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Mirror surface studies for Advanced Virgo OMC

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1 Introduction

In this document we will study mirror surface specifications for the Advanced Virgo Output Mode Cleaner (OMC). It is well known that flatness defects and micro-roughness on mirror surfaces affect the resonant mode in a cavity. Here we would like to determine the mirror surface specifications in order to have more than 99 % of transmission through the OMC for a perfect Gaussian input beam TEM_{00}. To do this determination we will simulate the cavity with the code OSCAR [3]. Two kinds of losses will be characterised. The first one is the part of resonant beam which go out of mirrors, such losses are called sometimes "diffraction losses" or "clipping losses". The second, which can be the most important loss, is the reflection of the incident beam which does not match the imperfect cavity.

2 Short elements of theory

The aim of this part is not to give the complete theory of optical cavity with defect surfaces but to give some definitions in order to be understandable, coherent and clear on what we are speaking about in this document. Surface defects correspond to small deviations of mirror surfaces from the ideal one. Even if such deviations are much smaller than the considered laser wavelength it can affect strongly results of optical systems. Usually these deviations are basically characterised by the Root Mean Scare (RMS) but can be detailed with the Power Spectral Density (PSD). The PSD correspond to the amplitude of defects as a function of its spatial frequency. There are several kind of PSD depending on which coordinate system we are dealing with. We will use in the document only the one dimensional PSD as a function of the radial frequency. In order to fix variables and avoid mistakes we express the 1 dimensional PSD with the 2 dimensional PSD in the following equation 1.

$$PSD1D(f_r) = \int_0^{2\pi} PSD2D(f_r, \theta)\, f_r \, d\theta \tag{1}$$

with $f_r = \sqrt{f_x^2 + f_y^2}$. Usually there are no angular dependence and we obtain the relation equation 2.

$$PSD1D(f_r) = 2\pi \, PSD2D(f_r) \, f_r \tag{2}$$

We remind the definition of the RMS and its relation with the PSD in equation 3

$$RMS^2 = \frac{\iint \tilde{h}^2(x, y)\, dxdy}{S} = \int PSD1D(f_r)\, df_r \tag{3}$$

where S is the area of the considered surface and $\tilde{h}(x, y)$ is the deviation from the ideal surface. With the PSD it is possible to characterise the distinction between flatness defects and micro-roughness. The first one can be associated to the difficulty to keep control of a surface on a large size for less than few nm. The second one is more associated to the local process of polishing.

With the PSD curve we can pass continuously from one kind of defects to the second kind but usually we define a transition border between them at around $f_r \simeq 1000\,\mathrm{m}^{-1}$. This distinction is really important since specification of polishing mirrors can be much different depending on the size of the mirror, as we will see. In our case of OMC mirrors, the diameter is supposed to be 5 mm, therefore the minimal frequency defects can not be less than $f_{rmin} = \frac{1}{0.005}\,\mathrm{m}^{-1} = 200\,\mathrm{m}^{-1}$. Such minimal frequency is close to the transition frequency and this optical system would be more sensible to micro-roughness specifications than flatness defects.

3 The simulation

Simulations of OMC had been done with the FFT code OSCAR. Details about this code can be found in following references Degallaix 2010 [3] and on the web page dedicated to OSCAR [4]. The main idea is to take the z axis for beam propagation and express all functions of x and y in a 2D matrix averaging values on each pixel. Propagation of any fields on z axis are computed by calculating two dimensional FFT on x and y axis in order to decompose such fields in a sum of plane waves. More explanation can be found in the manual associated to the code OSCAR [4] or in the article of Vinet et al. [5]. For the real OMC cavity the beam follow the path figure 1.

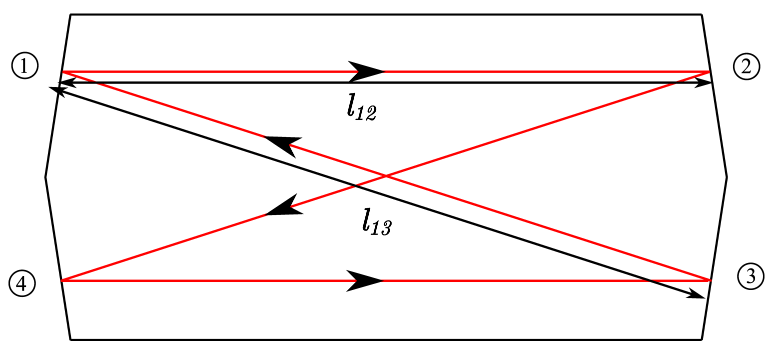


Figure 1: Path of the beam in the real OMC

But for the simulation in OSCAR we will approximate the path of the beam as in figure 2. In this case no angles are taken into account, so no astigmatism can be studied here. Basically, the code works with input parameters such as the grid size N (usually $N = 256$ or 512), the beam waist at entrance of the cavity w_0 and the diameters of mirrors d. Additionally to these basic input parameters it is possible to define other characteristics of incoming beam such as taking an arbitrary position of the waist in the cavity. But the most important supplementary function of OSCAR is to add surface defects on mirrors from known maps or from created fake maps with a chosen PSD. We will use all of these functionalities to calculate resonant, transmitted and reflected beams and losses for the OMC in order to study its optical properties.

