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Advanced Virgo INJ: Radiation pressure effects in the Advanced Virgo IMC - longitudinal and angular directions

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E. Genin, M. Mantovani, P. Ruggi

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VIRGO * A joint CNRS-INFN Project
Via E. Amaldi, I-56021 S. Stefano a Macerata - Cascina (Pisa)
Secretariat: Telephone (39) 050 752 521 * FAX (39) 050 752 550 * Email W3@virgo.infn.it

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P_{in}	180 [W]	
IMC cavity length	L	144 [m]
c	$3 \cdot 10^8$ [m/s]	
g_1	$1 - L/R_1 = -0.6$	$R_1 = 180$ [m]
g_2	$1 - L/R_2 = 1$	$R_2 = \infty$ [m]
IMC end mirror θ_x	resonant frequency	3.284 [Hz]
IMC end mirror θ_y	resonant frequency	1.315 [Hz]
injection bench θ_x	resonant frequency	0.65 [Hz]
injection bench θ_y	resonant frequency	0.01 [Hz]
IMC end mirror	mass	1.4 [kg]
injection bench	mass	145 [kg]
IMC end mirror	radius	0.07 [m]
IMC Finesse		1100
Mirror density	$2.201 \cdot 10^3$ [kg/m^3]	

Table 1: Optical and mechanical parameters [1]. The mechanical resonant frequencies of the IMC end mirror suspension listed above are the measured ones.

1 Introduction

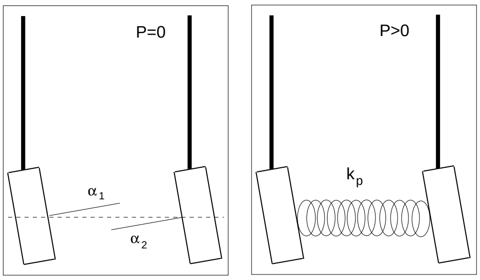

Figure 1: Effect of the radiation pressure in a Fabry-Perot cavity.

In the Advanced Virgo configuration the input power will be raised up to $\sim$ 180 W, thus in order to have a suitable design of the IMC cavity it is important to study the radiation pressure effects on the longitudinal and angular directions. The effect of the radiation pressure in an high power cavity is to act as a spring connected to the two mirrors, thus the mechanical responses of the system will change with power increase in the cavity. In the following the IMC optical and mechanical configuration has been modeled and analyzed starting from the parameters listed in Table 1.

2 Radiation pressure effect in the longitudinal direction

The IMC and ITF mirrors, for the Advanced Virgo optical configuration, will be subjected to incremental radiation pressure which changes as the input power is changed.
An internal Virgo note, by S. Hebri [2], has demonstrated that it should be possible to lock the IMC with an incident power of 180 watts with a IMC end mirror mass at least of 900 g. Thus, one should understand that taking an IMC end mirror heavy enough, there should not be any problems to lock the IMC even with the presence of the radiation pressure phenomenon.
Nevertheless, it is necessary to study a bit more in detail the radiation pressure effects that will raise according to the Advanced Virgo parameters, such as the cavity finesse F and the laser power coupled in the cavity.
The radiation pressure, by definition, is the pressure exerted upon any surface exposed to electromagnetic radiation. If the radiation is absorbed, the pressure will correspond to the power flux density divided by the

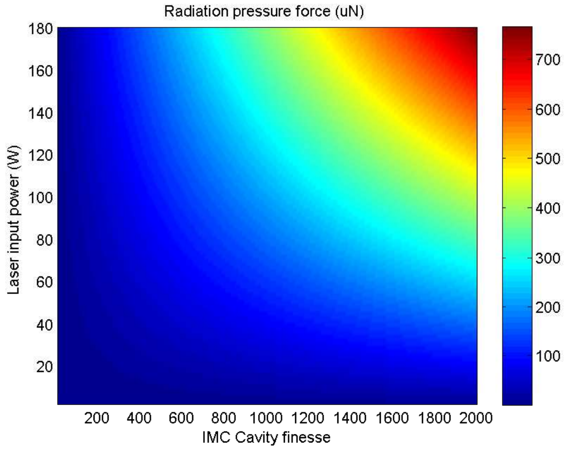

Figure 2: Radiation pressure in the 144m triangular IMC cavity respect to laser input power and cavity finesse.

