The Stochastic GW background

Nick van Remortel
University of Antwerp, Belgium

ET community workshop, Aachen

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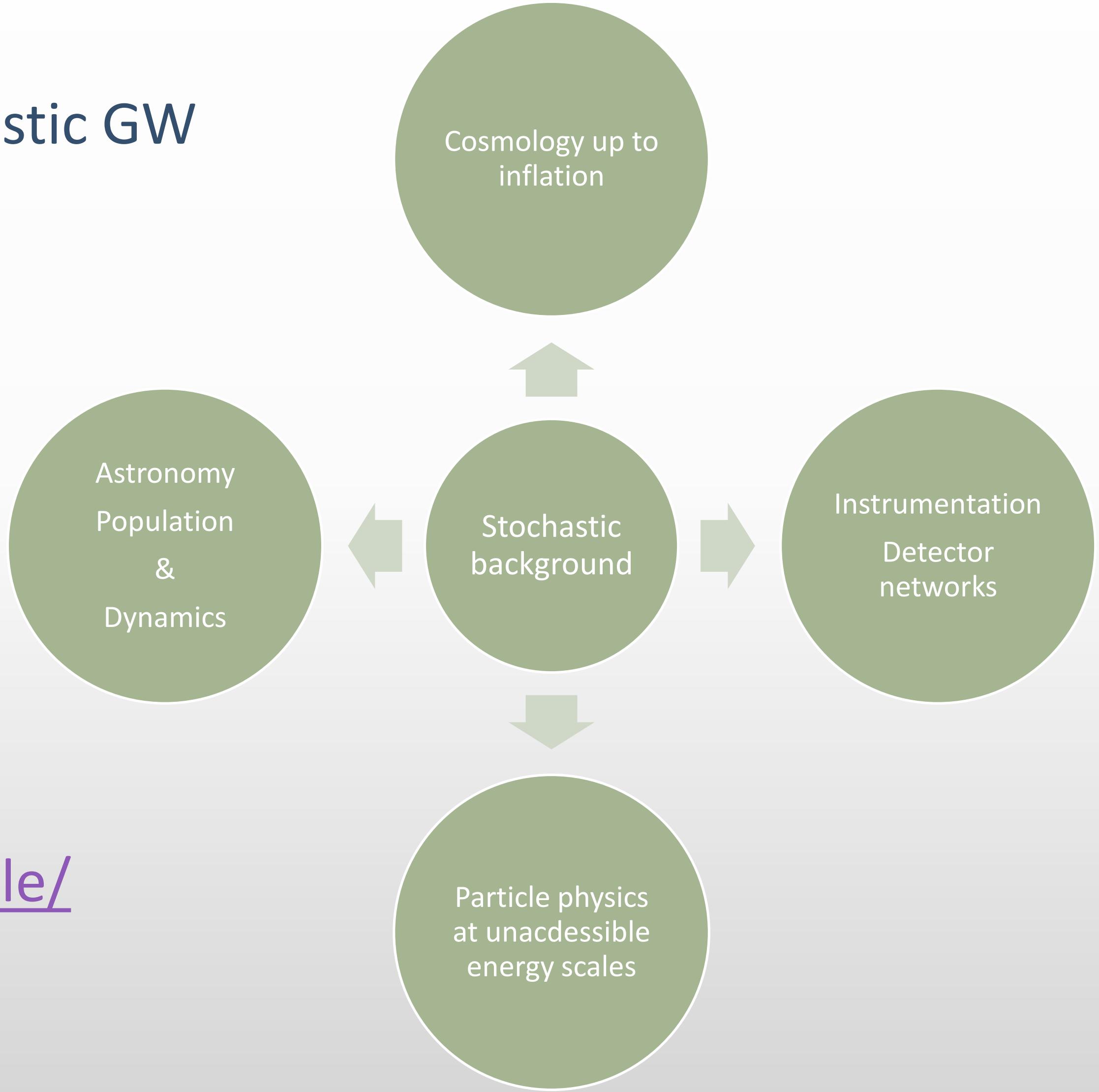
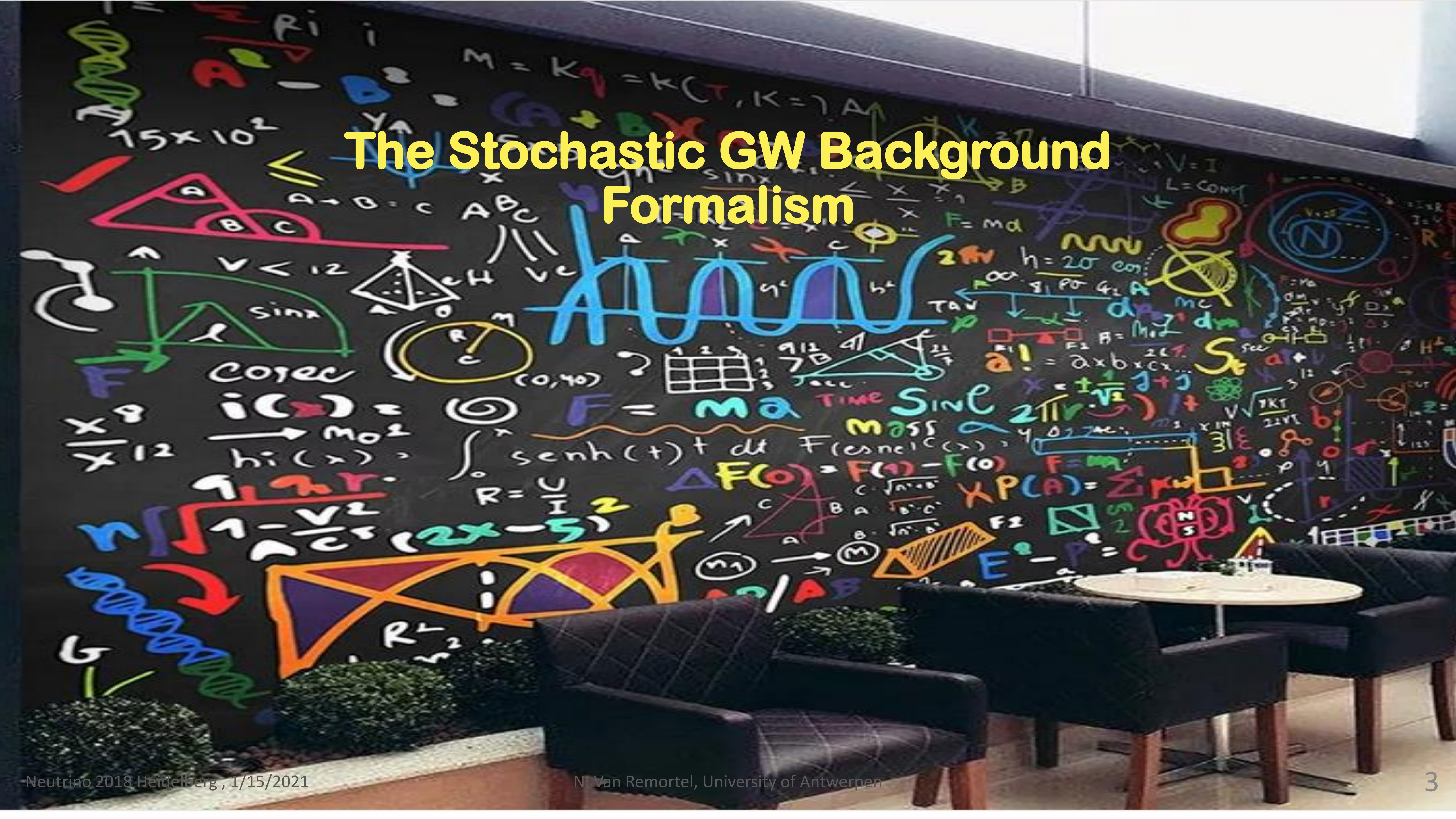
Outline