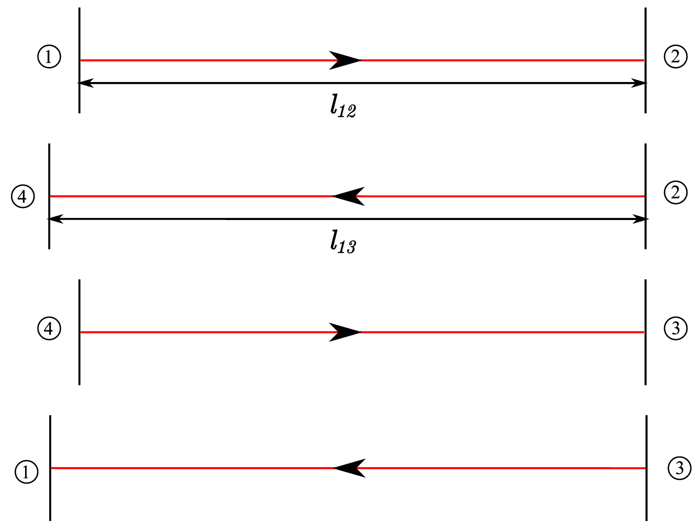


Figure 2: Path of the beam in the OSCAR OMC

4 Results for preliminary specifications

4.1 RMS effects

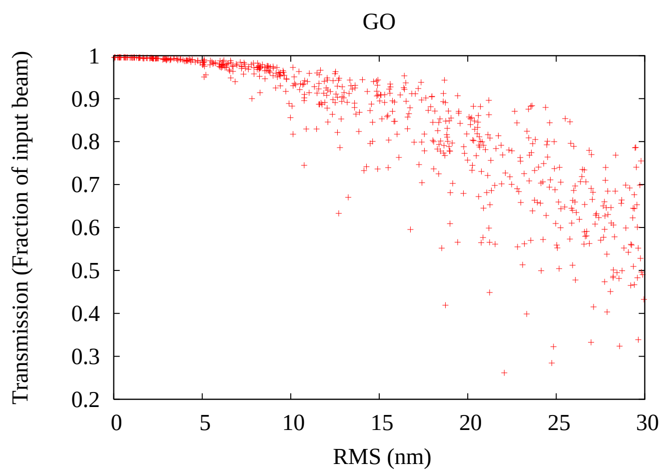
Preliminary pecifications which we would apply to the OMC are a pick to valley less than $\lambda/10 \simeq 100\,\mathrm{nm}$ for the flatness defects. This specification correspond approximately to $20\,\mathrm{nm}$ RMS. We would have also a micro-roughness less than 0.3 nm RMS. In this section we will test the viability of these specifications but since these specifications are not enough to determine the PSD, we will take PSDs obtained for Virgo mirrors made by General Optics (GO) and LIGO mirrors made by CSIRO. We remark that PSDs are approximated on three range of frequencies by a power law $PSD1D(f_r) = A\, f_r^{-n}$ and details about its parameters are given in the table 1.

	$f_r < 82.82\,\mathrm{m}^{-1}$	$82.82\,\mathrm{m}^{-1} < f_r < 400\,\mathrm{m}^{-1}$	$400\,\mathrm{m}^{-1} < f_r$
A_{GO}	$4.7897\,10^{-17}$	$1.115\,10^{-19}$	$1.4782\,10^{-16}$
n_{GO}	-2.8731	-1.5	-2.7

	$f_r < 53.43\,\mathrm{m}^{-1}$	$53.43\,\mathrm{m}^{-1} < f_r < 1089\,\mathrm{m}^{-1}$	$1089\,\mathrm{m}^{-1} < f_r$
A_{CSIRO}	$7.2452\,10^{-16}$	$1.6678\,10^{-24}$	$3.0188\,10^{-16}$
n_{CSIRO}	-4.5219	0.4775	-2.2415

Table 1: Parameters used for GO and CSIRO PSD.

This is enough for a first approach in our study. We did a simulation for 600 random fake maps of GO and 1200 random fake maps of CSIRO for a RMS varying between 0 and 30 nm.



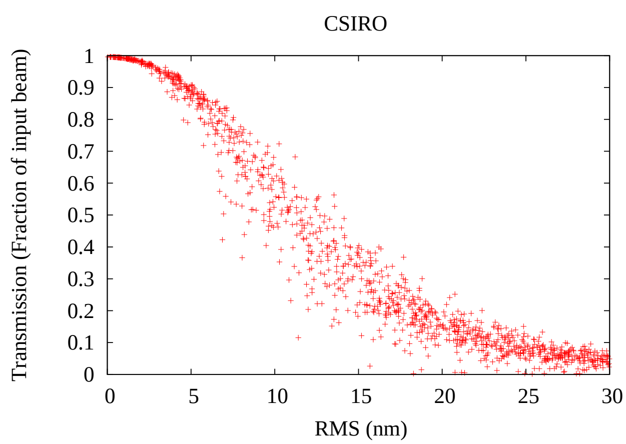


Figure 3: Transmission by the OMC for PSD of GO and CSIRO as a function of the RMS

The figure 3 show the power of transmitted beam in a fraction of the input beam power. We observe for the two PSD laws that transmission is dramatically low at 20 nm RMS and is already too much small even at few nm RMS. The decreasing is quite different for the two PSD laws but the conclusion about specifications is the same, the chosen RMS is too high. We will give more details in the next section about how to define specifications for flatness defects and micro-roughness. Next figures are still interesting since they help us to understand the