speed of light, while if it is totally reflected, the radiation pressure will be doubled. Hereafter is considered only the static radiation pressure effect.
The application of a laser beam on a mirror generates a force on it that follows the equation:

$$F_{rad} = 2P_{stored}/c \tag{1}$$

where $c = 3.10^8$ m/s is the speed of light and P_{stored} is the power stored in the IMC cavity.
We can write Equation 1 as:

$$F_{rad} = \frac{2}{c} \cdot \left(\frac{FP_{Laser}}{\pi}\right) \tag{2}$$

The radiation pressure force has been computed respect to the cavity Finesse and laser power coupled in the IMC cavity.

At maximum, it can be obtained a F_{rad} of about 400 μN for a finesse of 1000 and a laser input power of 180 W. Considering the mirror suspension as a simple pendulum it can be computed the correspondent longitudinal position variation due to the acting of the F_{rad} as:

$$\frac{l}{F_{rad}} = \frac{1}{M}\frac{1}{\omega_0^2 - \omega^2 + i\omega_0^2\phi_P} \tag{3}$$

where $\omega_0 = 2\pi f_0$, M is the mirror mass and ϕ_P is the inverted pendulum loss angle.
The parameters used in the following numerical evaluation have been extracted from the Michele Punturo's note [1], as:

- $f_0 = 0.657$ Hz
- $\phi_P = 5.4 \cdot 10^{-8}$
- M=1.4 kg (Virgo + IMC curved mirror weight).

For a radiation pressure force of $F_{rad} = 400$ μN (Finesse = 1000 and Laser power = 180 W), the longitudinal variation of the IMC terminal mirror position will be $\delta l = 1.676.10^{-5}$ m.
In order to evaluate the actuation range needed for the locking control scheme of the Advanced Virgo IMC cavity, it has to be determined the frequency shift due to the variation of the cavity length

$$\frac{\delta l}{L} = \frac{\delta f}{f} \tag{4}$$

where L is the IMC cavity length, 144 m, and $f = c/\lambda$, where $\lambda = 1.064 \cdot 10^{-6}$ m. The correspondent frequency shift is $\delta f = 35$ MHz which results to be feasible with the achievable actuation range (the master laser piezo actuator actuation is 1.75 MHz/V for a range of 100 V, thus reaching a maximum of 175 MHz). It can be concluded then that the lock can be easily acquired and the radiation pressure should be not an issue for the IMC longitudinal control if the initial Virgo locking strategy will be kept [3].

3 Radiation pressure effect on the angular directions

An other issue for the design of the IMC is the angular control. Analogously to what happens in the longitudinal control the optical torque will vary with the increase of the stored power. In this note, for the angular d.o.f., two different approaches have been considered: the analytical computation, based on the Sidles-Sigg small angle propagation [4], and numerical computation, by using the simulation tool Optickle [5].
In the case of the IMC, since it is a triangular cavity, we have a strong difference between the θ_x and θ_y modes. For the θ_x mode the IMC can be modeled as a simple Fabry-Perot cavity, while for the θ_y the analytical computation becomes too complex and the present simulation tools do not allow to model this configuration (in Optickle only the θ_x has been taken into account).
Thus in the following we will consider only the θ_x direction, only some approximate results, based on experimental measurements, will be given for the θ_y.

