CNRS
Centre National de la Recherche Scientifique

INFN
Istituto Nazionale di Fisica Nucleare

FoM for end benches telescope optimization for AA purposes

VIR-0058A-12

M. Mantovani

Issue: 1

Date: February 21, 2012

VIRGO * A joint CNRS-INFN Project
Via E. Amaldi, I-56021 S. Stefano a Macerata - Cascina (Pisa)
Secretariat: Telephone (39) 050 752 521 * FAX (39) 050 752 550 * Email W3@virgo.infn.it

Contents

1 Introduction

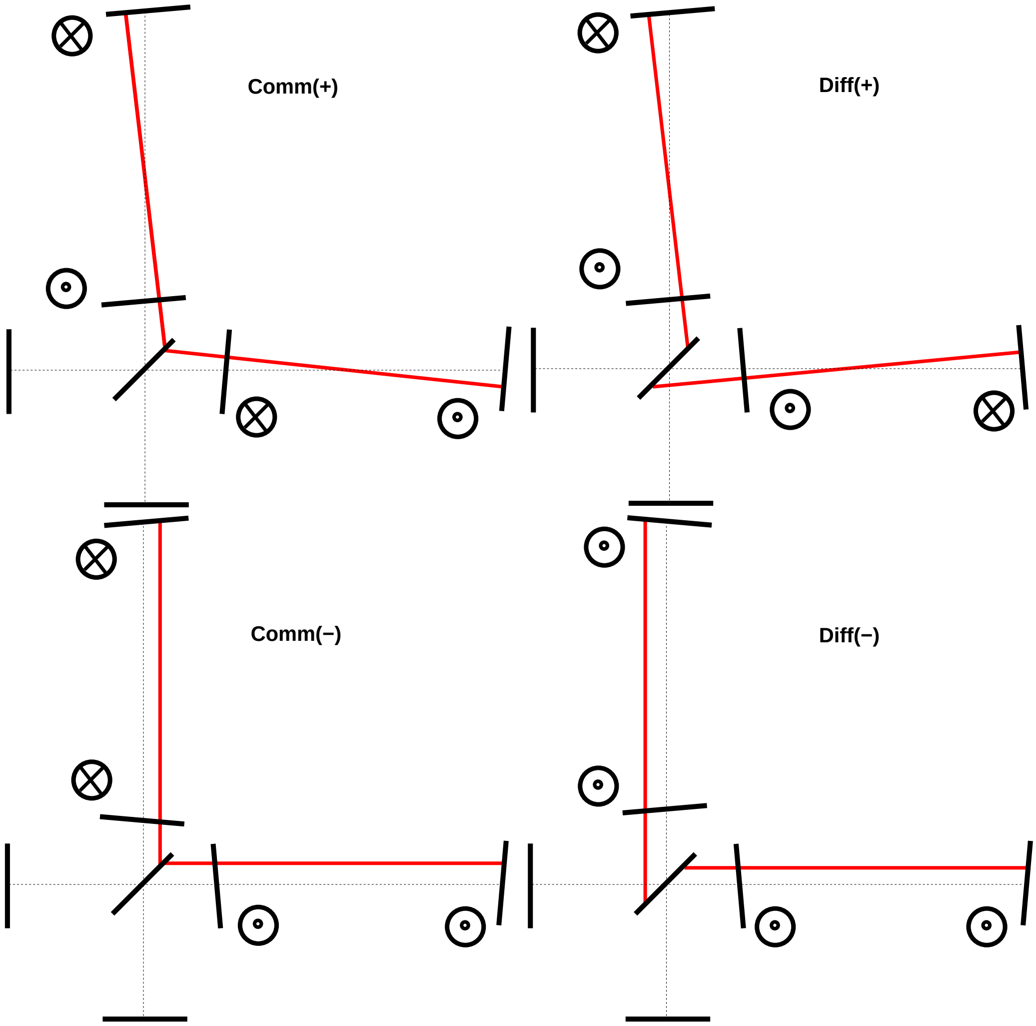

Figure 1: Drawing of cavity angular modes.