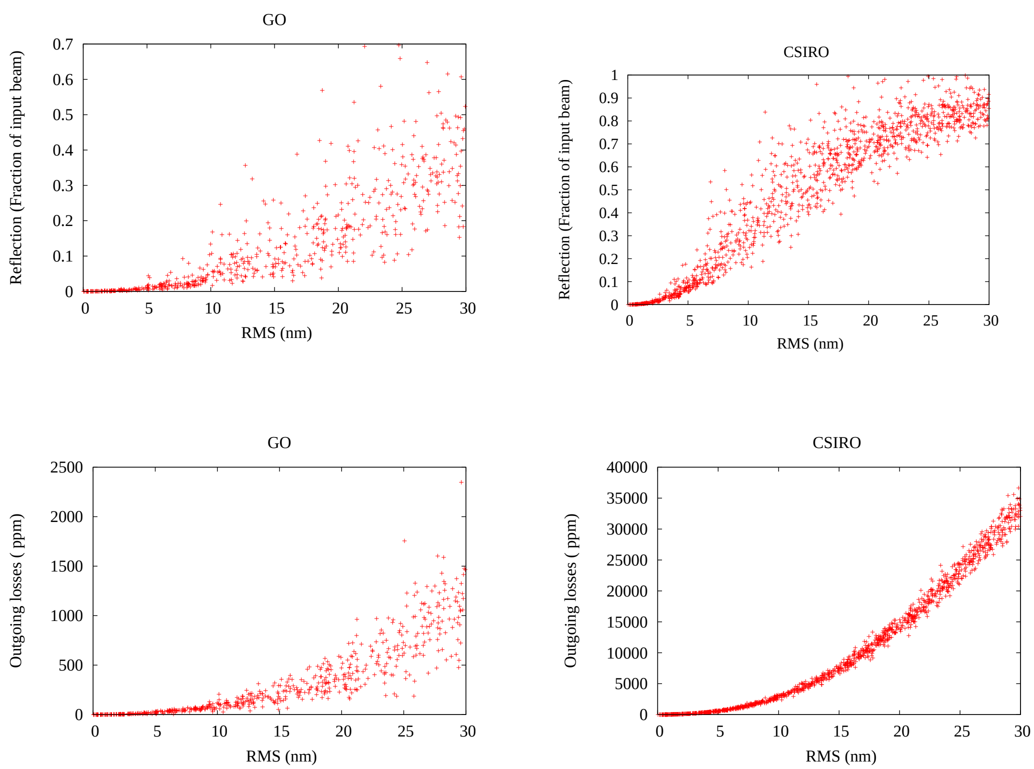


Figure 4: Reflection and outgoing losses by the OMC for PSD of GO and CSIRO as a function of the RMS

behaviour of the cavity. The top of the figure 4 show the fraction of reflected beam and bottom of this figure shows outgoing losses, both as a function of the RMS and for the two PSD laws. The reflection can be interpreted as a consequence of the non matching of the input beam with the cavity. The more the RMS increases, the more the mirrors are deformed and the more the resonant mode moves away from a TEM_{00}mode. The reflection is exactly the difference between the resonant mode and the input mode.To illustrate this phenomenon, the figure 5 show different fields calculated in OSCAR for a set of fake mirrors at 20 nm RMS with the GO PSD.

The outgoing losses are mainly due to the diffusion on mirror micro-roughness defects which is proportional to the square RMS [2]. But as we can see on the figure 4 such losses are very small and reach few percent only for CSIRO PSD at high RMS. We point out the simulation has been done with a grid size of $N = 256$ for a mirror size of $d = 5$ mm, therefore the resolution is $\delta d = 1.95\,10^{-5}$ m. This resolution is enough to consider micro-roughness effect in this simulation.

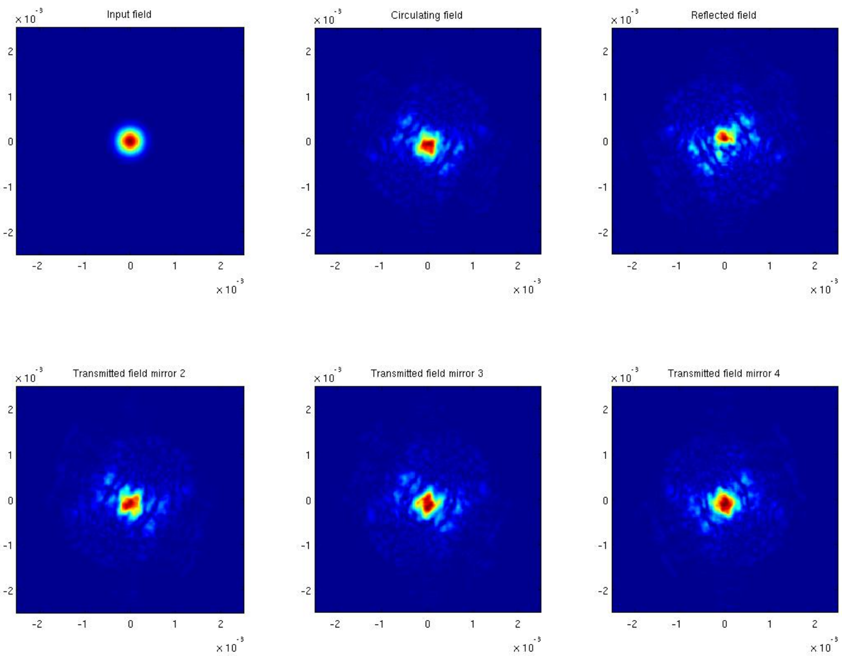


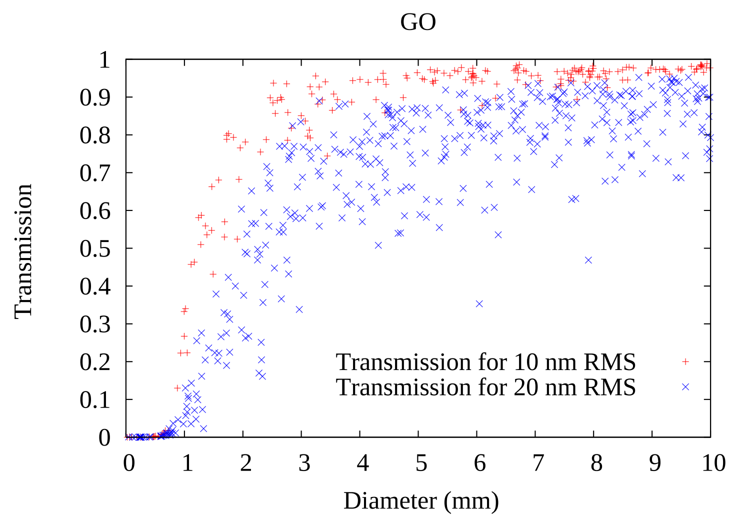
Figure 5: Fields calculated in OSCAR at 20 nm RMS with the GO PSD

4.2 Mirror diameter effects

To be sure we are not dealing with size effects of mirrors we can study losses and transmission dependence on the mirror diameter. We did a 200 (resp. 400) simulations for a RMS of 10 nm (resp. 20nm) with a diameter varying between 0 and 1 cm. We took only GO PSD for mirror maps.

We observe on the top of the figure 6 that the transmission starts to increase dramatically when mirrors size is 2 times greater than the beam size. But for both RMS 10 nm and 20 nm we notice that we have reached optimal size for transmission at around 4 mm of diameter even if points are spread, therefore 5 mm of diameter is enough for the OMC. For outgoing losses we observe that they decrease with the increasing diameter. This results is coherent with the origin of such losses. Then at 5 mm of diameter we observe losses around 100 ppm for 10 nm RMS and 1000 ppm for 20 nm.

We conclude this section by assuming a diameter of 5 mm is optimal for the transmission and high enough to have small outgoing losses. This conclusion means the origin of very small transmission comes from low frequency spatial defects, which are too large for a specification of 20 nm RMS. We will explore more in details this problem in the following section.



