Density of the material used for the first O4 NCal rotors. VIR-0160A-22

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Contents

1 Introduction

During the O3 calibrator test for Virgo the rotor relative density uncertainty was of 0.18%. This uncertainty directly propagates to the NCal signal and therefore to the calibration uncertainty. To reduce this uncertainty for O4, a careful set of measurements on the material used to machined the new rotors has been made. To simplify the density measurements, four blocks of aluminum coming from the same longer cylinder have been machined to a simple cylindrical shape. This note describes the measurements made on those aluminum 7075 cylinders that will later be machined into the O4 rotors. The main result is the density value and uncertainty of this aluminum.

2 Measurement method of the four cylinders

Figure 1 shows the four aluminum cylinders machined[1]. The cylinders have been machined to be 209 mm of diameter and 120 mm of height.

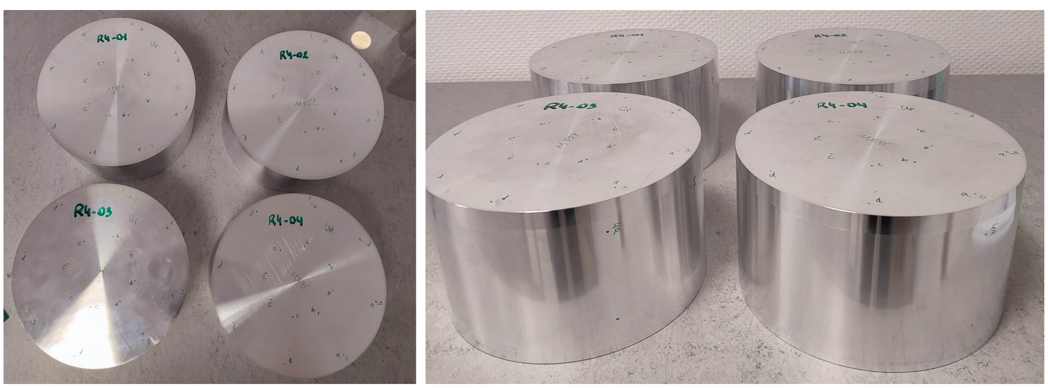

Figure 1: Aluminum cylinders machined. Left is a top view and right is a side view.

Figure 2 shows an outline of the faces and side of the cylinders with each measurement points shown as colored dots. There is a total of 40 points of measure on each cylinder to determine the volume. Points on one face are mirrored on the opposite face meaning for instance that "x_u" point on face up is associated to "x_d" on face down.

[1]The cylinder labelled "R4-04" on figure 1 has been renamed "R4-00" in this note.

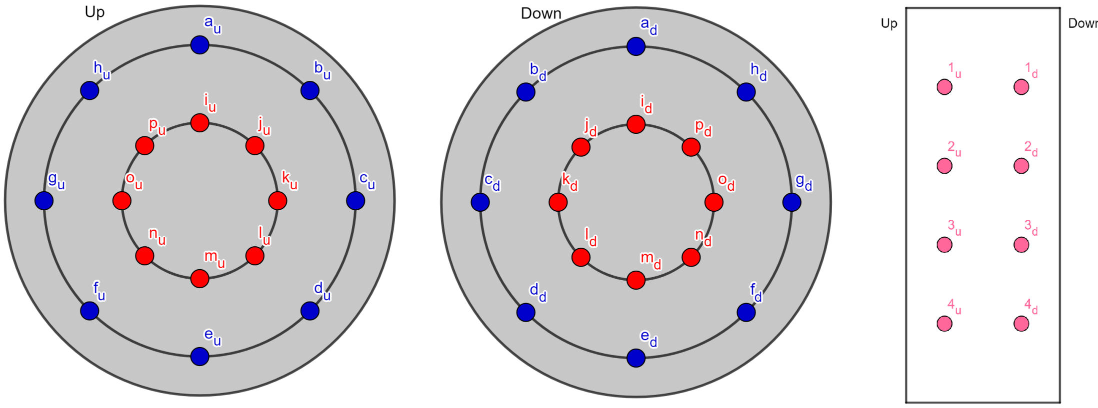

Figure 2: Outline of the faces and side of the cylinders, colored dots represent where the measures will be taken. From left to right, face up, face down, side.

In figure 2 we notice that the inner points (red colored points) are closer to each other than the outer points (blue colored points), the perimeter described by the outer points compared to the inner points is larger by a factor 2. To not bias the density we will then apply a weight of 0.5 on the interior points while computing the mean height.

3 Measuring tools used

To determine the mass of the cylinders we used a scale (figure 15) given with a linearity of ± 0.3 g. To measure the diameter and the height of the cylinders, we used two measuring columns, the first one located in the metrology room at IPHC (figure 17) with a given error of 1.8+L/600 μm at 95% CL (L being the measured length in mm) labelled later as "Column 1" and a second one located in the mecanical room at IPHC (figure 18) with a given error of 2+L/600 μm at 95% CL labelled as "Column 2".