- Definition and general characteristics of stochastic GW background
- Detection method
- Sources of various origins:
  - Cosmological/Primordial
  - Astrophysical
- Recent results from LIGO&Virgo
- Future avenues

Check us out on: https://www.virgo-gw.be/people/
We are often recruiting!



The Stochastic GW Background Formalism



Stochastic Gravitational waves

- Stochastic:
  - random character (in time and frequency) , and should therefore originate from large number of independent sources
- Background:
  - generally perceived as a ‘weak’ signal, often comparable to detector noise levels or smaller

- General properties: remember in the TT gauge a set of freely propagating GWs can be expanded in plane waves

$$h_{ij}(t,\bar{x}) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int d^2\hat{n}\, \tilde{h}_A(f,\hat{n}) e_{ij}^A(\hat{n}) \exp\left[-2\pi i f\left(t - \frac{(\hat{n}\cdot\bar{x})}{c}\right)\right]$$

- Stochastic: The amplitudes, $\tilde{h}_A(f,\hat{n})$, are random variables, characterized statistically by their ensemble averages.
- Ensembe averages: Will be practically replaced by time averages over set of observation sequences of duration ΔT, corresponding to a frequency resolution of $\sigma_f = 1/\Delta T$

General properties

- <u>Stationary</u>:
  - all quantities of interest (ie. Correlators) depend only on time differences

  $\langle h_A(t)h_{A'}(t')\rangle$ depends only on (t-t'), or

$$\langle \tilde{h}_A(f,\hat{n})\tilde{h}_{A'}(f',\widehat{n'})\rangle \sim \delta(f-f')$$

- <u>Gaussian:</u>
  - Consequence of central limit theorem: sum of many random processes produces gaussian stochastic distribution, regardless of underlying distributions
  - all N-point correlators reduce to sum or products of 2-point correlators or $\langle h_A\rangle$.

- <u>Isotropic:</u>
  - If similar to CMB, then highly isotropic, but precision science in the very small deviations
  - Motivates directional stochastic backgound search

$$\langle \tilde{h}_A(f,\hat{n})\tilde{h}_{A'}(f',\widehat{n'})\rangle \sim \delta(\phi-\phi')\delta(cos\theta-cos\theta')$$

- <u>Unpolarised</u>: Fair assumption is many astrophysical components or cosmological

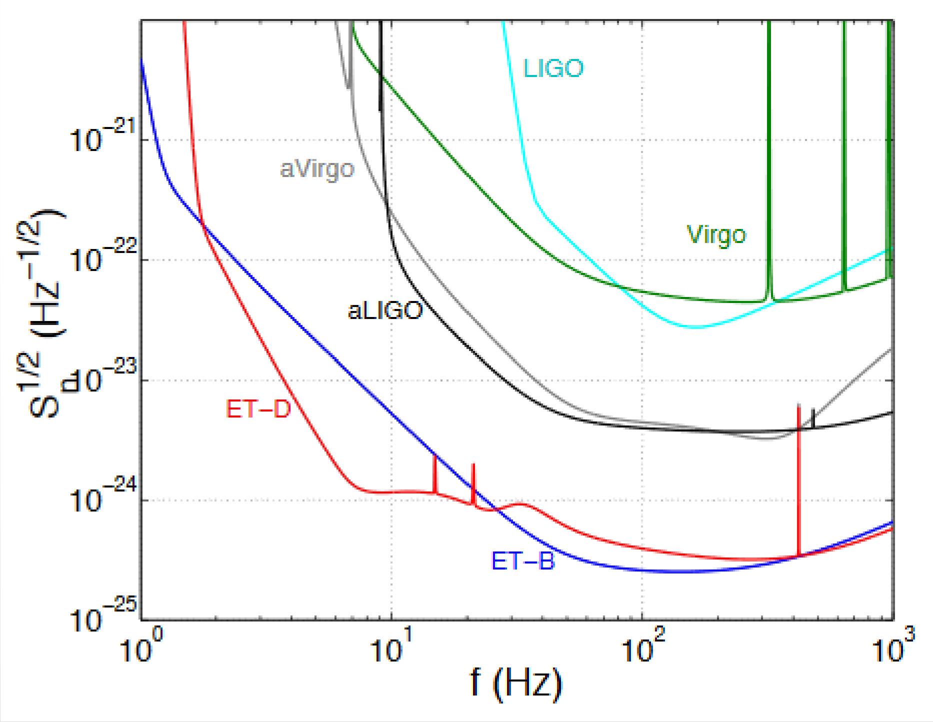
$$\langle \tilde{h}_A(f,\hat{n})\tilde{h}_{A'}(f',\widehat{n'})\rangle \sim \delta_{A,A'}$$