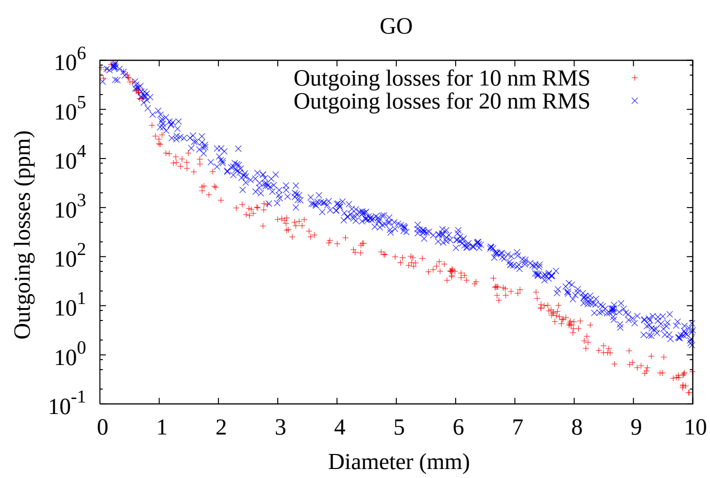


Figure 6: Reflection and outgoing losses dependence on the diameter of OMC mirrors for the GO PSD

5 Redefining specifications

In this section we will try to define new specifications in order to obtain 99 % of transmission. To understand the cause of small transmission and define good specifications we will give generalities about the behaviour of PSD curves and what is done in OSCAR to create fake maps of mirrors. On the figure 7 are shown PSD for several polishers and we observe in general

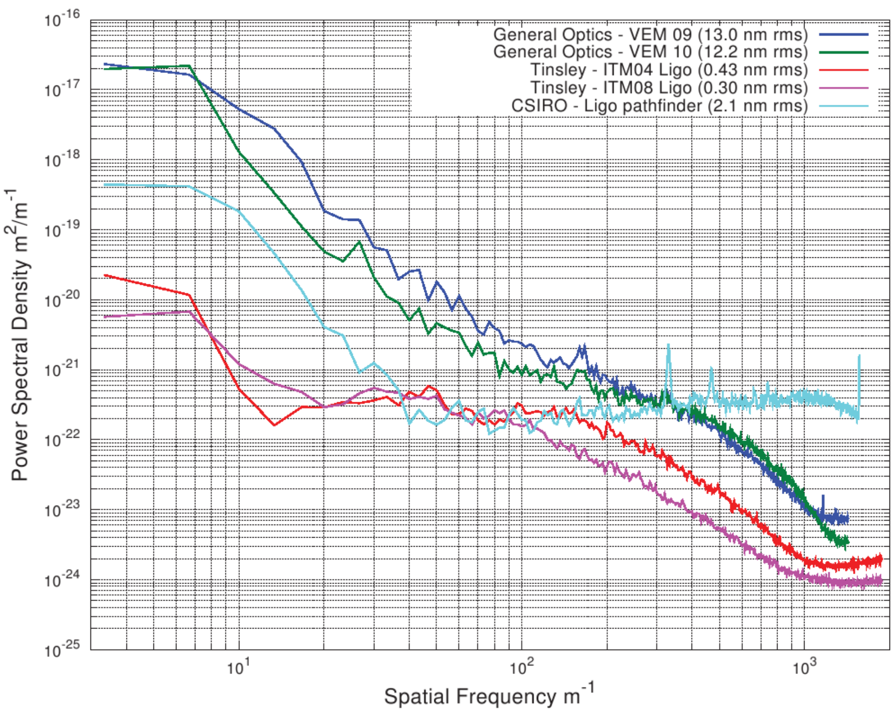


Figure 7: PSD of mirrors for several polishers, Bonnand et al. 2011 [1]

a fast decreasing amplitude of defects with the spatial frequency. But the main point to be mentioned here is the very small amplitude of defects at high frequency $f > 100\text{m}^{-1}$, thus independently of the polishers and the resulting RMS. This means the PSD is dominated by flatness defects and this part of the PSD determine the order of the RMS. The micro-roughness is sensibly independent of the resulting RMS. But we have to remind we create fake maps of mirrors for a given RMS in OSCAR by rescaling the PSD. Taking a mirror of 5 mm and assuming a 20 nm RMS for the fake map would say we have very very bad micro-roughness specifications. This is the reason why we can easily take a much smaller RMS for OMC mirrors. Now, let us finely study the OMC specifications. Since we observed the high impact of PSD on the simulation, the best thing would be to know the PSD shape expected for a given polisher. But unfortunately we have only maps of mirrors large of few hundreds of μm made in sapphire from the THALES-SESO factory, one of the candidate polisher. Therefore the best we could do is to take a simplified PSD as a function of only one power law and try to determine which is the limit for having 99% of transmission.

5.1 Simplified PSD

For this study we will take the following simplified PSD over all frequencies:

$$PSD1D(f_r) = \frac{A}{f_r^n} \tag{4}$$

Since the coefficient A is not relevant for polishers and can be a complex function of the RMS [6], we will fix this coefficient and rescale fake maps in order to obtain a chosen RMS. We did 1300 simulations for the power law $n \in [-0.5, 3]$ and the RMS $\in [0.1\,\text{nm}, 10\,\text{nm}]$ randomly distributed on these intervals.

