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Thermal Noise Constraints for the Advanced Virgo Non-Degenerate Recycling Cavity Design

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1 Scope of this document

Currently the replacement of marginally stable recycling cavities (as they are used in LIGO and Virgo) by non-degenerate recycling cavities [1] is under discussion for Advanced Virgo. Due to the fact that most thermal noise contributions scale with the inverse of the beam radius and non-degenerate recycling cavities featuring strongly reduced beam sizes, we need to investigate whether any problems arise from thermal noise in the case of non-degenerate recycling cavities.

For the recycling mirrors the strongest thermal noise contribution is assumed to originate from coating Brownian noise, while for the beam splitter the main suspect is thermo-refractive noise of its substrate.

2 Thermo-refractive noise in the beam splitter

The thermo-refractive noise of the Advanced Virgo beam splitter can be calculated using Equation 5.3 from [2]. Division by k^2, where $k = 2\pi/\lambda$, yields the the power spectral density of the corresponding efficient beam splitter displacement noise:

$$S_R(\omega) = \frac{4\beta^2 l_c k_B T^2 \kappa}{(\rho C)^2} \frac{1}{\pi (R_b/\sqrt{2})^4 \omega^2} \quad (1)$$

Here $\omega = 2\pi f$ is the frequency, β the temperature dependence of the refraction index, l_c the geometrical path length inside the substrate, k_B Boltzmann's constant, T the temperature, κ the thermal conductivity, ρ the density, C the specific heat and R_b the beam radius. Please note that in his original version Braginski uses the rather unconventional definition of the beam radius R_b, being the distance from the center where the intensity of the beam decreased to $1/e$, instead of the commonly used $1/e^2$. In order to keep compatibility with our normal definition of beam size we had to introduce the additional factor $1/\sqrt{2}$ into Equation 1.

temperature dependence of refraction index	β	$-1.5 \cdot 10^{-5}$ K^{-1}
substrate density	ρ	2200 $\frac{\mathrm{kg}}{\mathrm{m}^3}$
geometrical path length inside substrate	l_c	0.0736 m
thermal conductivity	κ	1.38 $\frac{\mathrm{W}}{\mathrm{m \cdot K}}$
temperature	T	290 K
specific heat	C	746 $\frac{\mathrm{J}}{\mathrm{kg \cdot K}}$

Table 1: Material properties of Suprasil 3001 from [4] or [5].

The Advanced Virgo beam splitter will be made of Suprasil 3001 [3]. The relevant material properties have been extracted from [4, 5] and are listed in Table 1. The value for l_c was derived by considering a beam splitter substrate of 6.5 cm thickness, an index of refraction $n = 1.45$ and an angle of 45 degrees. Using the values from

Table 1 we can now calculate the thermo-refractive noise of the beam splitter for various beam sizes R_b[1]. This can then be compared to the requirements for the relevant length degrees of freedom, which are in this case the differential arm length of the small Michelson (MICH) and the length of the signal recycling cavity (SRCL). Figure 1 shows the result using the length stability requirements as given in Figure 3 of [6] and the definition of MICH and SRCL from [7]. It is found that a beam radius of only 1 mm would not fulfill the length requirement for the MICH and SRCL degrees of freedom.[2] **In order to achieve the usual safety factor of 10, the beam radius needs to be increased to about 5 mm.**