In the Advanced Virgo configuration important angular modes, such as the Differential and Common End minus modes, have to be controlled with DC signals coming from the external benches in transmission from the Arm cavities. The (-)-modes have been evaluated to be the worse DOFs in terms of control noise, their control noise is at the level of the Advanced Virgo safety curve (a factor 10 below the design sensitivity) [1]
This request leads to define the suspension requirements for the optical benches where the sensor used to control the (-)-modes are placed, since a DC signal is directly affected by the bench displacement detecting only the relative position of the beam with respect to the diode. The plane of the interferometer is indeed fixed by three DC signals and if they are affected by the displacement of the benches where they are placed the whole ITF will move erroneously as the bench moves.
The most critical requirement is on the bench tilt residual displacement, which results to be $\langle\theta\rangle_{RMS} \simeq 0.03$ [μrad] (a factor 3 below the LC performances) and it can be relaxed by optimizing the telescopes.

2 Optimization of the telescopes

The (-)-modes are pure translation modes as it is shown in Figure 1. The AA error signal will be computed by digitally combining the two quadrant signals, placed at ±zR, in order to maximize the signal coming from the (-)-mode, i.e. having a virtual quadrant which makes the image at a precise Gouy phase ϕ_G. For the angular control system it is mandatory to chose the Gouy phase which maximizes the (-)-mode error signal otherwise the angular control noise will not be anymore compliant with the Advanced Virgo sensitivity requirements. The error signal is proportional to:

$$S_{((-)-mode)} \propto TF_{shift}|_{\phi_G} \tag{1}$$

where $S_{(-)-mode}$ is the angular signal coming from the (-)-mode and $TF_{shift}|_{\phi_G}$ is the telescope TF for the beam shift at the ϕ_G which maximizes the error signal. While for the bench tilt and shift the fake signals on the virtual quadrant are:

$$S_{bench\ tilt} \propto TF_{tilt}|_{\phi_G} \tag{2}$$

$$S_{bench\ shift} \propto TF_{shift}|_{\phi_G} \tag{3}$$

The requirements on the bench displacement ($\langle h \rangle_{RMS}$ and $\langle \theta \rangle_{RMS}$) are set to be below the AA error signal by a factor 5. The requirement for the bench shift is independent from the telescope design while the bench tilt requirement is proportional to:

$$\langle \theta \rangle_{RMS} \propto \frac{TF_{shift}|_{\phi_G}}{TF_{tilt}|_{\phi_G}} \tag{4}$$

The telescope transfer function ratio between the shift and the tilt at the Gouy phase which maximizes the error signal is the parameter which has to be maximized to relax the requirement on the bench tilt.

2.1 How to practically optimize the telescope

This can be obtained by knowing the telescope parameters as the beam displacement on the quadrants (placed at -/+zR from the waist) due to cavity beam tilt and shift and bench tilt and shift.
The effect of the telescope can be described by two set of parameters. The first is the beam displacement on the quadrants (placed at -/+zR), due to the cavity beam tilt and shift at the waist, it is given by:

$$\begin{pmatrix} x_{Qpd1} \\ x_{Qpd2} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ \theta \end{pmatrix}_{beam}$$

The second is the beam displacement on the quadrants, due to bench tilt and shift at the waist, is given by:

$$\begin{pmatrix} x_{Qpd1} \\ x_{Qpd2} \end{pmatrix} = \begin{pmatrix} a_b & b_b \\ c_b & d_b \end{pmatrix} \cdot \begin{pmatrix} x \\ \theta \end{pmatrix}_{bench}$$

The Gouy phase ϕ_G which maximizes the (-)-mode sensing can be computed, considering $(x\ ,\ \theta)_{beam} = (1\ ,\ 0)$, as :

$$\begin{cases} a = A\cos(\pi/4 + \phi_G) \\ c = A\cos(\pi/4 - \phi_G) \end{cases}$$

Thus the Gouy phase is equal to:

$$\phi_G = \arctan\left(\frac{c-a}{c+a}\right) \tag{5}$$

While the amplitude is:

$$A = \frac{a}{\cos\left(\arctan\left(\frac{c-a}{c+a}\right) + \pi/4\right)} \tag{6}$$