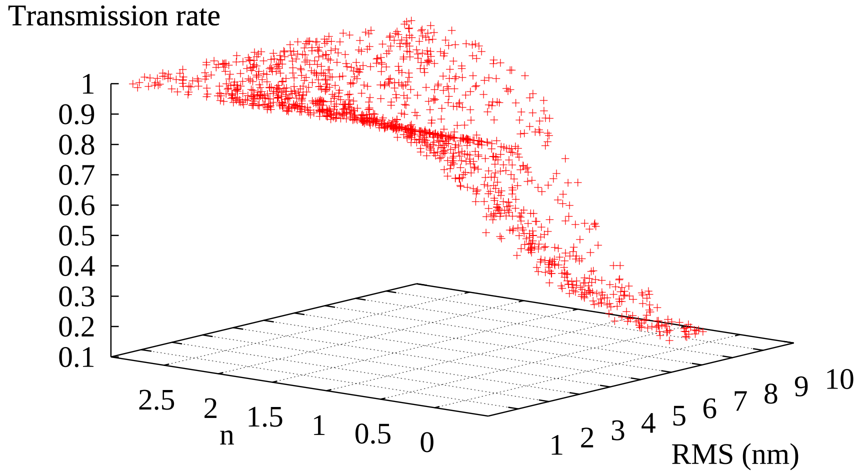


Figure 8: Rate of transmission as a function of the parameter n and the RMS

On the figure 8 are presented the rate of transmission as a function of the the parameter n and the RMS. On this figure we observe that the smaller the RMS is or the higher the power n is, the more the transmission rate is high. The figure 9 show three bands of points (n, RMS) for 99%±0.2, 98%±0.2 and 94%±0.5 of transmission. This figure is much more interesting and gives a lot of information about how to define new specifications. First, we confirm the dependence of the transmission on parameters (n, RMS) we obtained in the previous figure. These bands are representing approximately the contour line for 99 %, 98 % and 94 % of transmission. A better

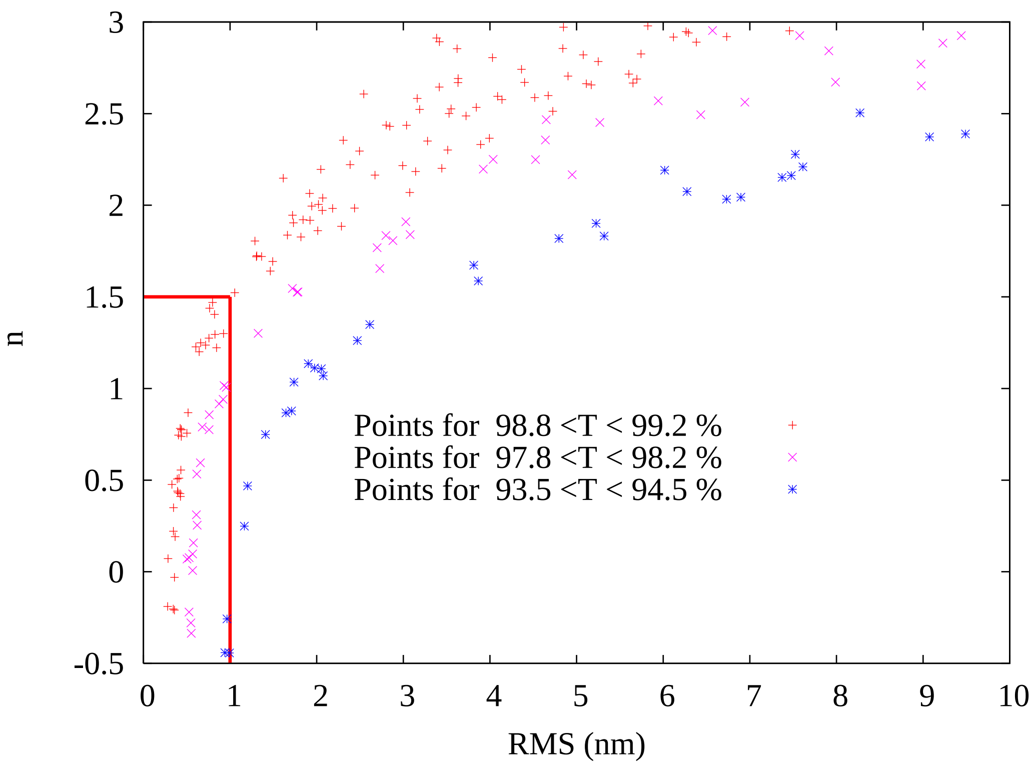


Figure 9: Band of points (n, RMS) for $99\% \pm 0.2$, $98\% \pm 0.2$ *and* $94\% \pm 0.5$ *of transmission.*

performance than a given percentage is obtained by taking parameters above the corresponding contour line. Then, we observe a threshold phenomenon on the RMS. At a RMS less than the threshold we have performance better than the corresponding percentage in any case of power law n. For example at around 0.3 nm RMS we have more than 99 % of transmission for any power law n. Such RMS correspond to the micro-roughness specification and is too excessive for the global RMS down to $f_{rmin} = 200\,\mathrm{m}^{-1}$. A more acceptable specification would be 1 nm RMS for OMC mirrors. At this RMS, we can reach 99 % of transmission by taking a power law n upper than 1.5. Furthermore, with this RMS the threshold ensures us to have more than 94 % of transmission for any power law n. Referring to the figure 7 on the range of frequencies upper than $f_{rmin} = 200\,\mathrm{m}^{-1}$, the power limit $n = 1.5$ is much less than the corresponding slope of PSD law for GO polisher or Tinsley polisher ($n > 1.5$) but the limit is higher than the corresponding slope of PSD law for CSIRO ($n \simeq -0.5$). This means PSDs of polishers don't respect systematically our limit. But we would like also to not require specifications based on giving a power law to polishers. The way to do so is to give micro-roughness specifications. As we will see in the next part, imposing a global RMS of 1 nm and micro-roughness RMS of 0.3

nm over $f_r = 1000\,\mathrm{m}^{-1}$, the power law is forced to be higher than $n = 1.5$.

5.2 Simplified PSD with micro-roughness specifications

We can understand easily when we require a small enough micro-roughness at a given global RMS, we will obtain a high enough power n. But determining precisely such quantities, moreover if we consider realistic PSD, is a difficult problem to solve. Here, we will try to do a simple analysis by taking the simplified PSD we used in equation 4. With this simplified PSD it will be possible to integrate analytically the equation 3 which give the RMS. In this study we will calculate the RMS on a frequency range, such as $[f_1, f_2]$. From equations 3 and 4 we obtain:

$$RMS^2_{f_1 f_2} = \int_{f_1}^{f_2} \frac{A}{f_r^n}\, df_r = \frac{A}{-n+1}\left[f_2^{-n+1} - f_1^{-n+1}\right] \tag{5}$$