Quantities of interest

- Stochastic backgrounds are characterized by a single function $S_h(f)$: spectral density, analogous to spectral density of detector noise

$$\langle \tilde{h}_A^*(f,\hat{n})\tilde{h}_{A'}(f',\hat{n}')\rangle = \delta(f-f')(4\pi)^{-1}\delta^2(\hat{n},\hat{n}')\delta_{A,A'}\frac{1}{2}S_h(f)$$

$$\Rightarrow \langle h_{ij}(t)h^{ij}(t)\rangle = 4\int_0^\infty df\, S_h(f)$$



- Energy density carried by stochastic background

By definition

$$\rho_{GW} = \frac{c^2}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$$

Which is then compared with critical density, $\rho_c = \frac{3c^2H_0^2}{8\pi G}$

to obtain a dimensionless quantity

$$\Omega_{GW} = \frac{\rho_{GW}}{\rho_c}$$

Quantities of interest

- Define the normalised energy density, $\Omega_{GW}(f)$, per logaritmic interval in frequency to maintain dimensionless quantity

$$\Omega_{GW} = \int_{f=0}^{f=\infty} d(logf)\,\Omega_{GW}(f)$$

- Similarly

$$\rho_{GW} = \frac{c^2}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$$

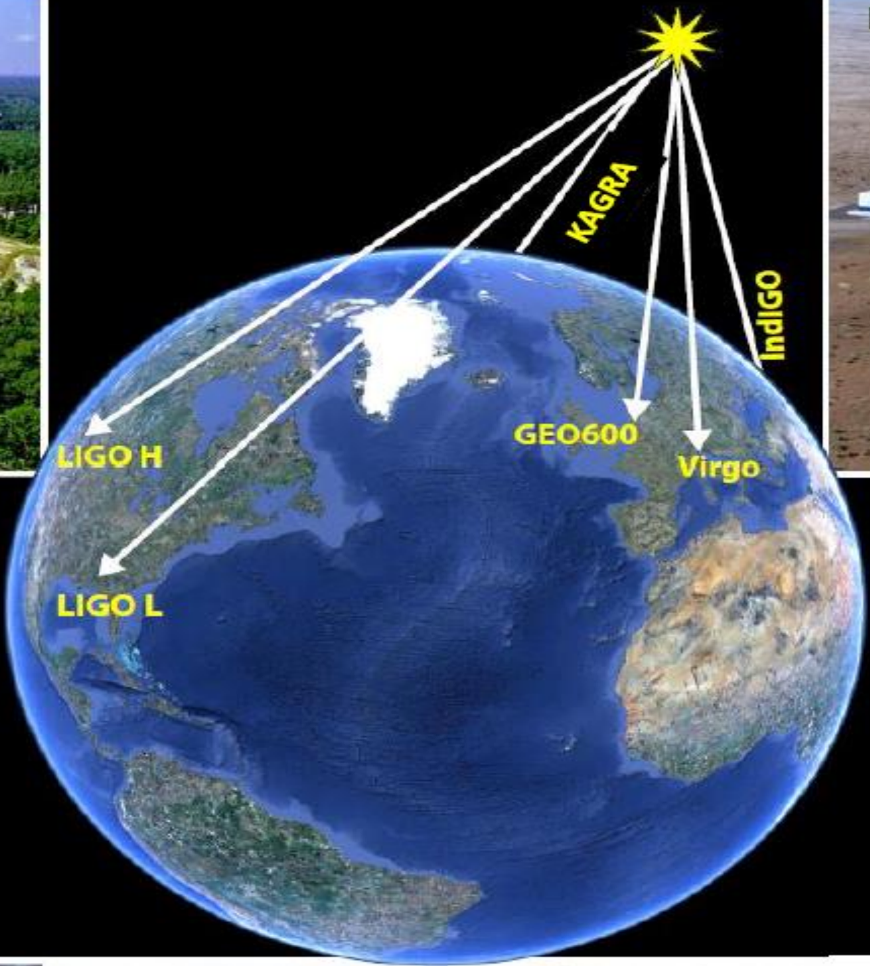
$$= \frac{c^2}{8\pi G}\int_{f=0}^{f=\infty} d(logf)\, f(2\pi f)^2 S_h(f) \Rightarrow \frac{d\rho_{GW}}{d(logf)} = \frac{\pi c^2}{2G} f^3 S_h(f)$$

