Propagation through a prism (POP, CP,BS…)

And numerical simulations

VIR-0224A-15

Does propagation through an optical element having a wedge angle cause astigmatism ? We conclude that the effect exists in principle, but is well below the accuracy of current numerical propagation codes at least for the existing wedges.

The wedge is α, the incidence angle is β (see Fig.1 below)

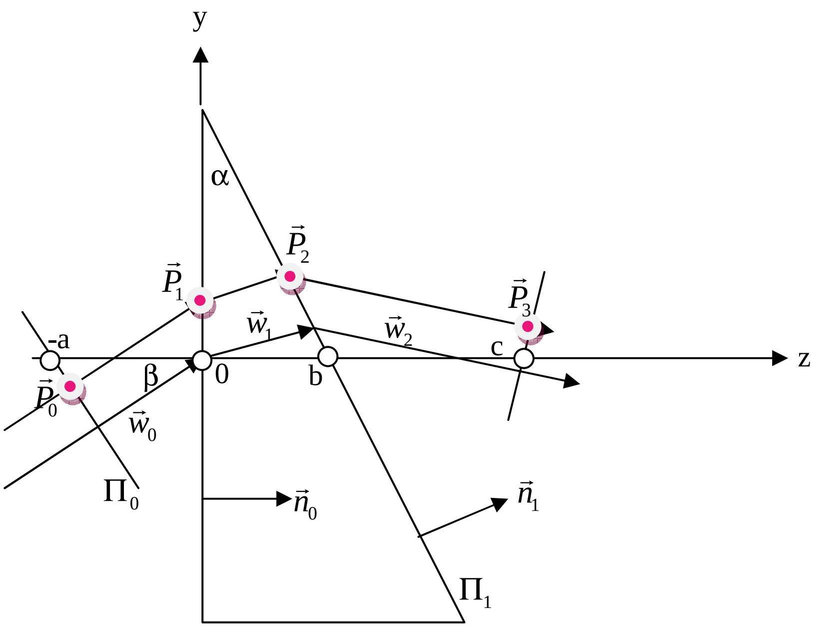

Fig.1

The direction of incidence is determined by the unit vector $\vec{w}_0 = (0, \sin\beta, \cos\beta)$. The starting point is $\vec{P}_0$. We assume $\vec{P}_0$ to be in a plane Π_0 orthogonal to $\vec{w}_0$, of equation $y\sin\beta + (z+a)\cos\beta = 0$ where a is an arbitrary distance ; so that $\vec{P}_0 = (x, y, -a - y\tan\beta)$. The next point on the path, $\vec{P}_1 = \vec{P}_0 + \lambda_0 \vec{w}_0$ is obtained by solving for λ_0 the intersection with the plane $z = 0$:

$$\lambda_0 = \frac{a + y\tan\beta}{\cos\beta} \Rightarrow \vec{P}_1 = (x, a\tan\beta + \frac{y}{\cos^2\beta}, 0)$$

Then the refraction law gives the new unit vector $\vec{w}_1$:

$$\vec{w}_1 = \frac{1}{N}\vec{w}_0 - \left[\frac{1}{N}\vec{w}_0.\vec{n}_0 - \sqrt{1-\frac{1}{N^2}\left(1-(\vec{w}_0.\vec{n}_0)^2\right)} \right]\vec{n}_0 = \left(0, \sin\gamma, \cos\gamma\right)$$

with $\sin\gamma \equiv \frac{1}{N}\sin\beta$ (as expected !)

Then the next point $\vec{P}_2$ is obtained by solving for λ_1 the intersection of $\vec{P}_1 + \lambda_1\vec{w}_1$ with the plane Π_1 of equation $y\sin\alpha + (z-b)\cos\alpha = 0$. A new refraction at $\vec{P}_2$ gives the new direction :

$\vec{w}_2 = N\vec{w}_1 - \left[N\vec{w}_1.\vec{n}_1 - \sqrt{1-N^2\left(1-(\vec{w}_1.\vec{n}_1)^2\right)} \right]\vec{n}_1$, and a new point $\vec{P}_3 = \vec{P}_2 + \lambda_2\vec{w}_2$ in the plane Π_3 orthogonal to $\vec{w}_2$, of equation $\vec{w}_2.(\vec{r} - c\vec{n}_0) = 0$ (c being an arbitrary distance). This gives :

$$\vec{P}_3 = \vec{P}_2 - \left(\vec{P}_2.\vec{w}_2\right)\vec{w}_2 + c\left(\vec{n}_0.\vec{w}_2\right)\vec{w}_2$$