Three characteristic frequencies will be considered, the smallest f_{rmin}, the cut-off frequency f_c above which we are dealing with micro-roughness defects and the maximum frequency f_{rmax}. The minimal frequency is limited by the size of mirrors and is given in the first section, *i. e.* $f_{rmin} = 200\,\mathrm{m}^{-1}$. A more elaborated theory would say the minimal frequency is higher but the one we choose correspond to the worst case.Then, the cut-off frequency f_c is not well determine, often it is assumed $f_c = 1000\,\mathrm{m}^{-1}$ which correspond to defects of the order of the 1 mm. But we will take three other cut-off frequencies $f_c = 2000\,\mathrm{m}^{-1}$, $f_c = 4000\,\mathrm{m}^{-1}$ and $f_c = 10000\,\mathrm{m}^{-1}$ in order to make complete this study. Finally, the maximum frequency should be infinite but $f_{rmax} = 51200\,\mathrm{m}^{-1}$, which is the maximum frequency considered in our configuration of OSCAR, is enough. We observed that higher frequencies do not change significantly the results. We will now take the previously defined specifications, so 1 nm RMS for mirror surfaces defects and 0.3 nm RMS for the micro-roughness. The first specification corresponds to the whole integration, $RMS^2_{total} = RMS^2_{f_{rmin} f_{rmax}} = 1\,\mathrm{nm}^2$ and the second corresponds to a partial integration $RMS^2_{micro} = RMS^2_{f_c f_{rmax}} = 0.09\,\mathrm{nm}^2$. The aim of this part is to study the ratio $\frac{RMS^2_{total}}{RMS^2_{micro}}$ as a function of n in order to confirm that for the value $\frac{RMS^2_{total}}{RMS^2_{micro}} = 1/0.09 = 11.11$ we have a power $n > 1.5$. On the figure 10 are represented the ratio $\frac{RMS^2_{total}}{RMS^2_{micro}}$ as a function of n for different cut-off frequencies. The smallest power n for the ratio at a value 11.11 correspond to the cut-off frequency $f_c = 10000\,\mathrm{m}^{-1}$. In this case we have $n = 1.47$ which is close to the minimal limit for having 99 % of transmission. But we can say such frequencies are much higher than the limit for micro-roughness. At $f_c = 1000\,\mathrm{m}^{-1}$ we find a value of $n = 2.49$ which is much higher than the minimal limit. The conclusion for this study based on a simple PSD is if we take a micro-roughness specification of 0.3 nm at higher than $f_c = 1000\,\mathrm{m}^{-1}$ and a global RMS defect of 1 nm we would ensure us approximately to have 99 % of transmission by the OMC.

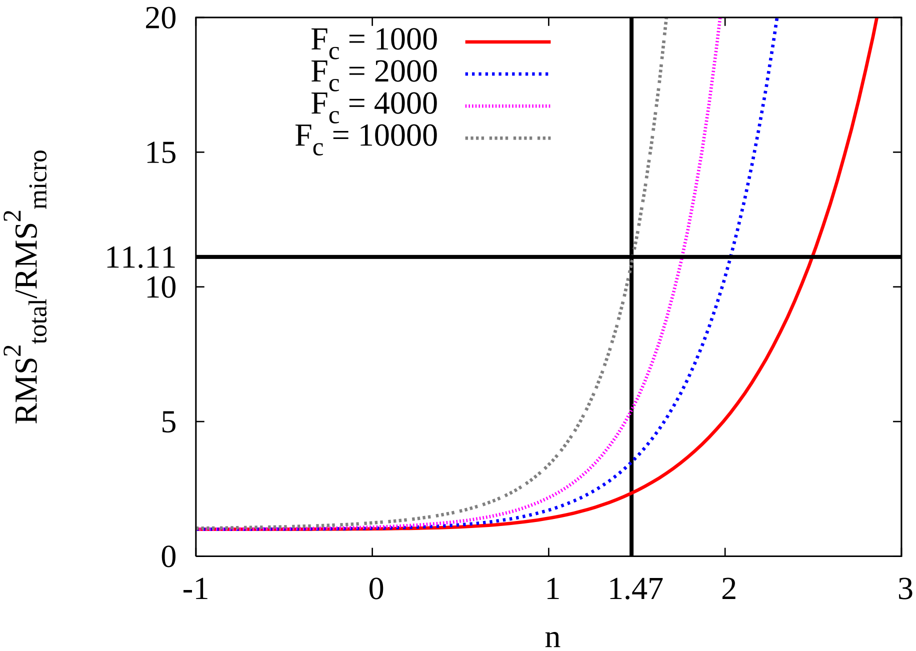


Figure 10: Ratio $\frac{RMS^2_{total}}{RMS^2_{micro}}$ as a function of n for several cut-off frequencies.

5.3 Extrapolation of SESO PSD

In this section we will try to take advantages of having mirror maps from the SESO factory, one of the candidate polisher. We are confronted to two difficulties with these maps. First, the map for the plane mirror has a size of 800 μm and the map for the spherical mirror has a size of 200 μm which are much less than OMC mirrors and this will oblige us to extrapolate PSD obtained from these maps. The second difficulty comes from the substrate of such mirror maps which is the sapphire. This is a much different substrate than the silica, *i. e.* the OMC substrate, but the SESO confirmed us their specifications for polishing silica are better than for the sapphire. Therefore we will take such maps as an upper limit. The good point is even if we are dealing with very smal surfaces made of sapphire we have very small RMS for each mirror, *i. e.* 0.165 nm for the plane one and 0.214 nm for the sopherical one.