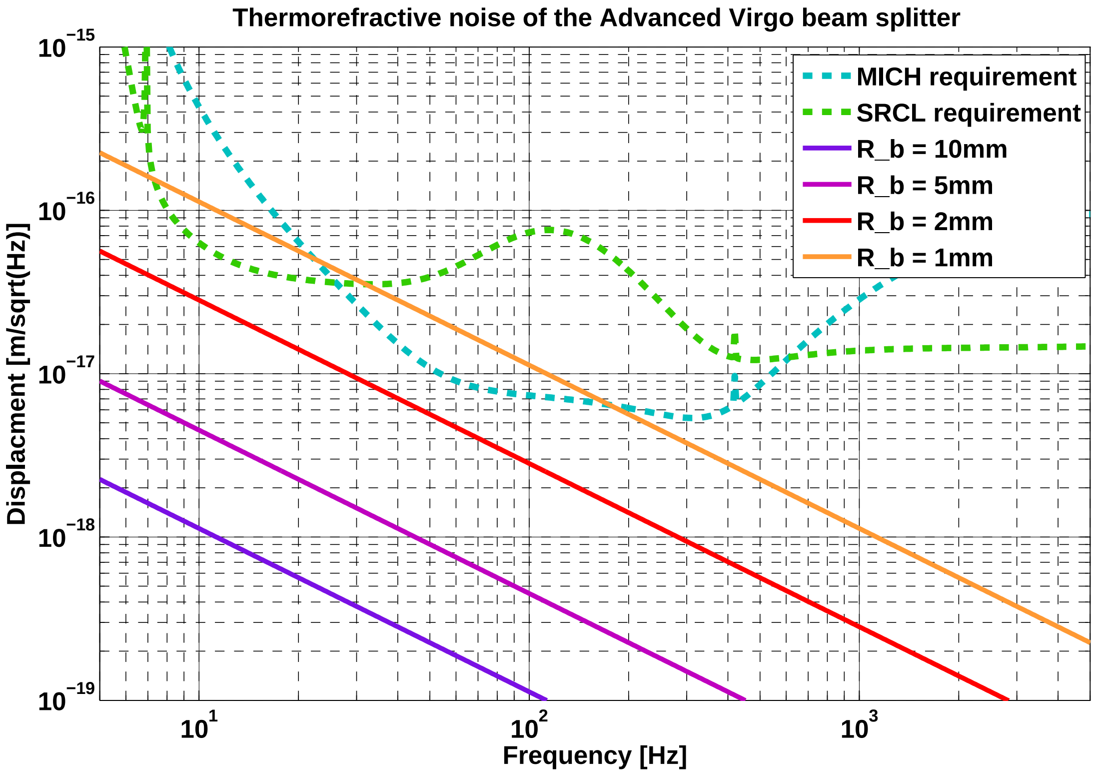

Figure 1: Beam splitter thermo-refractive noise for various beam radii and a beam splitter thickness of 6.5 cm. Obviously a beam radius of 1 mm would not fulfill the length requirement for the MICH and SRCL degrees of freedom. In order to achieve the usual safety factor of 10, the beam radius needs to be increased to about 5 mm. (Please note: In principle the beam splitter substrate thickness could be reduced. However, the ASD of the thermo-refractive noise goes down only proportional to the square-root of the beam splitter thickness.)

3 Coating Brownian noise of the Signal Recycling mirrors

For calculating the coating Brownian noise of the signal recycling mirrors in the case of non-degenerate recycling cavities we made use of the existing GWINC [8] model for advanced Virgo [9]. GWINC calculates the coating Brownian noise of a mirror, based on the formular given in G. Harry et al [10]. The following material properties, extracted from the GWINC configurations file **adv_270608.m**, have been considered:

[1] Please note that in the literature currently a variety of values in the range from about $-1.5 \cdot 10^{-5}$ K^{-1} to $-0.8 \cdot 10^{-5}$ K^{-1} is used for β. In order to keep our analysis conservative we chose the worst case value for β (see Table 1).