$$\Omega_{GW}(f) = \frac{4\pi^2}{3H_0^2} f^3 S_h(f)$$

Or, when using characteristic strain

$$h_c(f) = \sqrt{f S_h(f)}$$

$$h_0^2\Omega_{GW}(f) = \frac{4\pi^2}{3(100km/s/Mpc)} f^2 h_c(f)$$



Detection methods

Advanced LIGO and Virgo
run simultaneously

Kagra joined in 2020
LIGO India approved

Minimal detectable values of Ω

Bibliography:
[M. Maggiore, “Gravitational Waves, Vol 1: Theory and experiments”, Oxford university Press(2008), chap. 7]
[B. Allen, and J. D. Romano, 1999, Phys. Rev. D, 59, 102001]
[E. Thrane, and J. D. Romano, 2013, Phys. Rev. D 88, 124032]

- Output strain, $h(t)$, of a single detector with sinusoidal response function,$R^A(f,\hat{k})$, as response to a GW wave metric perturbation $h_{ab}(t,\bar{x})$

$$h(t) = \sum_A \int_{-\infty}^{\infty} df \int d^2\Omega_k\, \tilde{h}_A(f,\hat{k}) R^A(f,\hat{k}) \exp\left[2\pi i f(t - \hat{k}\cdot\bar{x}/c)\right]$$

$$h(f) = \sum_A \int d^2\Omega_k\, \tilde{h}_A(f,\hat{k}) R^A(f,\hat{k}) \exp\left[-2\pi i f \hat{k}\cdot\bar{x}/c\right]$$

- Total strain measured by a detector $s(t) = n(t) + h(t)$
- If detector measures just noise $\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \frac{1}{2}\delta(f-f')S_n(f)$ (power spectral density)
- One can carefully model the noise and when $\langle s^2(t)\rangle$ in excess of $\int_0^\infty df\, S_n(f)$ above a treshold, observation! (naive approach)

SNR for detection

- In the presence of a signal $\langle s^2(t)\rangle = \langle n^2(t)\rangle + \langle h^2(t)\rangle$

$$= \int_0^\infty df [S_n(f) + R(f) S_h(f)]$$

With $R(f)$, the detector response function integrated over all angles and polarisations

- When working in discrete frequency bins with width $\Delta f \sim 1/T$

$$\int_0^\infty df S_{h,n}(f) = \sum_i S_{h,n}(f_i) \Delta f$$

- Signal-To-Noise in a given frequency bin , *i*, becomes $\boxed{\left(\frac{S}{N}\right)^2 = R(f) \frac{S_h(f_i)}{S_n(f_i)}}$

- Note: no $\Delta f \sim 1/T$ dependence, so no improvement by measuring longer!
- Only benefit of measuring longer is improved frequency resolution