In the figure 11 are shown the PSD calculated from SESO maps for plane and spherical mirrors made in sapphire. We will interpolate these PSD with three segments for different range of frequencies and we will extrapolate the line for low frequencies down to 200 m^{-1} for OMC mirrors simulations. On the figure 12 are shown the PSD from SESO maps, the approximated

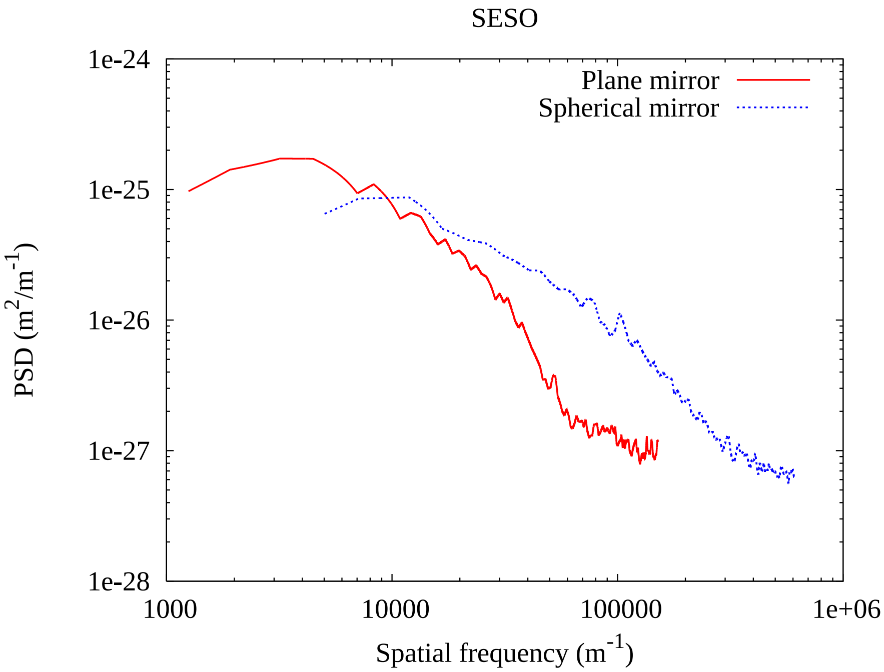


Figure 11: SESO PSD for plane and spherical mirrors made in sapphire

PSD and the PSD resulting from one example of fake map obtained with OSCAR, all of these for plane mirrors. And the same are shown on the figure 13 but for spherical mirrors. Only here the PSD is extrapolated for a larger range of frequencies than for plane mirrors.

	RMS Plane (nm)	RMS Spherical (nm)	Transmission (rate of input)	Outgoing losses (ppm)
Mean	0.58924	0.27131	0.99442	16.920
Min	0.42171	0.24915	0.99428	16.248
Max	0.80032	0.30853	0.99460	17.679

Table 2: Summary table of results for 20 simulations with extrapolated SESO PSD

On the table 2 are summarised results for 20 simulation of OMC for approximated SESO PSD. We observe from these simulations the resulting RMS for mirrors is less than we would specify for the SESO polisher. The deviation of RMS between plane and spherical mirrors is not realistic and is caused for sure by the extrapolation. These RMS values should be regarded more as a order of magnitude as well determined values. The main result emerging from these simulations is the transmission rate which is no less than 99.4 %. We can conclude even if the

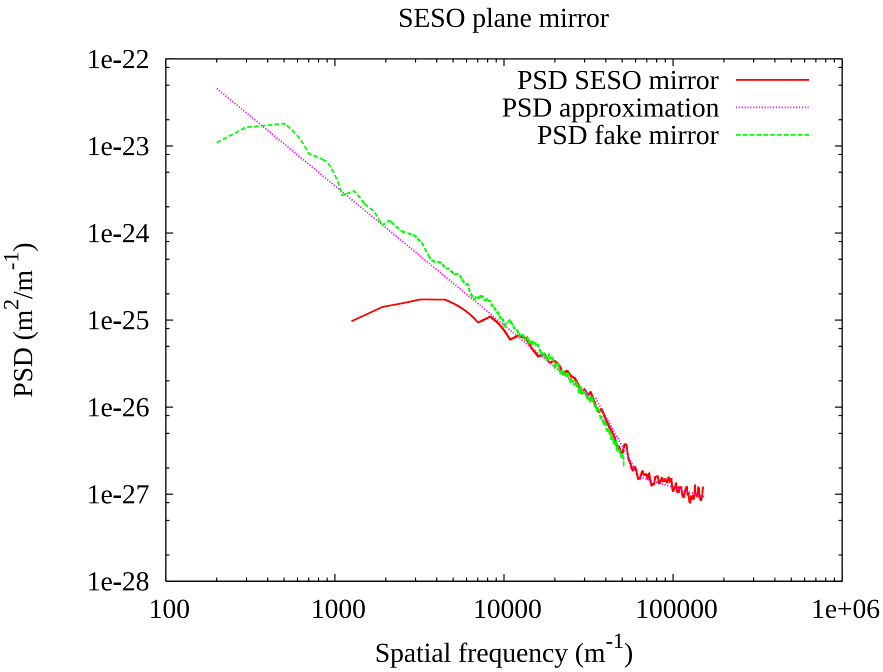


Figure 12: PSD, approximate PSD and fake mirror PSD for SESO plane mirrors made in sapphire

RMS change a little in the reality, we can expect to stay in our specification of more than 99 % of transmission. The outgoing losses are in any case very low and confirm to us that PSD give very good results.

To finish the study of surface specification we can rescale the approximate SESO PSD for a chosen RMS in order to estimate the transmission of the OMC as a function of the RMS.

We observe on the figure 14 a transmission around 99 % for a RMS $\simeq 0.73\,\mathrm{nm}$ and a transmission around 98.6 % at 1 nm. This results should leads us to define a less RMS than 1 nm for the specification but we should take care with this conclusion because the PSD used for the SESO maps are strongly extrapolated and for spherical mirrors we have a power law $n = 1$ which is less than the value defined in the previous section 5.2. We remind also the SESO can do a better polishing for silica. This mean, we can easily ask for polisher a specification which ensure us a higher transmission than 98.6 %.