[2] Actually the beam in the x arm pathes twice the beam splitter substrate. For all frequencies within the detection band the thermo-refractive noise of the two passes adds coherently, thus resulting in a factor 2. However, due to the definitions of of MICH and SRCL, which includes terms of $l_x - l_y/2$ and $l_x + l_y/2$ this factor two is compensated again.

```
%% high index material: tantala
ifo.Materials.Coating.Yhighn = 140e9;
ifo.Materials.Coating.Sigmahighn = 0.23;
ifo.Materials.Coating.CVhighn = 2.1e6;                % Crooks et al, Fejer et al
ifo.Materials.Coating.Alphahighn = 3.6e-6;            % 3.6e-6 Fejer et al, 5e-6 from Braginsky
ifo.Materials.Coating.Betahighn = 1.4e-5;               % dn/dT, value Gretarrson (G070161)
ifo.Materials.Coating.ThermalDiffusivityhighn = 33;   % Fejer et al
ifo.Materials.Coating.Phihighn = 2.3e-4;
ifo.Materials.Coating.Indexhighn = 2.06539;

%% low index material: silica
ifo.Materials.Coating.Ylown = 72e9;
ifo.Materials.Coating.Sigmalown = 0.17;
ifo.Materials.Coating.CVlown = 1.6412e6;              % Crooks et al, Fejer et al
ifo.Materials.Coating.Alphalown = 5.1e-7;             % Fejer et al
ifo.Materials.Coating.Betalown = 8e-6; % dn/dT,  (ref. 14)
ifo.Materials.Coating.ThermalDiffusivitylown = 1.38; % Fejer et al
ifo.Materials.Coating.Philown = 4.0e-5;
ifo.Materials.Coating.Indexlown = 1.45;
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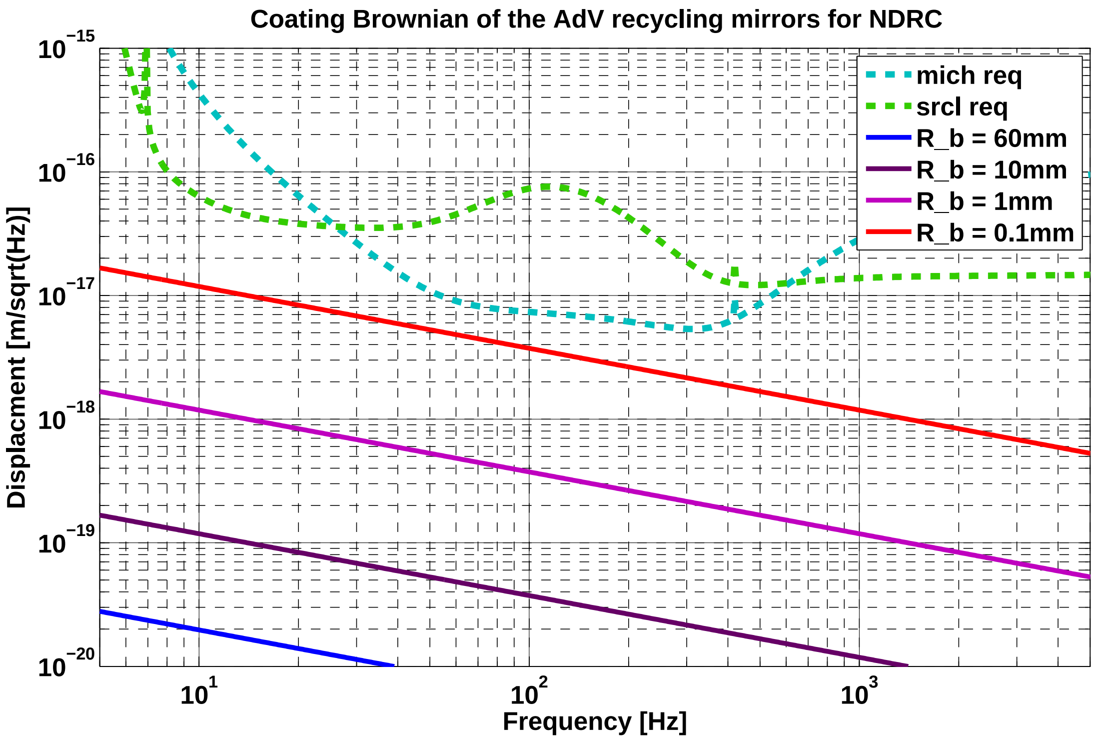

Figure 2: Coating Brownian noise of the a single recycling mirror with 100 percent reflectivity (PRM2, PRM3, SRM2 and SRM3) in dependence of the laser beam radius R_b. In order to achieve the usual safety factor of 10 for the SRCL degree of freedom, the beam radius needs to be at least about 0.3 mm.

Finally, for the overall thickness of the low-refractive and high-refractive coating meaterials we applied the same values as currently used for the end mirrors of the Advanced Virgo arm cavities: 3.5e-6 m for the low index material and 2.5e-6 m for the high index material [11]. Figure 2 shows the result for the coating Brownian noise of a single recycling mirror with 100 percent reflectivity. We find that only for beam radii much smaller than currently considered for any design option of the Advanced Virgo non-degenerate recycling cavities, coating Brownian noise would become a problem. **In order to achieve the usual safety factor of 10 for the SRCL degree of freedom, the beam radius needs to be larger than 0.3 mm.**

4 Summary

The beam radius at the beam splitter needs to be at least 5 mm (for a beam splitter thickness of 6.5 cm) in order to not spoil the MICH and SRCL degrees of freedom with thermo-refractive noise.[3] In addition a minimum beam radius of 0.3 mm is required at SRM2 and SRM3 in order to avoid problems from coating Brownian noise inside the signal recycling cavity.

References

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[9] S. Hild and G. Losurdo: *Advanced Virgo design: Comparison of the Advanced Virgo sensitivity from Bench 4 and GWINC (v1)*, Virgo note, VIR-055A-08 (2008). 2

[10] Harry G. M et al ”Titania-doped tantala/silica coatings for gravitational-wave detection” *Classical and Quantum Gravity* **24** (2007) 405-415 2

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[3]Please note that in some of the currently discussed NDRC layouts also the compensation plates might be transmitted by a rather small laser beam. In this case additional thermo-refractive noise from these CP needs to be considered.