SNR for detection

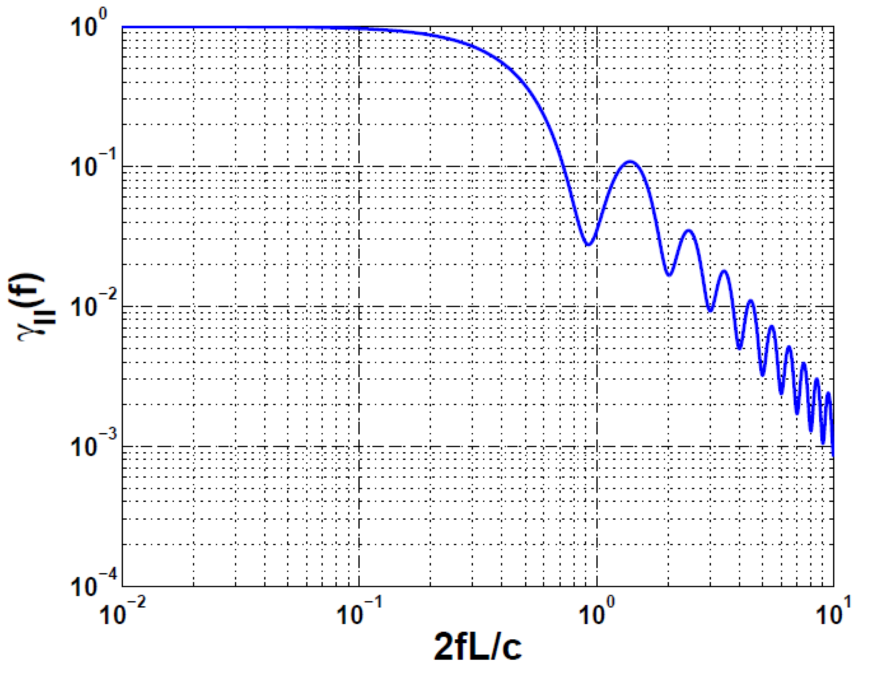
- Minimal detectable value of $S_h(f)$ measurable with single detector with a noise spectral density $S_n(f)$, at a given treshold value of SNR is

$$[S_h(f)]_{min} = \frac{S_n(f)(S/N)_T^2}{R(f)}$$

- Or, corresponding minimal detectable value of Ω_{GW} is

$$[\Omega_{GW}(f)]_{min} = \frac{4\pi^2}{3H_0^2} f^3 \frac{S_n(f)(S/N)_T^2}{R(f)}$$

- Note: For a constant value of S_n, your detection sensitivity increases as f^3
- Detection of stochastic GW backgrounds is always easier at low frequencies!
- To increase your single detector efficiency:
  - Push your noise spectra density down at low frequencies!



Note: dips occur at multiples of $f = \frac{c}{2L}$

With *L*= arm length

Need for detector network: cross correlation!

- Typical numerical values for earth based interferometers make detection impossible!

$$[h_0^2\Omega_{GW}(f)]_{min} = 0.12\left(\frac{f}{100Hz}\right)^3\left(\frac{S_n^2(f)}{4.10^{-23}Hz^{-1/2}}\right)^2\left(\frac{2/5}{R(f)}\right)\left(\frac{(S/N)_T}{5}\right)^2$$

$h_0 \cong 0.7$, absorbs uncertainties in Hubble parameter

- When correlating two detectors *I* and *J*, the detector noise contributions dissappear! (if noise is uncorreated)
- Detector overlap reduction function: note $\Gamma_{II}(f) = R_I(f)$