4 Uncertainties from the measuring tools

To compute the relative density uncertainty coming from the measuring tools we assume that the uncertainty on the mass and the distances are uncorrelated. Therefore we add the relative uncertainties quadratically. However the diameter and height values are assumed to be correlated since we used the same measuring tool. The relative density uncertainty is computed using:

$$\frac{\sigma_\rho}{\rho} = \sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{2\sigma_D}{D} + \frac{\sigma_h}{h}\right)^2} \tag{1}$$

with:

- D the measured diameter of the cylinder
- h the measured height of the cylinder
- m the measured mass of the cylinder
- σ_D the uncertainty on the diameter being the precision of the column

- σ_h the uncertainty on the height being the precision of the column
- σ_m the uncertainty on the mass of the cylinder being the linearity of the scale

In this note we will present the mean values of diameter and height for the measurements made.

Table 1 shows the relative uncertainties on the measurements. The relative uncertainty on the density is compatible with all of the four cylinders since they have been machined to be identical.

	Column 1	Column 2
σ_D/D	1.03×10^{-5}	1.12×10^{-5}
σ_h/h	1.67×10^{-5}	1.83×10^{-5}
σ_m/m	2.60×10^{-5}	
σ_ρ/ρ	4.6×10^{-5}	4.8×10^{-5}

Table 1: Relative uncertainties associated to the measurement tools.

5 Measurements on R4-01

This section concerns the cylinder labelled as R4-01.

5.1 Shape of the cylinder

After machining the cylinders are not perfect. The following figures show the difference between the measurements and the theoretical values amplified by a factor 500 to better see the deformations of the cylinder.

The figure 3 represents the shape of the side of R4-01. The dotted lines represent the theoretical height of the cylinder (120 mm), the colored points represent the different measures of the height done (outer and inner points from figure 2).

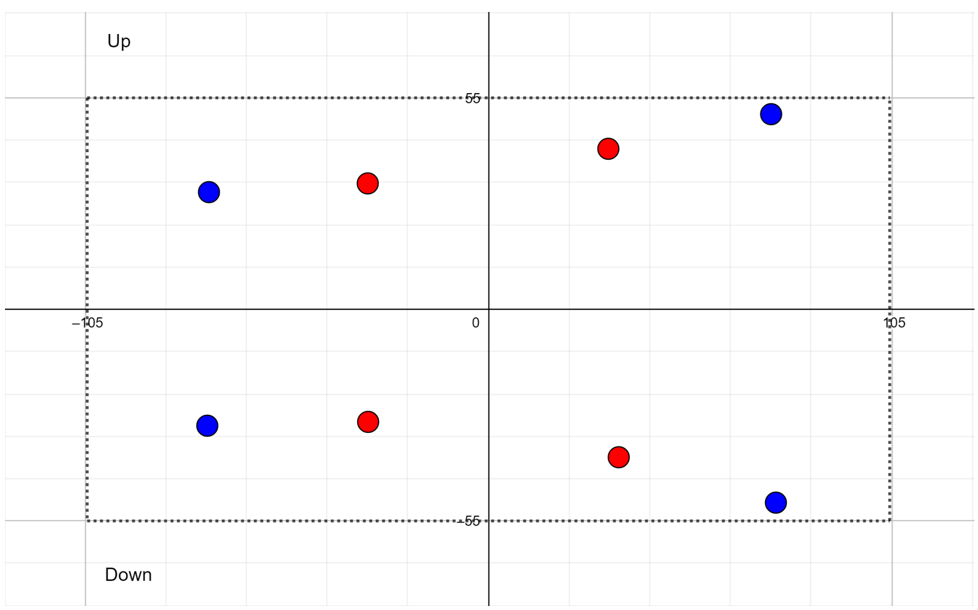

Figure 3: Outline of the side of the R4-01 cylinder.

On figure 3 we notice the tilt of the surfaces describing a trapezoidal shape, with a slight curvature on both surfaces.

The figure 4 represents the shape of both faces of R4-01. The circle represents the theoretical diameter of the cylinder (209 mm), the colored points represent the different measures of the diameter done (right side of figure 2).

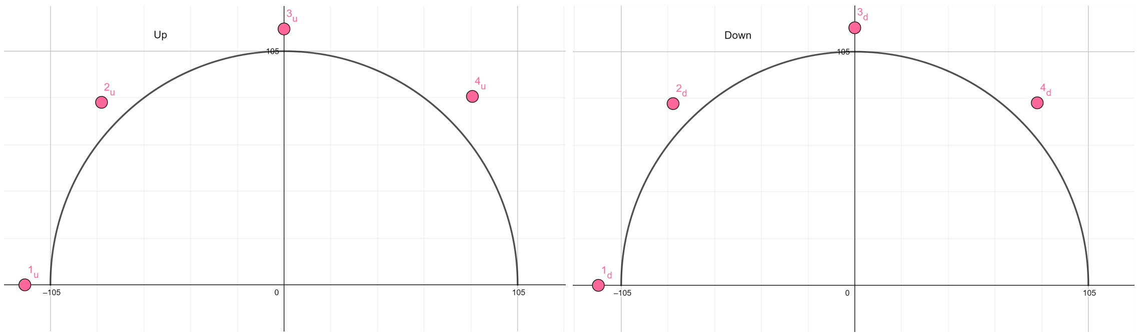

Figure 4: Outline of the diameters of the R4-01 cylinder. Left is face up, right is face down.