Deformation

We obtain the effect of the propagation on a bundle of rays by varying $\eta = y\cos\beta$ in the plane Π_0 , the transverse extension of the bundle being : $(d\xi = dx, d\eta = dy\cos\beta)$. The variation of the extension in the final plane Π_3 is $d\vec{P}_3 = (\vec{u}_0 \times \vec{w}_2).\left(\vec{P}(y+dy) - \vec{P}(y)\right)$ ($\vec{u}_0$ being the unit vector of the x axis) giving :

$$\delta\xi = d\xi, \delta\eta = d\eta\frac{N\cos\gamma\sqrt{1-\left(N\sin\alpha\cos\gamma - \sin\beta\cos\alpha\right)^2}}{\cos\beta\left(\sin\alpha\sin\beta + N\cos\alpha\cos\gamma\right)}$$

Obviously, there is no dilatation in the x direction, whereas the dilatation in the orthogonal direction is

$$\Delta(\alpha,\beta) = \frac{N\cos\gamma\sqrt{1-\left(N\sin\alpha\cos\gamma - \cos\alpha\sin\beta\right)^2}}{\cos\beta\left(N\cos\alpha\cos\gamma + \sin\alpha\sin\beta\right)} \quad \left(\cos\gamma \equiv \sqrt{1-\frac{\sin^2\beta}{N^2}}\right)$$

For β=0, this is simply

$$\Delta(\alpha,0) = \frac{\sqrt{1-N^2\sin^2\alpha}}{\cos\alpha}$$

And for small α :

$$\Delta(\alpha,0) = 1 + \frac{N^2-1}{2}\alpha^2 + \mathcal{O}(\alpha^4)$$

For $N \sim 1.456$, we get $\frac{\Delta y}{y} \approx 0.56 \times \alpha^2$ (negligible for a CP).

For $\beta = \pi / 4$, this is

$$\Delta(\alpha, \pi/4) = m \frac{\sqrt{2-(m\sin\alpha - \cos\alpha)^2}}{m\cos\alpha + \sin\alpha} \quad \left(m \equiv \sqrt{2N^2 - 1}\right)$$

And for small α

$$\Delta(\alpha, \pi/4) = 1 + \frac{m^2-1}{m}\alpha - \frac{m^4-1}{m^2}\alpha^2 + \mathcal{O}(\alpha^4)$$

For the beamsplitter ($\alpha \simeq 400\,\mu\text{Rd}$, $\beta = \pi/4$) we get $\frac{\Delta y}{y} \approx 5 \times 10^{-4}$

For a POP (α =1 mRd, $\beta = \pi / 30$) we have $\frac{\Delta y}{y} \approx 8 \times 10^{-5}$

Deviation

We find the deviation angle φ by $\cos\varphi = \vec{w}_0 . \vec{w}_{2.}$ giving :

$$\cos\varphi = \sin(\alpha - \beta)\left[N\sin\alpha\cos\gamma - \cos\alpha\sin\beta\right] + \cos(\alpha-\beta)\sqrt{1 - N^2 + (N\cos\alpha\cos\gamma + \sin\alpha\sin\beta)^2}$$

For small α and $\beta = 0$, this is :

$$\cos\varphi = 1 - \frac{(N-1)^2}{2}\alpha^2 + \mathcal{O}(\alpha^4) \quad \Rightarrow \quad \varphi \approx (N-1)\alpha$$

For

For β =45° and small α :

$$\cos\varphi = 1 - (N^2 - m)\alpha^2 + (N^2-1)(m-1)\alpha^3 + \mathcal{O}(\alpha^4) \quad \Rightarrow \quad \varphi \approx \alpha\sqrt{2(N^2 - m)}$$

Numerical consequences

Consider a propagation code in which the beam is sampled in a square window $\left[-\frac{F}{2},\frac{F}{2}\right]$. The coordinates (x,y) are both obtained by $x_i, y_i = -\frac{F}{2}+\frac{i-1}{n-1}F \quad (i=1,...,n)$. If now we have a widening of the beam, or equivalently a narrowing $\Delta^{-1}<1$ of the y coordinate, we get for the same value of the beam amplitude a different index : $i' = i-(1-\Delta^{-1})\left[i-\frac{n+1}{2}\right]$. The maximum shift in index corresponds to $i=1$, or $i=n$ (the two extreme values of y). This gives a maximum shift of $\delta i_{\text{max}} = -(1-\Delta^{-1})\frac{n+1}{2}$. Even in the case of the beamsplitter $(1-\Delta^{-1}\sim 5\times 10^{-4})$ and with a $n=1024$ sampling, we have $\delta i_{\text{max}} \sim -0.25$, which is thus below the sampling accuracy.

Conclusion : **no effect in practice**.