Upper Limits and Constraints on Stochastic Backgrounds with non-Standard Polarizations

with Advanced Detectors

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The stochastic background of GWs

There are strong theoretical motivations to expect that the Universe is permeated by a

Stochastic Background of Gravitational Waves (SGWB),

similar to the cosmic electromagnetic background radiation (CMB), generated by the superposition of a large number of independent, uncorrelated and unresolved gravity-wave (GW) sources:

- **Astrophysical sources**, mostly located in our galaxy or within red-shift $z \lesssim 4$, such as:
  - supernovae collapses, rapidly rotating neutron stars, and coalescing binary systems of compact objects, e.g. neutron stars, white dwarfs and black holes; [Regimbau, 2011]
- **Cosmological processes**, that took place in the very early Universe, $\sim 10^{-22} \div 10^{-17}$ sec after the Big Bang, at energy densities of 10^{19} GeV.
  Large uncertainties arise due either to the fact that we must use physics beyond the Standard Model, or to uncertainties in the details of the cosmological mechanisms.

  Some possible models are: [Maggiore, 2000]
  - Inflation*: amplification of vacuum fluctuations at the transition between inflationary and radiation-dominated (RD) phase; [Turner, 1997]
  - "Stiff ($w > 1/3$) energy" between inflation and RD era; [Boyle and Buonanno, 2008]
  - Cosmic strings: one dimensional topological defects formed during phase transitions, produce GWs with their relativistic oscillations; [Siemens et al., 2007]
  - Pre-Big Bang models, based on superstring theories; [Mandic and Buonanno, 2006]
  - Axion infl.: backreaction on the inflaton extends inflation; [Barnaby et al., 2012]
  - Phase transitions: GWs from bubbles collisions, turbulence or scalar field relaxation;
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Importance of studying the SGWB
and Alternative Theories of Gravity

Any SGWB takes trace of the process(es) that produced it. A detection would provide invaluable information about:

- the early Universe cosmology, far beyond the current understandings, at times and energy densities never accessible with any other means (*e.g.* the EM radiation: $t_{\text{dec}} \sim 10^5$ sec);

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- correspondingly high-energy physics, beyond the Standard Model of particle physics: strings, supersymmetries, higher dimensions, quantum gravity...;

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- **Alternative Theories of Gravity**, that is, those theories different from Einstein's General Relativity (GR). There are several reasons to introduce (and test) these theories:
  - they are important in order to test GR itself: it is useful to consider some alternative theories of gravity and see precisely how their physical predictions differ from those of GR;
  - Extended Theories of Gravity (ETGs), that is those theories based on corrections and enlargements of GR, "emerge" in effective actions describing the low energy limit of models for the unification of fundamental interactions (like superstrings, supregravity, GUTs);
  - ETGs are also introduced to correct some issues with GR, both cosmological and astrophysical (Mach's principle, dark energy, coincidence problem, monopole problem) and mathematical (Palatini formalism, minimal vs non-minimal couplings, extra spatial dimensions);
  - they could be a step toward the solution for the gravity quantization problem: since the efforts in unifying quantum field theory with GR have not been fully successful, it is important to look for other classical theories of gravity to quantize.

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Gravitational Waves in Alternative Theories of Gravity

There is an enormous variety of Alternative Theories of Gravity in literature (see [Clifton et al., 2012] and [Capozziello and Faraoni, 2010]). They can differ mainly through:

- the gravitational action and the equations of motion, ($f(R)$ theories)
- the presence of additional dynamical gravitational fields, (Brans-Dicke, Einstein-Æther theories)
- higher spatial dimensions, (Kaluza-Klein, DGP braneworld)
- prior geometries, (bimetric theories, stratifield theories)
- etc...

Nevertheless, most of the “viable” theories have in common that:

- they can be described by a symmetric rank-$(0,2)$ tensor, *the metric*, which completely determines the interaction of gravity with massive bodies;
- they must incorporate the request of local Lorentz invariance;
- they are based on second order differential equations, at lest through a suitable conformal transformation.

⇒ Since the *wave operator* (the D'Alambertian, $\Box$) is the Lorentz invariant 2-nd order differential operator, most gravitational theories admit wave-like solutions (GWs) [Will, 1993].