Both faces have a larger diameter than the theoretical value. There seem to be less deformation on the diameter than the height.

It is not possible to determine if any imperfection is a local deformation on one side or if it is on both sides of the cylinder.

5.2 Density

Table 2 shows the measurements made for R4-01 using the measuring columns and the scale. Three set of measurements have been performed on three different days. The yellow row shows the computed density (with weighted inner points) of the cylinders using the mean values for diameter and height and the mass measured.
The mean density will be discussed in section 9.3 after taking into account thermal effects.

	15-12-21		16-12-21		04-01-22	
	Column 1	Column 2	Column 1	Column 2	Column 1	Column 2
Mean diameter (mm)	209.047	209.045	209.045	209.045	209.044	209.043
Mean height (mm)	119.916	119.916	119.915	119.916	119.913	119.914
m (kg)	11.5578		11.5579		11.5583	
ρ (kg.m^{-3})	2808.16	2808.21	2808.25	2808.24	2808.42	2808.44

Table 2: Measurements made at IPHC on R4-01. A weight of 0.5 is applied on the inner points for the height.

5.3 Cross check with a vernier Caliper

A vernier caliper has been used to check the similarity with the measurements made using the columns. The vernier caliper used (figure 16) has a precision of 20 μm at 95% CL, which is more than eight times larger than the precision reached with the columns in the same confidence interval.

Table 3 shows a set of measurements made on R4-01 on the diameters "D$_u$" and "D$_d$". This table shows that the vernier caliper confirmed the more precise measurements made with the column.

	Column 1	Caliper
D_u (mm)	209.028	209.00
D_d (mm)	209.034	209.02

Table 3: Preliminary measurements made on R4-01 made with column 1 and the vernier caliper.

6 Measurements on R4-02

This section concerns the cylinder labelled as R4-02.

6.1 Shape of the cylinder

As for the previous rotor, the figure 5 represents the shape of the side of R4-02. The dotted lines represent the theoretical height of the cylinder, the colored points represent the different measures of the height done.

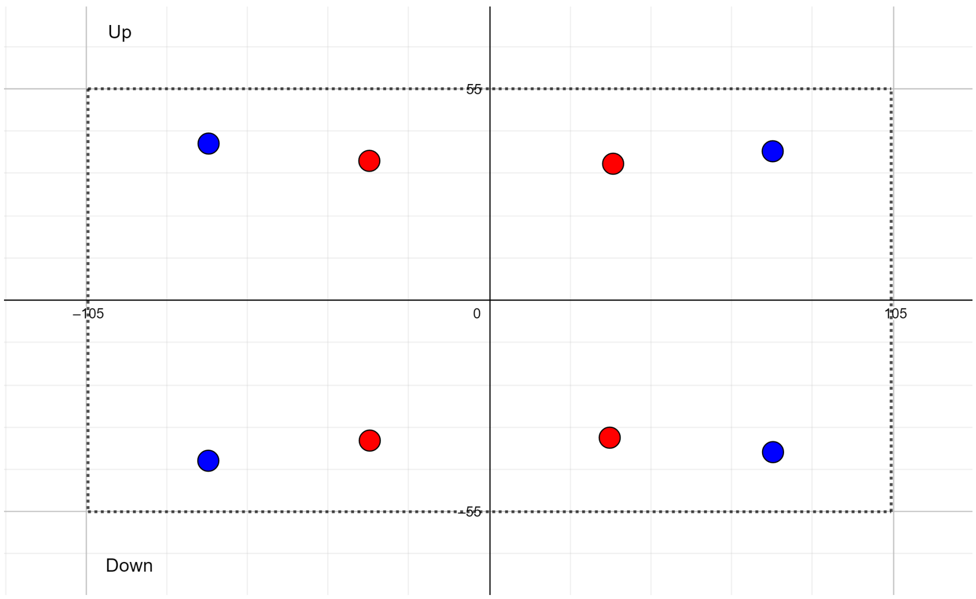

Figure 5: Outline of the side of the R4-02 cylinder.

On figure 5 we notice a hollowed shape of the surface of the cylinder.

The figure 6 represents the shape of both faces of R4-02. The circle represents the theoretical diameter of the cylinder, the colored points represent the different measures of the diameter done.