3.1 Sidles-Sigg analysis - θ_x mode

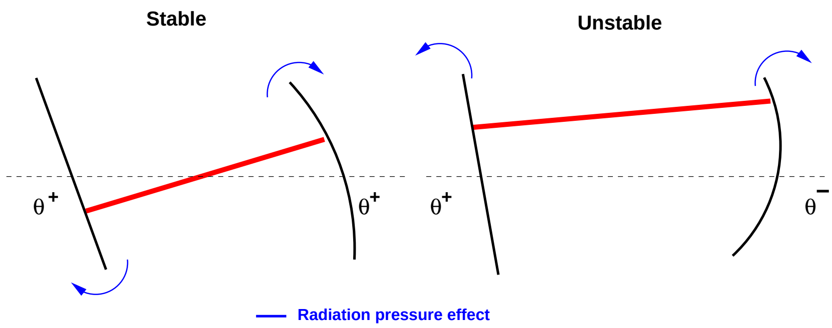

Figure 3: Effect of the radiation pressure in the IMC cavity considering the θ_x mode.

In presence of radiation pressure effect in a Fabry-Perot cavity the two mirrors became connected by an optical spring which modifies the mechanical transfer functions of the two suspension. It can be evaluated analytically by computing the system energy equations and numerically with Optickle. The kinetic and potential energy can be computed for the whole system as [6]:

$$K_{i,j} = \begin{pmatrix} I_1 & 0 \\ 0 & I_2 \end{pmatrix}$$

$$V_{i,j} = \begin{pmatrix} \mu_1 - \frac{2P}{c}\frac{g_2 L}{1-g_1 g_2} & -\frac{2P}{c}\frac{L}{1-g_1 g_2} \\ -\frac{2P}{c}\frac{L}{1-g_1 g_2} & \mu_2 - \frac{2P}{c}\frac{g_1 L}{1-g_1 g_2} \end{pmatrix}$$

where I_i is the momentum of inertia of the i-d.o.f., $\mu_i/I_i = \omega_i^2$ is the torsion pendulum natural frequency, P is the stored power inside the cavity, g_i is the cavity parameter , c is the light speed and L is the cavity length. Thus the eigenvalues can be computed as $det(V - \lambda K) = 0$:

$$\begin{aligned} I_1 I_2 \lambda^2 - I_1 \left(\mu_2 - \frac{2P}{c}\frac{g_1 L}{1-g_1 g_2}\right)\lambda - I_2 \left(\mu_1 - \frac{2P}{c}\frac{g_2 L}{1-g_1 g_2}\right)\lambda \\ + \left(\mu_2 - \frac{2P}{c}\frac{g_1 L}{1-g_1 g_2}\right)\left(\mu_1 - \frac{2P}{c}\frac{g_2 L}{1-g_1 g_2}\right) - \frac{4P^2}{c^2}\left(\frac{L}{1-g_1 g_2}\right)^2 &= 0 \end{aligned} \quad (5)$$

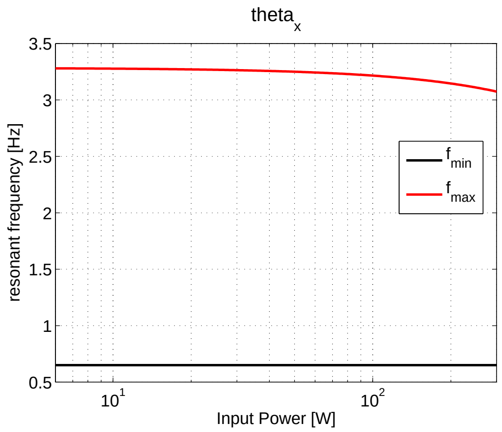

Figure 4: Effect of the radiation pressure on the system resonant frequencies, the 0.65 Hz frequency does not changes due to the strong massivity of the injection bench.

Since the momentum of inertia and the natural resonant frequencies of the two mirror suspension are strongly different, the diedron is placed on the injection bench which is a 145 kg massive object while the IMC end mirror is only 1.4 kg, this equation can not be further simplified.
Intuitively, due to the massivity of the injection bench, it can be guess that the only effect of the radiation pressure will be on the IMC terminal mirror. We can compute then the minimum and maximum resonant frequencies as a function of the input power.
As it is shown in Figure 4, the IMC θ_x resonant frequencies varies as a function of the input power lowering the f_{max} up to $\sim$ 3 Hz for 300 W of input power, while the lower frequency, 0.65 Hz, does not move since it is dominated by the injection bench.