GWs predicted by different theories could differ through:

- the propagation speed (*e.g.* in case of massive gravitons or extra-dimensions);
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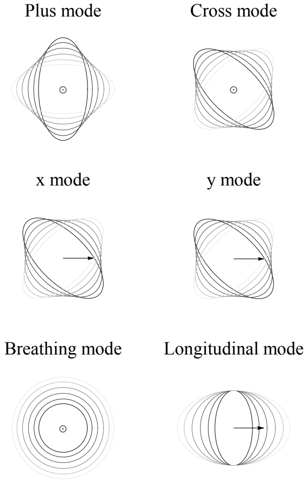
Non-standard polarizations for alternative theories

It can be shown that any general theory of gravity, with additional fields, degrees of freedom, massive gravitons, or extra dimensions (once projected on our 3-space) can allow, at most, six polarization modes of a GW [Eardley et al., 1973]:



Theoretical Model	e_{ij}^{+}	$e_{ij}^{\times}$	e_{ij}^{b}	e_{ij}^{ℓ}	e_{ij}^{x}	e_{ij}^{y}
Einstein General Relativity	*	*				
GR in a noncompactified 5D sp.	*	*	*[1]	*[1]	*	*
GR in a noncompactified 6D sp.	*	*	*	*	*	*
5D Kaluza-Klein theory	*	*	*		*	*
Randall-Sundrum braneworld	*	*				
DGP braneworld (normal branch)	*	*				
DGP braneworld (acceler. branch)	*	*	*[2]	*[2]		
Brans-Dicke theory	*	*	*[2]	*[2]		
$f(R)$ theory	*	*	*[2]	*[2]		
Bimetric theory	*	*	*[2]	*[2]	*	*

TABLE: GW polarization modes for various viable theories of gravity. Table taken from [Nishizawa et al., 2009].

Notes: [1]These modes are correlated and behave as 1 degree of freedom. [2]If $m_g = 0$ than the ℓ-mode vanishes; if $m_g \neq 0$ then the b- and ℓ-modes are correlated.

FIGURE: effect of different polarization modes on a circular array of test masses. Each polarization mode can be identified by its rotational symmetry around the GW propagation axis.

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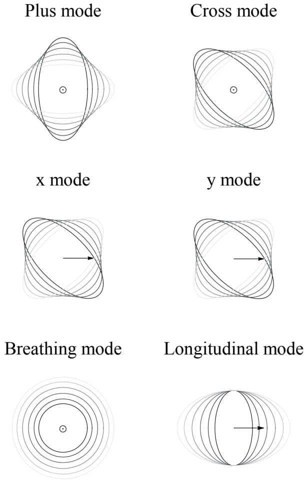

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SGWB: assumptions and characterization

We make some assumptions, and "first-order" approximations, in order to study a very general SGWB, produced by any mechanism within the paradigm of any generic theory of gravity:

- Stationarity: it means that its statistical properties must not change for all the duration of our experiments (usually several orders of magnitude shorter than the SGWB time scales);
- Gaussianity: justified by the central limit theorem if the number of independent sources that contribute to the SGWB is large enough;
- Isotropy: that is, no preferred directions, as it is, in first approximation, for the CMB.

All these assumptions are well justified for a background of cosmological origin. On the other hand, in increasing order of approximation, they may not hold for an SGWB of astrophysical origin if the number of sources is small and they are distributed mostly in our galaxy.

If we take these assumption as true, the most general SGWB we are looking for:

- can be described at most by six modes of polarization: two tensor circular polarizations (± 2), two vector circular polarizations (± 1) and two scalar modes (b and ℓ);
- it can be fully characterized by the two point correlator of the signal outputs of a sufficient number of detector pairs (ij):

$$h_i(t) = h_{ab}(t)F_i^{ab}\,; \qquad \langle \tilde{h}_i^*(f)\tilde{h}_j(f')\rangle = \delta(f-f')\sum_A \frac{1}{2}S_h^A(f)\Gamma_{ij}^A(f)$$

$A = \pm 2, \pm 1, 0$ and $i, j = 1, 2, \ldots, N$ for a network of N GW detectors.