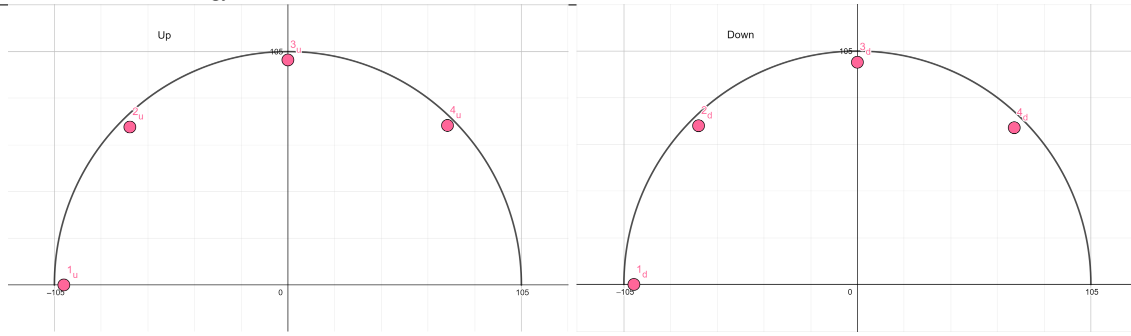

Figure 6: Outline of the faces of the R4-02 cylinder. Left is face up, right is face down.

Both faces have a smaller diameter than the theoretical value.

6.2 Density

Table 4 shows the computed density (with weighted inner points) for R4-02 using the measuring columns and the scale, the parameters are the same as in table 2.
The mean density will be discussed in section 9.3 after taking into account thermal effects.

	16-12-21		04-01-22		18-01-22	
	Column 1	Column 2	Column 1	Column 2	Column 1	Column 2
Mean diameter (mm)	208.986	208.986	208.984	208.983	208.980	208.983
Mean height (mm)	119.916	119.917	119.914	119.916	119.913	119.917
m (kg)	11.5521		11.5523		11.5522	
ρ (kg.m^{-3})	2808.39	2808.38	2808.54	2808.53	2808.66	2808.49

Table 4: Measurements made at IPHC on R4-02. A weight of 0.5 is applied on the inner points for the height.

7 Measurements on R4-03

This section concerns the cylinder labelled as R4-03.

7.1 Shape of the cylinder

As for the previous rotors, the figure 7 represents the shape of the side of R4-03. The dotted lines represent the theoretical height of the cylinder, the colored points represent the different measures of the height done.

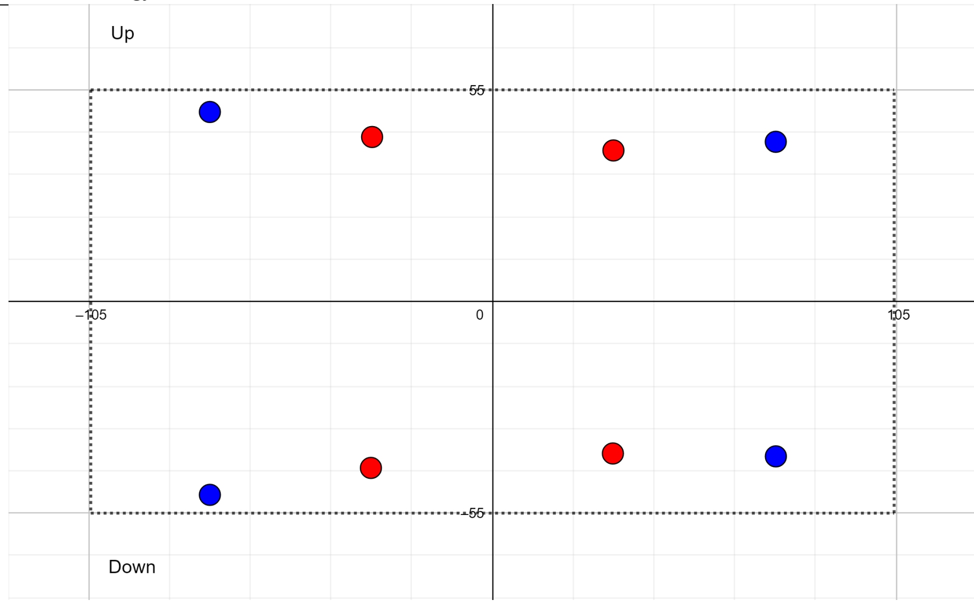

Figure 7: Outline of the side of the R4-03 cylinder.

On figure 7 we notice a tilted hollowed shape of the surface of the cylinder.

The figure 8 represents the shape of both faces of R4-03. The circle represents the theoretical diameter of the cylinder, the colored points represent the different measures of the diameter done.

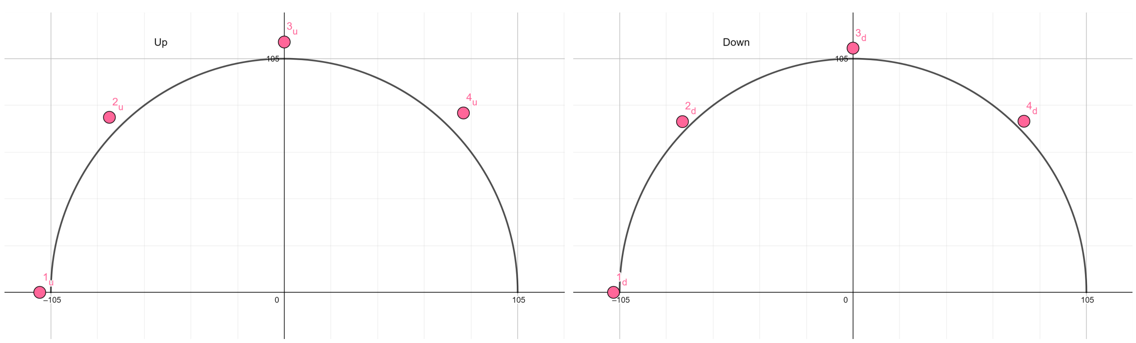

Figure 8: Outline of the faces of the R4-03 cylinder. Left is face up, right is face down.