Then after the Gouy phase has been computed the telescope TFs for the bench, which gives the effect of the bench shift and tilt on the virtual quadrant, will be computed by rotating the vectors $(a_b\ ,\ c_b)$ and $(b_b\ ,\ d_b)$ by the angle $\theta = -(\pi/4 + \phi_G)$. The telescope TF for the tilt, which is the one that it is the goal of this computation, is then:

$$\begin{pmatrix} b'_b \\ d'_b \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} b_b \\ d_b \end{pmatrix}$$

the parameter that has to be optimized to be maximum is $\left|\frac{A}{b'_b}\right|$. Where A gives the magnification of the (-)-mode error signal due to the telescope and b'_b is the magnification of the bench tilt due to the telescope.

2.2 Example with the baseline design

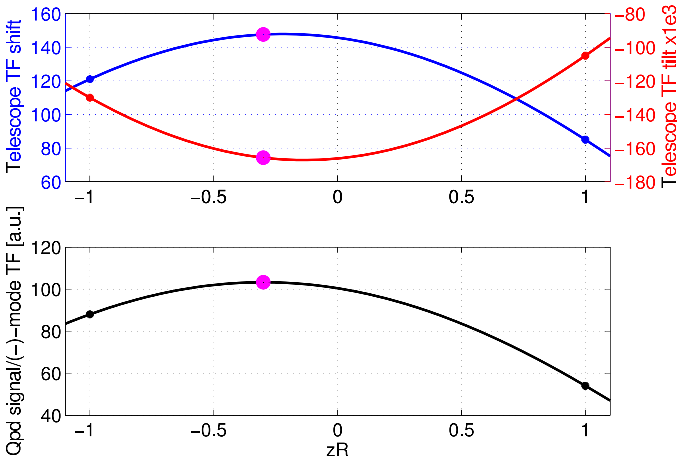

Figure 2: On the top plot Telescope TF as a function of the distance from the waist (blue curve and red curve for shift and tilt respectively). On the bottom the behavior of the quadrant/(-)-mode transfer function is shown as a function of the distance of the virtual quadrant from the waist. The magenta dot points the Gouy phase which will be reconstructed with the two quadrant diodes placed in -zR and +zR to obtain the maximum signal for the (-)-mode.

The beam displacement on the quadrants, due to bench tilt and shift, is given by, according to C. Buy:

$$\begin{pmatrix} x_{Qpd1} \\ x_{Qpd2} \end{pmatrix} = \begin{pmatrix} 121 & -130\cdot 10^3 \\ 85 & -105\cdot 10^3 \end{pmatrix} \cdot \begin{pmatrix} x \\ \theta \end{pmatrix}_{bench}$$

The beam displacement on the quadrants, due to cavity beam tilt and shift at the waist , is given by:

$$\begin{pmatrix} x_{Qpd1} \\ x_{Qpd2} \end{pmatrix} = \begin{pmatrix} 88 & 29\cdot 10^3 \\ 54 & -24\cdot 10^3 \end{pmatrix} \cdot \begin{pmatrix} x \\ \theta \end{pmatrix}_{beam}$$

The Gouy phase which maximizes the error signal is:

$$\phi_G = -13.4652\ [\text{deg}] \tag{7}$$

And the Amplitude is:

$$A = 103.2473 \tag{8}$$

The telescope transfer function at that precise Gouy phase for the bench tilt is, the magenta dot on the red curve shown in Figure 2:

$$b'_b = -165.7326\cdot 10^3 \tag{9}$$

thus the figure of merit ($|\frac{A}{b'_b}|$) of this telescope design is $6.23 \cdot 10^{-4}$ and the telescope will allow to relax the bench tilt residual displacement up to $\simeq 0.1$ [μrad], which will be feasible with the present LC, if the figure of merit will be larger than $1.869 \cdot 10^{-3}$.

References

[1] M. Mantovani, "Differential offset vs Automatic Alignment control", VIR-0737A-11 2