6 Conclusion

The OSCAR software developed by Jérome Degallaix [3] had been a tool very helpful for simulate and study mirror surfaces of the Advanced Virgo OMC. The first and general conclusion is global RMS specification for surfaces must be adapted to the size of the surface. Through our

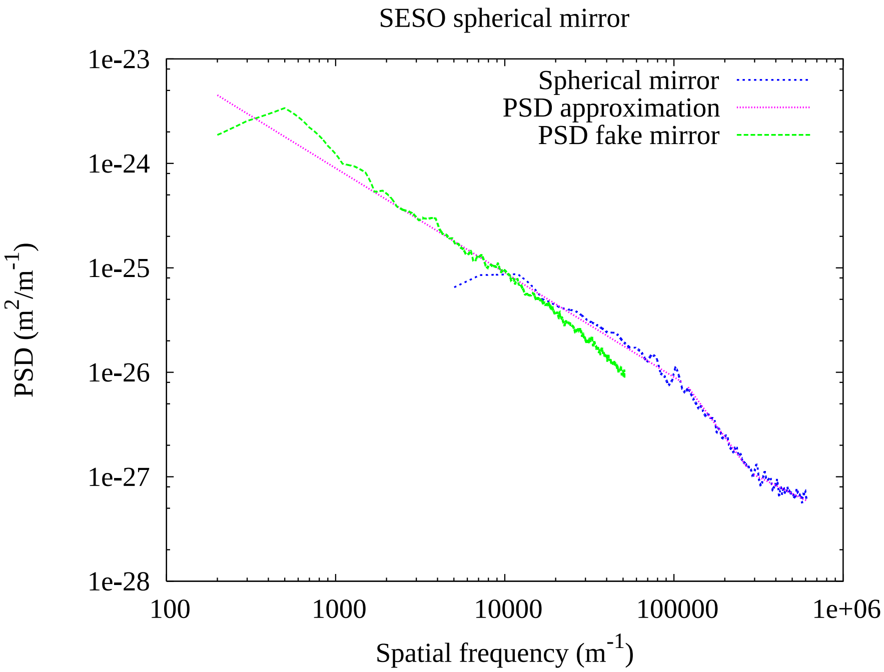


Figure 13: PSD, approximate PSD and fake mirror PSD for SESO spherical mirrors made in sapphire

study we redefined initial surface specifications in order to ensure a transmission of 99 % for the OMC with a TEM_{00}mode at the entrance. We have found three kind of specifications for this problem. The first one is given in the section 5.1, the best way to get 99 % of transmission would be to impose a RMS of 0.3 nm on the whole mirror of 5 mm. This specification seems to be independent to the behaviour of the PSD but could be too much strong for polishers. The second specification is to take a higher RMS and impose to the polisher to have a PSD lower than a determined power law. As a realistic example we could impose a RMS of 1 nm on the whole mirror and a PSD lower than the power law $PSD1D(f_r) = A\, f_r^{-1.5}$, *i. e.* $n > 1.5$. Since this specification is not usual for polisher we can force their PSD to be approximately lower than such power law by imposing micro-roughness specification. The previous example is reached by taking a RMS of 1 nm on the whole mirror and a micro-roughness RMS of 0.3 nm over the spatial defects frequency of $f_c = 1000\,\mathrm{m}^{-1}$. Finally we can ask to the SESO to do an equivalent or better polishing than they have done for sapphire mirrors. In conclusion we can show the fields calculated by OSCAR for a fake map created with the simplified PSD for 1 nm RMS and a power law $n = 1.5$. On the figure 15 we can see the "gaussianity" of all fields except the reflected field of very small power because of defects on surfaces.

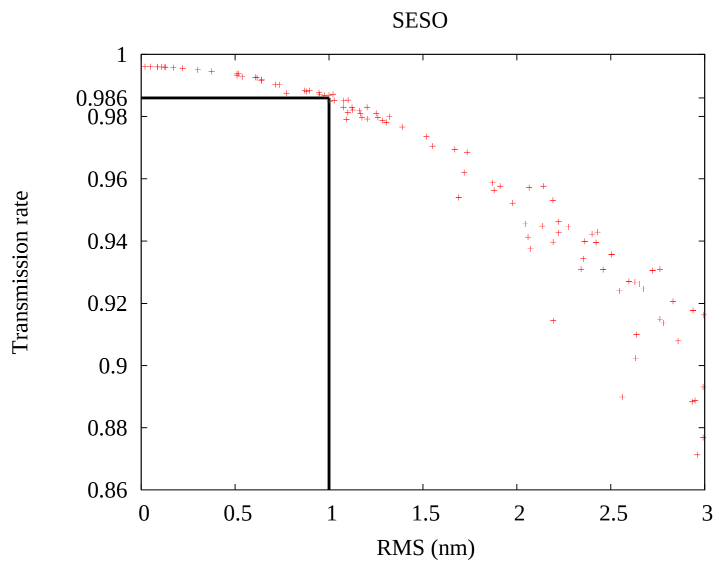


Figure 14: Transmission rate of the OMC for approximated SESO PSD as a function of the RMS

Acknowledgements

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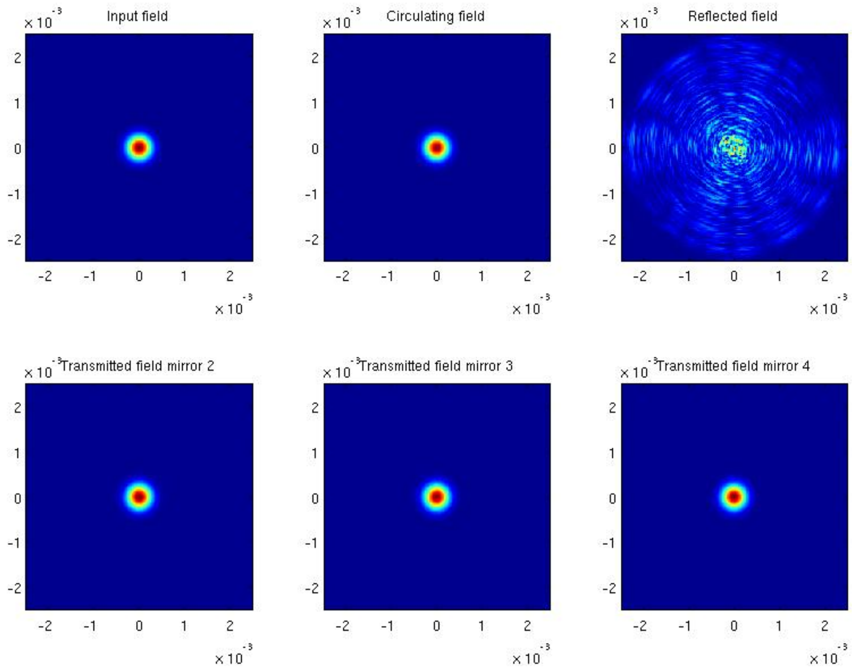

Figure 15: Fields calculated in OSCAR with the simplified PSD for 1 nm RMS and a power law $n = 1.5$.

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