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Cross-correlation analysis to detect an SGWB

Our aim is **to detect the SGWB measuring its power spectrum density** $S_h(f)$, or, equivalently, its energy density per logarithmic frequency: [Nishizawa et al., 2009]

$$\Omega_{\text{gw}}^A(f) \equiv \left(\frac{2\pi^2}{3H_0^2}\right) f^3 S_h^A(f), \qquad \text{for every mode } A = \pm 2, \pm 1, 0$$

If we also assume that the noises of the i-th and the j-th detector are stationary, gaussian and uncorrelated,

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where $s_i(t) = h_i(t) + n_i(t)$ is the output of the i-th detector, then we can perform the "standard cross-correlation analysis", first developed by Flanagan and Christensen.

For a power-law template for the SGWB energy density,

$$\Omega_{\text{gw}}(f) = \Omega_\nu \left(\frac{f}{f_0}\right)^\nu \qquad \text{where } \Omega_\nu \text{ is a constant and } f_0 \text{ a frequency of reference,}$$

we can find the minimum detectable SGWB [Allen and Romano, 1999]:

$$\Omega_\nu \geqslant \frac{1}{\sqrt{T_{\text{tot}}}} \frac{10\pi^2}{3H_0^2} \left[\int_{-\infty}^{+\infty} \mathrm{d}f \left(\frac{f}{f_0}\right)^{2\nu} \frac{\gamma^2(|f|)}{f^6 P_1(|f|) P_2(|f|)}\right]^{-1/2} \sqrt{2}\left(\operatorname{erfc}^{-1}(2\alpha) - \operatorname{erfc}^{-1}(2\gamma)\right)$$

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Non-standard cross-correlation analysis

Here we want to extend the Neyman-Pearson (NP) hypothesis test [Kay, 1998] for the detection of an SGWB signal, as discussed by Allen and Romano [1999], in order to include the possibility of non-standard polarizations.

The key ingredient that permits to perform this extension are the overlap reduction function for non-standard polarizations:

$$\gamma_{ij}^{A}(f), \qquad \text{for} \quad i,j = 1,\dots,N, \quad \text{and} \quad A = \pm 2, \pm 1, 0.$$

This is the algorithm we are using to investigate this SGWB within the LIGO S5 and VIrgo VSR 3 data:

- First of all, inverting the equation for $\langle \tilde{h}_i^*(f)\tilde{h}_j(f')\rangle$, we find the Maximum Likelihood Estimator (MLE) for the component $S_h^A(f)$ of the power spectrum density of the SGWB;
- Then, we construct a Generalize Likelihood Ratio Test statistics using the p.d.f.s for the detector output signals in the case of the Null hypothesis of no SGWB signal ($S_h(f) = 0$) and for the Alternative hypothesis of the presence of a signal ($S_h(f) \neq 0$);
- finally, we choose a false alarm probability (say, 5% or 1%) and we perform the NP test.

$$\Omega_\nu^A \geqslant \frac{1}{\sqrt{T_{\text{tot}}}}\frac{10\pi^2}{3H_0^2}\left[\int_{-\infty}^{+\infty} df \left(\frac{f}{f_0}\right)^{2\nu} \frac{{\gamma^A}^2(|f|)}{f^6 P_1(|f|)P_2(|f|)}\right]^{-1/2} \sqrt{2}\left(\text{erfc}^{-1}(2\alpha) - \text{erfc}^{-1}(2\gamma)\right)$$

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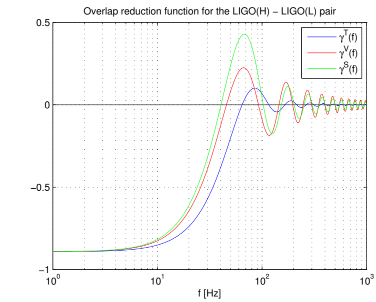
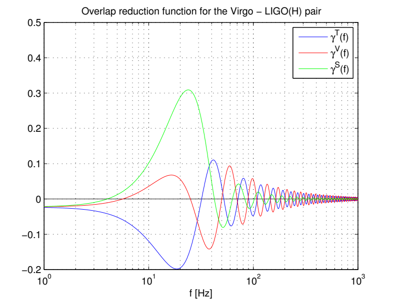
Overlap Reduction Function





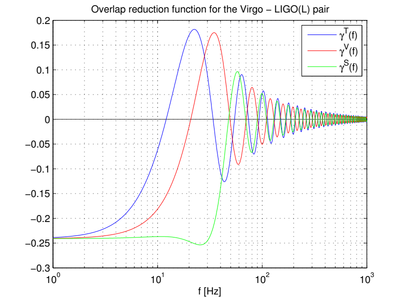


Figure: ORF for the different detector pairs Virgo - LIGO(L), Virgo - LIGO(H) and LIGO(H) - LIGO(L).Note how the difference of the behavior between the polarization modes appears at around the characteristic frequency $f_c \equiv c/2|\Delta\mathbf{x}|$, above of which the ORFs rapidly decrease to 0.