Both faces have a larger diameter than the theoretical value.

7.2 Density

Table 5 shows the computed density (with weighted inner points) for R4-03 using the measuring columns and the scale, the parameters are the same as in table 2.
The mean density will be discussed in section 9.3 after taking into account thermal effects.

	15-12-21		16-12-21		04-01-22	
	Column 1	Column 2	Column 1	Column 2	Column 1	Column 2
Mean diameter (mm)	209.023	209.021	209.022	209.022	209.020	209.019
Mean height (mm)	119.936	119.936	119.935	119.935	119.934	119.935
m (kg)	11.5580		11.5580		11.5583	
ρ (kg.m^{-3})	2808.38	2808.43	2808.44	2808.42	2808.57	2808.60

Table 5: Measurements made at IPHC on R4-03. A weight of 0.5 is applied on the inner points for the height.

8 Measurements on R4-00 - Reference cylinder

In this section we consider the cylinder R4-00 as a reference cylinder for future checks. It will therefore not be machined into a rotor.

8.1 Shape of the cylinder

As for the previous rotors, the figure 9 represents the shape of the side of R4-00. The dotted lines represent the theoretical height of the cylinder, the colored points represent the different measures of the height done.

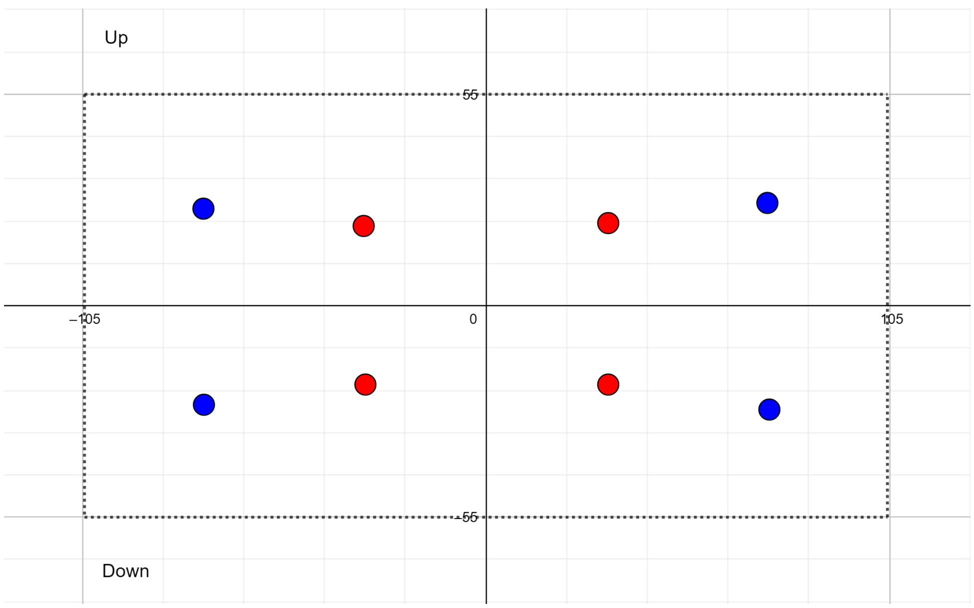

Figure 9: Outline of the side of the R4-00 cylinder.

On figure 9 we notice a hollowed shape of the surface of the cylinder.

The figure 10 represents the shape of both faces of R4-00. The circle represents the mean diameter of the cylinder, the colored points represent the different measures of the diameter done.

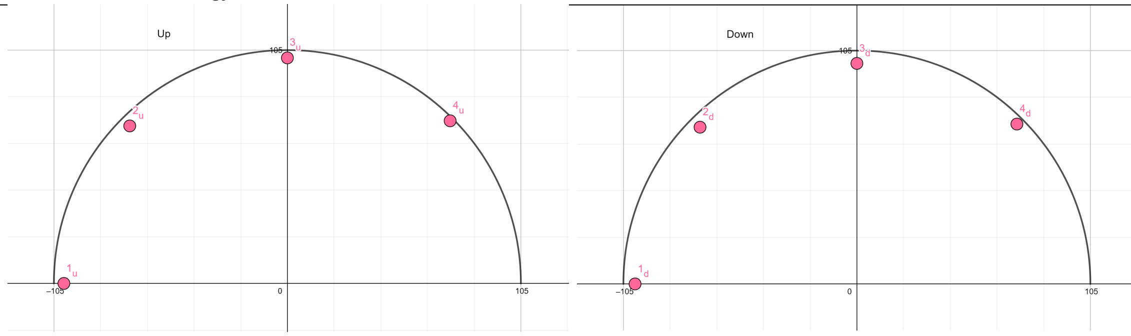

Figure 10: Outline of the faces of the R4-00 cylinder. Left is face up, right is face down.