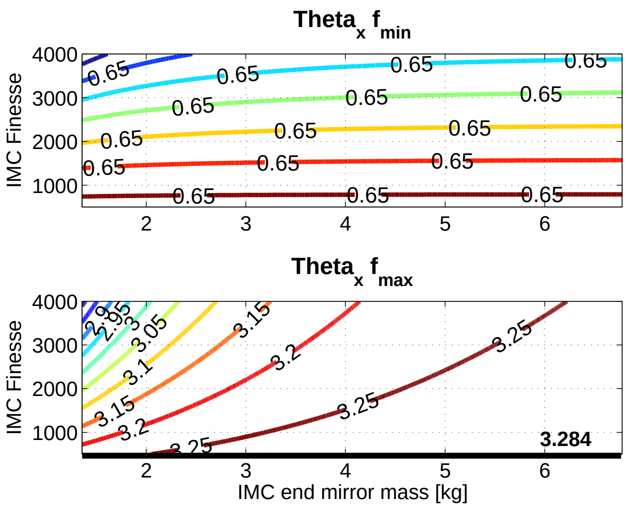

Figure 5: Resonant frequencies as a function of the IMC end mirror mass and as a function of the IMC Finesse. The lower frequency does not change at all, while the maximum frequency decreases but remaining positive, thus remaining stable.

It is interesting to evaluate the effect of the radiation pressure with different optical configurations, such as a different IMC end mirror mass or a different IMC Finesse. In order to understand which would be the best mirror mass to fight with the radiation pressure effects, the radius of the IMC end mirror will be maintained fixed, 7 cm as the present one, and the mass will be varied by simply increasing the thickness.

For the θ_x mode even if the finesse of the IMC cavity would be increased up 4000 the variation of the resonant frequencies will be acceptable, any variation for the lower frequency and a small variation for the higher frequency see Figure 5.

3.2 Modeling with Optickle

In order to validate the previous computation the IMC cavity has been modeled in the θ_x direction with the simulation tool *Optickle*.

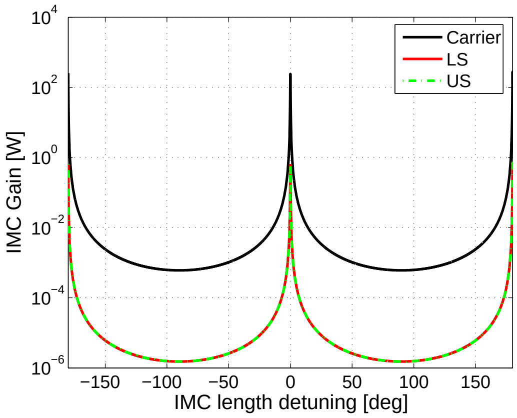

Figure 6: Sweep of the IMC terminal mirror along the z-direction in order to evaluate the resonant condition. The longitudinal position of the mirror has put on resonance and the cavity has a Finesse of about 1100.

The longitudinal position of the terminal mirror has put on resonance in order to have the maximum as possible effect of the radiation pressure, maximum stored power, see Figure 6. Thus the resonant frequencies of the mechanical transfer function have been evaluated as a function of the input power obtaining results coherent with the analytical computation, as it is shown in the Figures 7, 8.