Some results and prospects for the future

- NO DETECTION has been possible with the LIGO S5 and Virgo VSR 3 data , published in [Abbott et al., 2009], where the collaboration obtained an upper limit of:

$$h_0^2 \Omega_{\rm gw}^{T}{}^{95\%,5\%} = 6.9 \times 10^{-6}$$

for a flat SGWB energy density in the frequency band around 100 Hz;

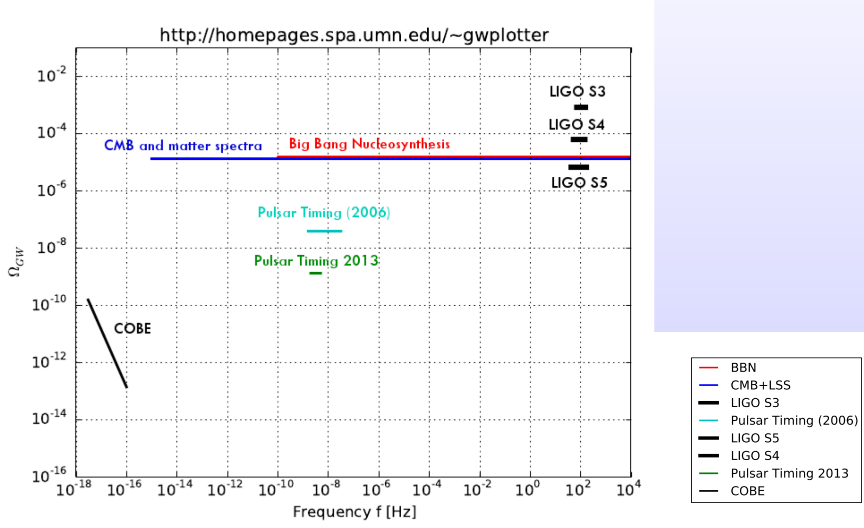
- we can evaluate the sensitivities that will be reach by the new network of Advanced Detectors AdVirgo and AdvLIGO, whose scheduled upgrades are planned for the years 2015-2021 [Aasi et al., 2013]:

Mode	Detector pair	Early ('15-'17)	Mid ('17-'18)	Late ('18-'19)	Designed ('19-'21)
$h_0^2 \Omega_{\rm gw}^{T}{}^{95\%,5\%}$	AdV - AdvLIGO(L)	2.11×10^{-7}	7.82×10^{-8}	3.13×10^{-8}	2.49×10^{-8}
	AdvLIGO(L) - (H)	4.23×10^{-8}	1.02×10^{-8}	2.87×10^{-9}	2.59×10^{-9}
$h_0^2 \Omega_{\rm gw}^{V}{}^{95\%,5\%}$	AdV - AdvLIGO(L)	2.00×10^{-7}	6.94×10^{-8}	2.48×10^{-8}	1.99×10^{-8}
	AdvLIGO(L) - (H)	5.21×10^{-8}	1.35×10^{-8}	3.72×10^{-9}	3.47×10^{-9}
$\xi h_0^2 \Omega_{\rm gw}^{S}{}^{95\%,5\%}$	AdV - AdvLIGO(L)	1.75×10^{-7}	5.87×10^{-8}	1.86×10^{-8}	1.47×10^{-8}
	AdvLIGO(L) - (H)	3.83×10^{-8}	1.22×10^{-8}	3.87×10^{-9}	3.53×10^{-9}

and these sensitivities seems to be good enough for testing several mechanism of production of an SGWB!! That is, there are many models that (if they were correct!) tell us that we will be able to see something during the next decade experiments...!