Both faces have a smaller diameter than the theoretical value.

8.2 Density

Table 6 shows the computed density (with weighted inner points) for R4-00 using the measuring columns and the scale, the parameters are the same as in table 2.
The mean density will be discussed in section 9.3 after taking into account thermal effects.

	15-12-21		16-12-21		04-01-22	
	Column 1	Column 2	Column 1	Column 2	Column 1	Column 2
Mean diameter (mm)	208.985	208.985	208.985	208.985	208.984	208.982
Mean height (mm)	119.858	119.858	119.856	119.858	119.856	119.857
m (kg)	11.5459		11.5458		11.5464	
ρ (kg.m^{-3})	2808.28	2808.29	2808.30	2808.26	2808.48	2808.50

Table 6: Measurements made at IPHC on R4-00. A weight of 0.5 is applied on the inner points for the height.

9 Combining measurements

9.1 Thermal effects

The measurements have been taken during five different days, the temperature and humidity of the room where the cylinders were stored are displayed on table 7.

	15-12-21	16-12-21	17-12-21	04-01-22	18-01-22
Temperature (°C)	21.9	21.8	21.8	21.2	21.0
Humidity (%)	37.70	35.90	36.10	47.00	34.90

Table 7: Temperature and humidity during the measurements at IPHC.

The thermal properties of the material have to be taken into account: for the Aluminum 7075 between 20°C and 100°C we have a thermal expansion coefficient of 23.6 μm.m^{-1}.°C^{-1}.
If we consider a maximum variation of temperature of about 0.9°C and a maximum measured length of 20 cm, we obtain a maximum of 4.3 μm of deformation on the machined cylinders throughout the measurement days. This is more than our measurement uncertainty and therefore should be corrected for.

9.2 Analysis

The densities have been computed using measurements made at a given temperature, we have to check if a normalization at a reference temperature (21.5°C) will correct the deviations of computed densities.

Figure 11 shows the mean density computed and the temperature with weighted values, the circular points correspond to densities at different temperatures while triangular points correspond to those same densities normalized at a reference temperature of 21.5°C. For this figure, all the points of measurements (see 2) were used to compute the densities.

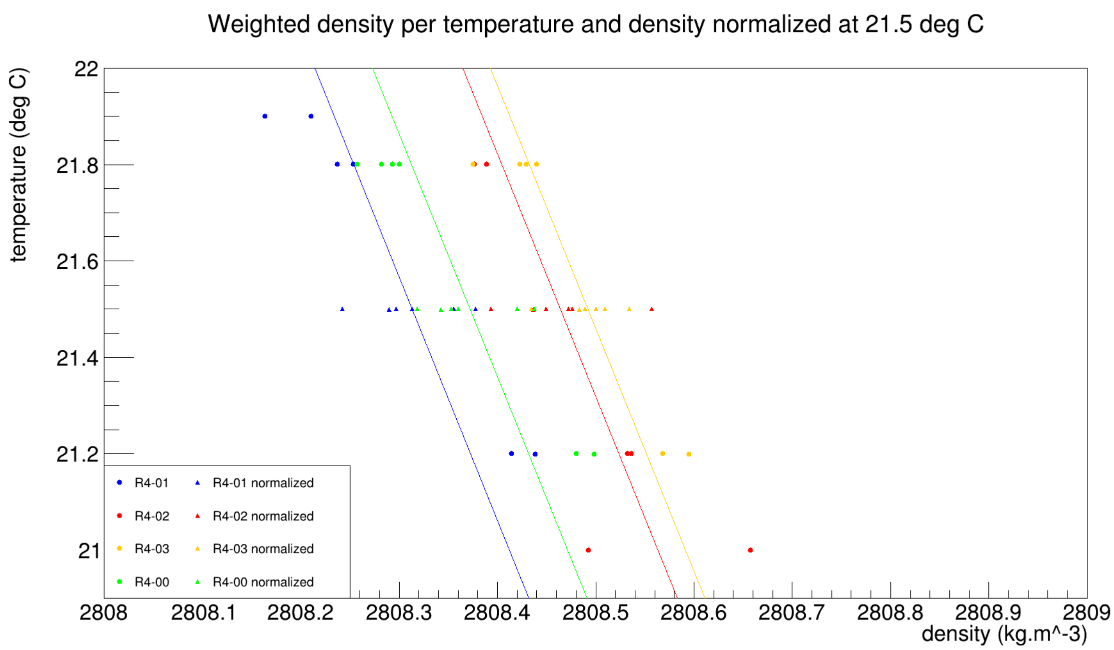

Figure 11: Weighted density of the cylinders per ambiant temperature. Each color corresponds to a named rotor and each line to its theoretical slope.