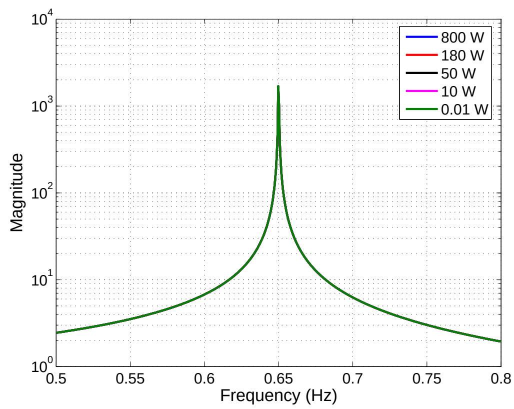

Figure 7: Mechanical Transfer function of the lower resonant mode θ_x mode.

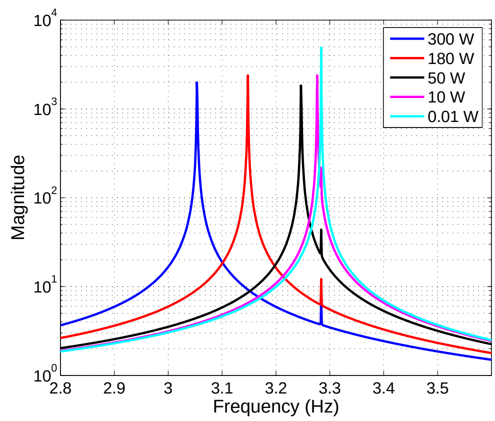

Figure 8: Mechanical Transfer function of the higher resonant mode θ_x mode.

The lower frequency does not change while the higher decreases with the input power.
In the comparison of the behavior of the higher resonant frequency computed with the Sigg analysis and with the modeling by Optickle a very good agreement is obtained, see Figure 9.

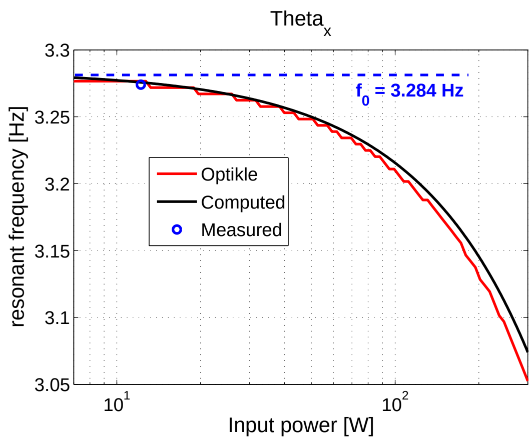

Figure 9: Higher resonant frequency behavior comparison between the result of the analytical computation and the simulation done with Optickle. It shows a very good agreement.