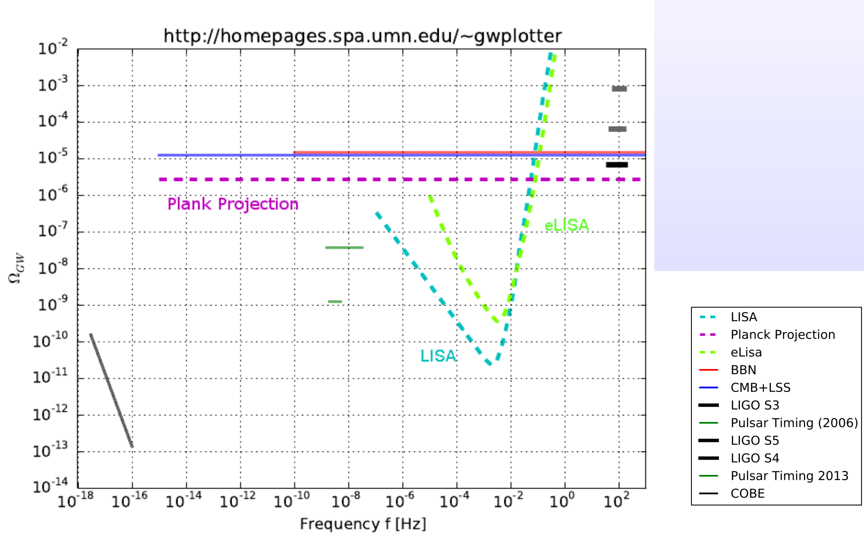
Some results and prospects for the future

Current experimental limits



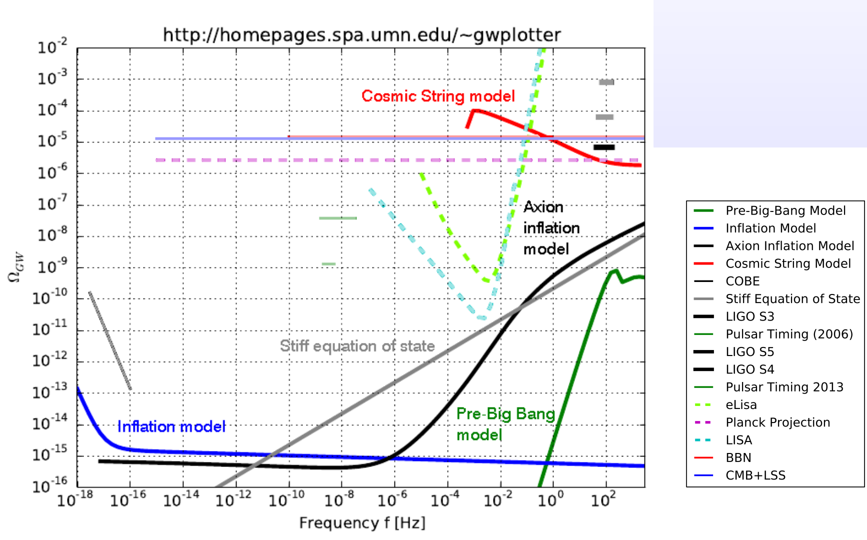
Some results and prospects for the future

Projected sensitivities



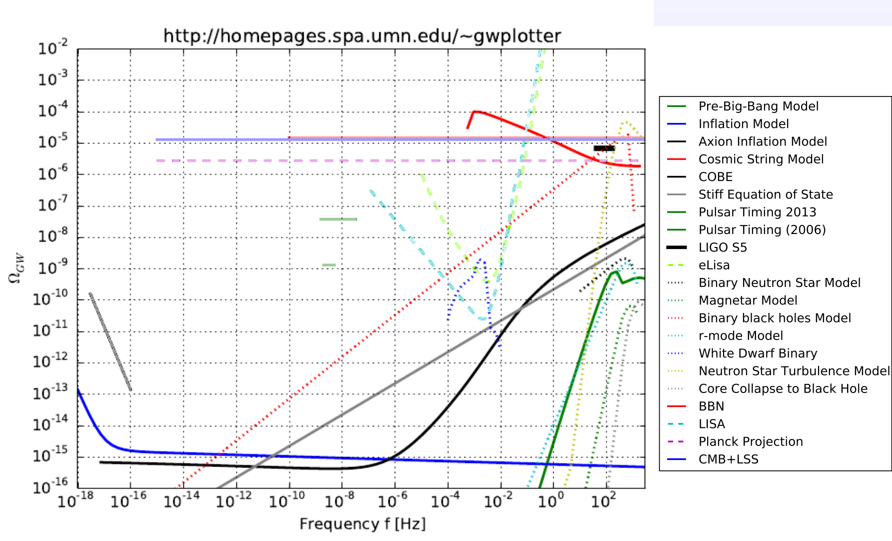
Some results and prospects for the future