Figure 12 shows the same kind of data with the unweighted points.

We notice that for each rotor at a given temperature, the data is clustered by rotors. Also the weighted plot shows overall lighter densities. This can be explained due to the hollowed shape of the cylinders (represented in figures 3, 5, 7 and 9), as those central points are given a lighter weight in the computation of the density the mean height of the cylinder is then slightly greater and so the density gets lower. This confirms the necessity to apply weight on measurement points.

It is useful to check if the deformation of the cylinders is responsible for the differences between the densities. Figure 13 shows a similar histogram as in figure 11 with weighted values but removing some of the measurement points that were causing most of the deformation for each cylinder.

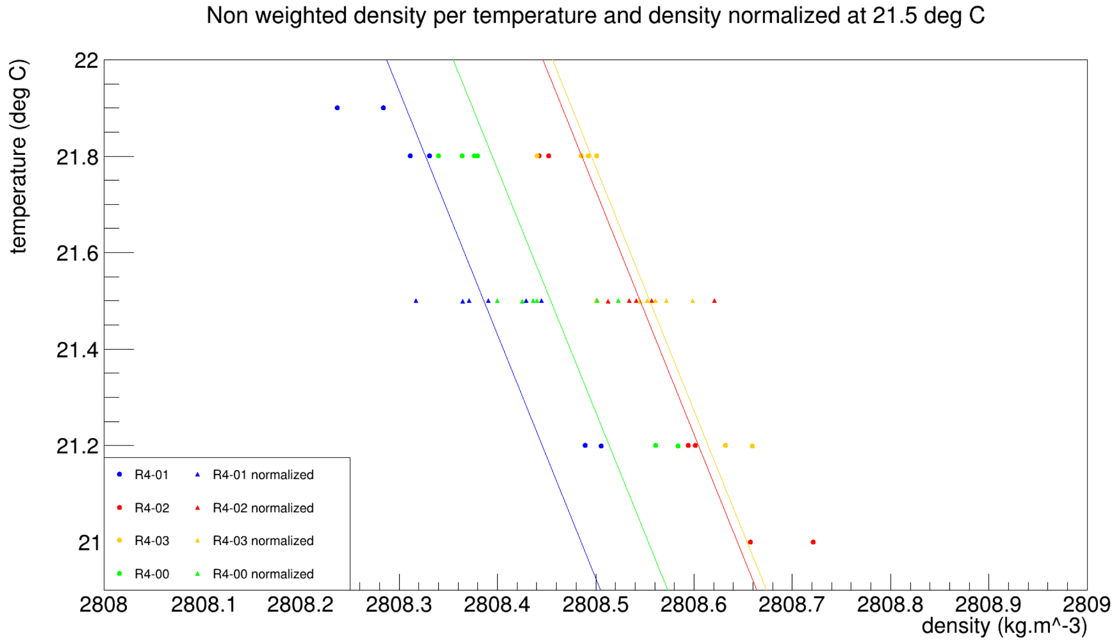

Figure 12: Unweighted Density of the cylinders per ambiant temperature. Each color corresponds to a named rotor and each line to its theoretical slope.

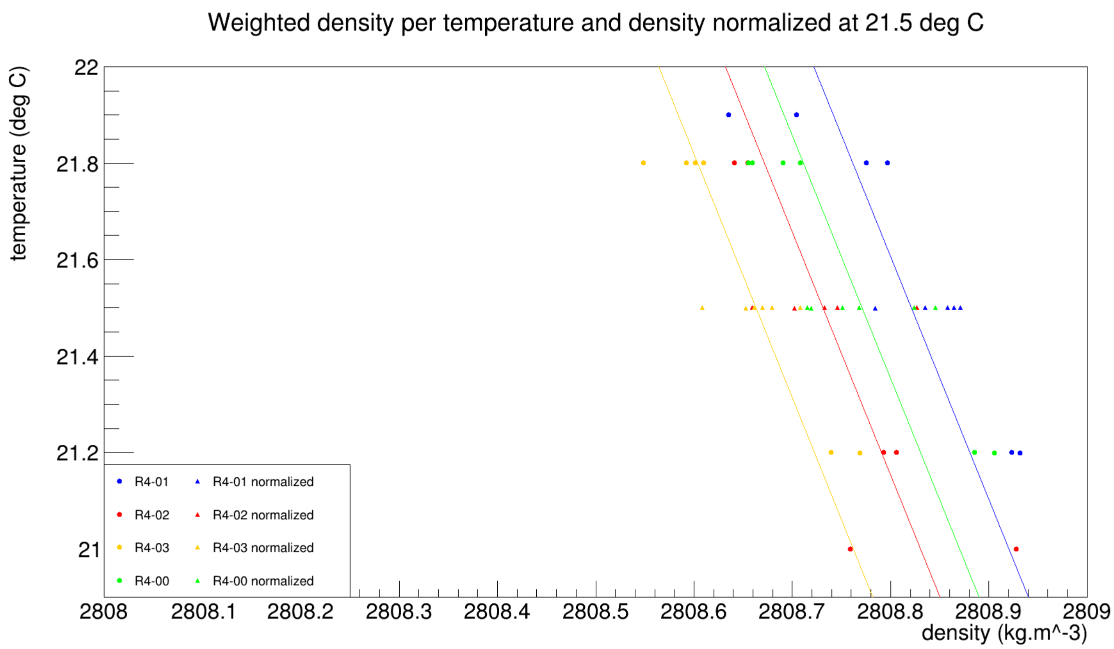

Figure 13: Weighted density of the cylinders per ambiant temperature without including large deformations. Each color corresponds to a named rotor and each line to its theoretical slope.

