from displacement to strain, while the factor 2 describes the conservative sum of the length noise of both input mirrors.<sup>3</sup>



Figure 2: Vertical component of the seismic,  $\tilde{V}_{sei}$ , present in the Virgo central lab [2].

Using the actual parameters planned for Advanced Virgo,  $F \approx 750$  and  $\alpha = 3 \text{ deg}$  as well as the actual vertical seismic in the central lab (see Figure 2) and the actual vertical isolation performance of a Virgo super attenuator (see Figure 3), we get the noise projection shown in Figure 4.

For all frequencies above 2 Hz the projection turns out to be clearly below the advanced Virgo design sensitivity [1]. Except for a small frequency band between 35 and 50 Hz, the projected wedge noise is at least 5 orders of magnitude below the design sensitivity within the detection band.

<sup>&</sup>lt;sup>2</sup>Small Michelson interferometer formed by the beam splitter and the two input test masses.

<sup>&</sup>lt;sup>3</sup>Since both input mirrors are located close to each other inside the same building, it seems to reasonable to assume that they both encounter similar seismic noise. In that case (both mirrors see correlated noise) there would ideally be no differential arm length noise in the small Michelson from the wedges, but only a common mode length change. However, in our equation we account for the worst case scenario, which is correlated seismic of opposite phase at the two input mirrors.



Figure 3: Vertical transfer function of a Virgo super attenuator,  $TF_{SA}$ , as it was measured in [3].

#### **3** Summary and Discussion

From the investigation presented in this document it seems to be possible to include wedged input mirrors into the Advanced Virgo design without spoiling the the overall sensitivity at any frequency within the detection band.

In addition this result might be interesting regarding the design of the thermal compensation system. From our analysis it seems to be reasonable to also investigate compensation plates featuring wedges. This might strongly decreases the isolation requirements of a potential compensation plate suspension.

Finally it has to be pointed out, that the projection carried out in this document only includes seismic as source for vertical mirror motion. This assumption might not reflect the reality<sup>4</sup> and therefore lead to an underestimated result.

#### References

- [1] The Advanced Virgo Team, "Advanced Virgo Conceptual Design", Virgo note VIR-042A-07 1, 2, 4
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- [3] G. Ballardin et al, "Measurement of the VIRGO superattenuator performance for seismic noise suppression", Rev. Sci. Instrum., Vol. 72, No. 9 (2001) 3
- [4] R. Abbott et al, "AdvLIGO Interferometer Sensing and Control Conceptual Design ", LIGO-T070247-00-I, 2008/02/29

 $<sup>^{4}</sup>$ Longitudinal and alignment control actuation on the mirror and Marionetta stage might introduce additional vertical motion of the test mass.





Figure 4: Projection,  $h_{\text{wedge}}$ , of the differential arm length noise from vertical movement of wedged input mirrors. A wedge of 3 degree was assumed. For all frequencies above 2 Hz the project turns out to be clearly below the advanced Virgo design sensitivity [1]. Except form a small frequency band between 35 and 50 Hz, the projected wedge noise is at least 5 orders of magnitude below the design sensitivity within the detection band.