3.3 Extrapolation for the θ_y

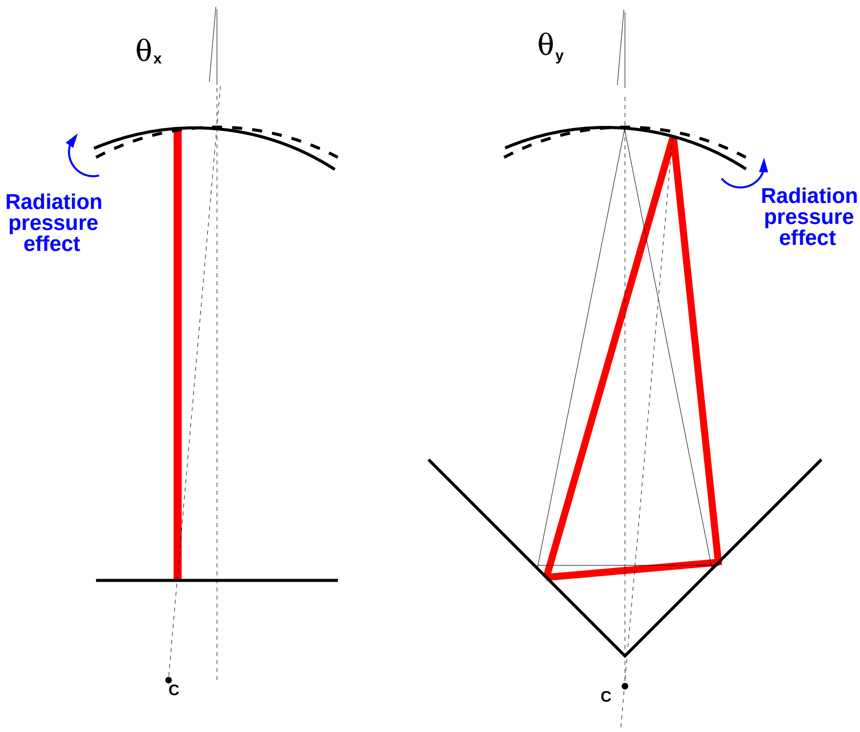

Figure 10: Effect of the radiation pressure in a triangular cavity on the θ_x and θ_y directions. The effect of the radiation pressure on the two directions is completely different: on the θ_x the radiation pressure acts on the system lowering the stiffness, while on θ_y the radiation pressure increase the stiffness.

In the previous sections the behavior of the resonant frequency in the pitch direction has been studied both with an analytical computation and with a numerical modeling. This was possible since in the pitch direction the IMC behaves like a simple Fabry-Perot cavity while in the yaw direction the system is much more complex. The effect of the terminal mirror misalingment on the resonant mode in a triangular cavity is completely different

from the behavior in a single F-P cavity. This implies that the radiation pressure will act on the stiffness of the mirror in different ways, as it is shown in Figure 10, restoring the alignment in the yaw direction, thus increasing the stiffness of the system, and worsening the misalignment in the pitch direction.
The radiation pressure effect can be valuated by simply fitting the experimental data with the square root of the input power. Moreover in first approximation it is possible to geometrically extrapolate the effect on the yaw direction, considering that the miscentering is more or less equivalent with respect to the pitch direction and the opto-mechanical parameters are similar. The result is shown in Figure 11.

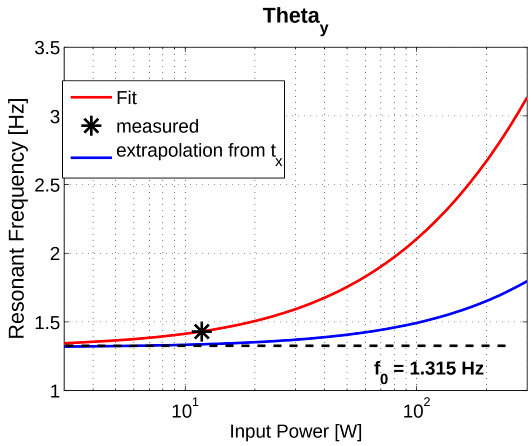

Figure 11: Radiation pressure effect on the IMC terminal mirror yaw direction.

From Figure 11 a big discrepancy between the measured data and the extrapolated results it is found, the measured effect is much much stronger The behavior of the yaw resonant frequency is more critical with respect to the pitch mode since it varies more than 1 Hz at full power, 180 W, thus the development of the angular control could be more critical during the lock acquisition. Moreover it has to be considered that the behavior on the

4 Conclusions

In this note the effect of the radiation pressure in the IMC for the Advanced Virgo design has been studied. The radiation pressure will not affect strongly the longitudinal control, see section 2, thus it can be stated that the radiation pressure will not be an issue for longitudinal control. The situation becomes a little bit more critical for the angular directions. The radiation pressure effects in the IMC cavity have been studied in both analytically and numerical approaches for the θ_x mode. The results shows that also with the present configuration the effects of radiation pressure will modify the mechanical transfer functions but the unstable configuration will not be reached, see Figure 11.
The effect on the yaw direction has been evaluated by using measured data, since the modeling is quite complex, and as in the pitch direction the radiation pressure can not bring the system in the unstable configuration, resonant frequency lower than zero. The only concern is that the situation is more critical since the variation of resonant frequency is much larger, more than 1 Hz, which could introduce some technical problems in the angular control of this d.o.f. during the locking acquisition, thus would be preferable to increase the mirror mass, up to about 3 kg, in order to have smaller effects.

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