Cosmological Models



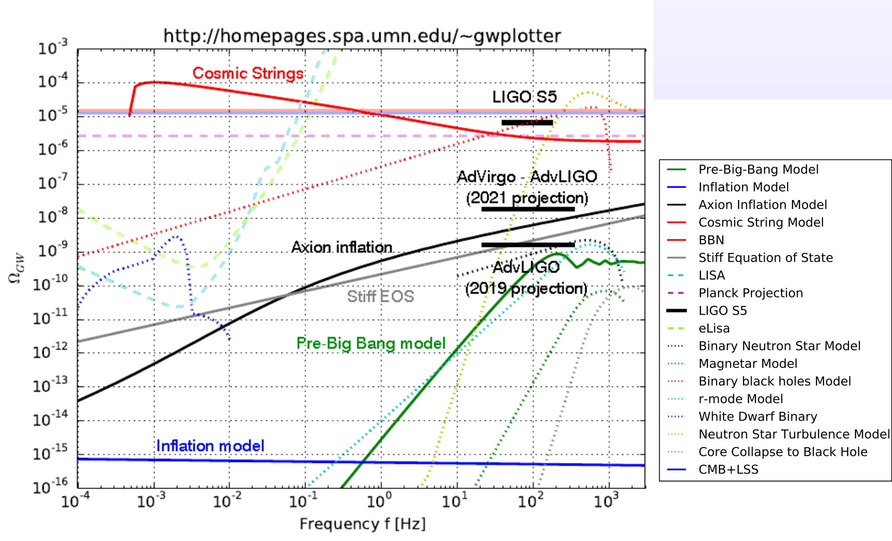
Some results and prospects for the future

Astrophysical Models



Some results and prospects for the future

Cosmological Models



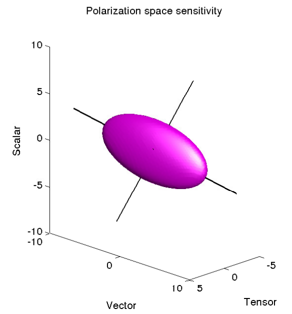
Summary and Conclusions

- Many theoretical models, both astrophysical and cosmological, predict a stochastic background of gravitational waves. This background is of great interest in the study of the early Universe cosmology and very-high energy physics, since it provides access to early-times and energy densities never accessible with any other means;
- for the study of the SGWB, we must include the possibilities of Alternative Theories of Gravity, different form General Relativity. The SGWB signal can also be used to test these theories;
- the most general SGWB predicted by a generic theory of gravity can admit at most 6 modes of polarization: detecting these modes can be a valuable, "model-independent" test for alternative theories of gravity;
- we generalize the standard cross-correlation analysis developed by [Allen and Romano, 1999] to include the possibility of these non-standard polarizations;
- we use the data of the predicted sensitivities of the advanced GW detectors to evaluate projections on their sensitivities to the SGWB;
- this projections lie under the energy densities expected by many cosmological and astrophysical models;
- if these models are true, we will be able in the next decade to detect an SGWB signal. Otherwise, we will improve upper limits and bounds on the models predicting the SGWB. In any case, the study of the SGWB still remains a valuable testing ground for cosmological models and very-high energy physics.

Backup material

Some notes on the proposed algorithm for studying the SGWB

- The MLEs for the components of the power spectrum density $S_h^A(f)$ are equal to those obtained by Seto and Taruya and Nishizawa et al. in the particular cases of circular tensor polarization and tensor, vector and scalar modes with no circular polarizations, respectively;
- we can resolve all the five components of the power spectrum density if we have a large enough number N of detectors ($\binom{N}{2} > 5$, or 3 if we exclude circular polarizations) or if we assume some power-low model for these spectra: $S_h^A(f) = S_h^A \cdot f^{\nu}$. With less detectors we can only find some directions, in the polarization space, where the detectability is most favorable, that is, our apparatus is most sensitive;
- we also recovered the algorithm described by Allen and Romano, where they considered only unpolarized tensor modes, as a special case. Respect to their algorithm, the introduction of other degrees of freedom, in the form of non-standard polarizations, reduce the statistics and hence the sensitivity we can reach;



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