For this last figure, we notice that the density can change up to 0.5 kg.m^{-3} compared as in figure 11. This change is larger than the difference between rotor to rotors. Therefore the dispersion of the rotor density observed in figure 11 gives us the uncertainty due to the limitation of the number of points used to measure the shape of the cylinders: 0.1 kg.m^{-3}. This is of the same order as the uncertainty coming from the measuring tools (0.14 kg.m^{-3}). Overall the density uncertainty is the quadratic sum of these two numbers rounded to 0.2 kg.m^{-3}.

As expected and shown by figures 11 to 13 the temperature dependance of the density follows the theoretical slope.

9.3 Mean density of Aluminum 7075 at 21.5°C

Table 8 shows the mean density retrieved from the normalized data at 21.5°C for each rotor of figure 11 to 13. **This table gives a mean density of 2808.41 $\pm$ 0.2 kg.m^{-3} for the aluminum 7075 at 21.5°** using weighted values from figure 11.

	R4-01	R4-02	R4-03	R4-00
Weighted density (kg.m^{-3})	2808.31	2808.46	2808.49	2808.37
Non weighted density (kg.m^{-3})	2808.39	2808.55	2808.56	2808.45
Weighted density corrected (kg.m^{-3})	2808.82	2808.73	2808.66	2808.77

Table 8: Mean densities computed with weighted and non weighted measures at a reference temperature of 21.5°C.

A better mapping of the surfaces of the cylinders would give a better estimation of the density but it would take a serious amount of time using the columns.

10 Homogeneity check

There are no significant differences between the density observed for the four cylinders.

Another check has been made on the material that has been cut from a cylinder as shown in figure 14. Both sectors have the same geometry and were cut from opposite sides of the cylinder and so they theorically have the same mass if the density is homogeneous in the material. The scale has given a mass of 1.1402 kg for both sectors meaning that the density is homogeneous at least at 10^{-4}.

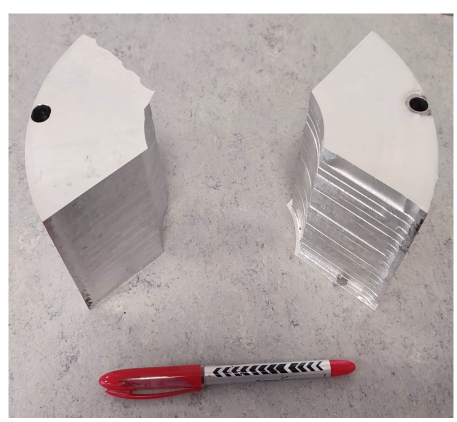

Figure 14: Material cut from one of the cylinders.

Appendix

Figure 15: Scale used to measure the mass of the cylinder. The model is a KFB 36K0.1.

Figure 16: Vernier caliper used. The model is a TESA-CAL IP67.

Figure 17: Measuring column used to measure the diameter and height of the cylinders. The model is a Garant 44 5350_600 HC1.

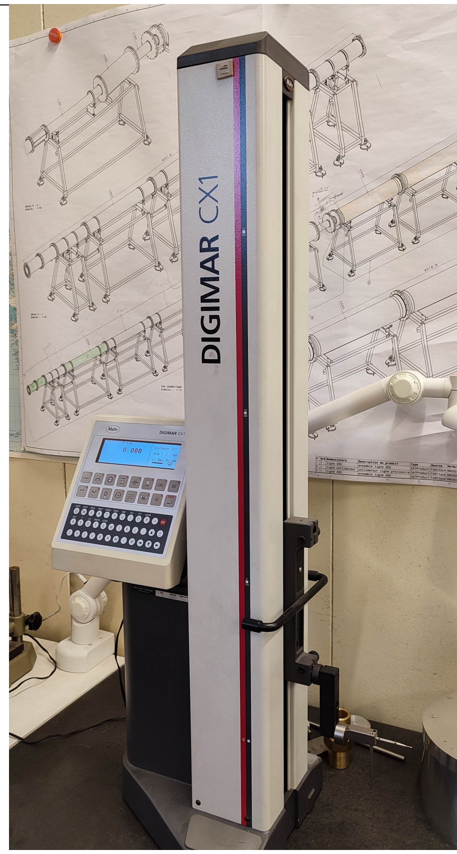

Figure 18: Measuring column used to measure the diameter and height of the cylinders. The model is a Mahr CX1.