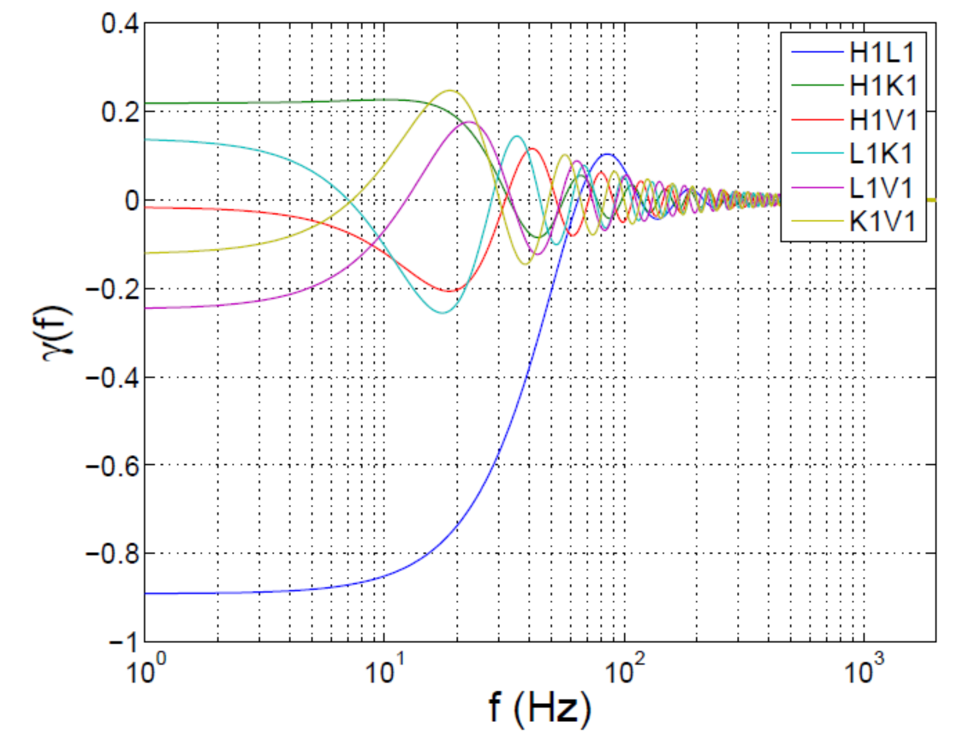
$$\langle \tilde{h}_I(f)\tilde{h}_J^*(f)\rangle = \frac{1}{2}\delta(f-f')\Gamma_{IJ}(f)S_h(f)$$

$$\Gamma_{IJ}(f) = \frac{1}{8\pi}\int d^2\Omega_k \sum_A R_I^A(f,\hat{k})R_J^{A*}(f,\hat{k})\exp\left[-2\pi i f\hat{k}\cdot(\bar{x}_I-\bar{x}_J)/c\right]$$

- Note also: exponential oscillates and dampens rapidly if

$$2\pi f\frac{\Delta x}{c} \gg 1 \quad \text{or} \quad \Delta x \gg \frac{\lambda}{2\pi} = 500\text{km (100Hz)}$$

($\gamma_{IJ}(f)$ is normalised to be 1 for co-aligned and co-located detectors)



SNR for correlation measurement

- Equivalently, the SNR for a cross correlation search yields an observation time, *T*, dependent

$$\left(\frac{S}{N}\right)^2 = 2T\int_{f_{min}}^{f_{max}} df\, \frac{\Gamma_{IJ}^2(f) S_h^2(f)}{S_{n,I}(f) S_{n,J}(f)}$$

Where the integration limits f_{min} and f_{max} define the detector bandwidths

- This (S/N) is the optimally filtered version of a detection with Wiener filter $\tilde{Q}(f) = \frac{\Gamma_{IJ}(f)\, S_h(f)}{S_{n,I}(f) S_{n,J}(f)}$

- For a binned correlation analysis of M detectors, averaged over the total bandwidth, one gets

$$\left(\frac{S}{N}\right)^2 = 2T\delta f N_{bins} \left\langle \frac{S_h^2}{S_{eff}^2} \right\rangle$$

with $S_{eff}^2(f) = \sum_{I=1}^{M}\sum_{J>I}^{M} \frac{\Gamma_{IJ}^2(f)}{S_{n,I}(f) S_{n,J}(f)}$ which reduces to $S_{eff}^2(\mathrm{f}) = \frac{2}{M(M-1)} S_n^2(f)$

for co-located and co-aligned detectors

$$BW = \Delta f = \delta f N_{bins}$$

Remarks on cross correlation technique

- The minimally detectable stochastic energy density now becomes dependent on observation time and bandwidth

$$[\Omega_{GW}(f)]_{min} = \frac{4\pi^2}{3H_0^2} f^3 \frac{\sqrt{S_{nI}(f)S_{nJ}(f)}}{\Gamma_{IJ}(f)} \frac{(S/N)_T}{(2T\Delta f)^{1/2}}$$

- Numerically $\frac{1}{(2T\Delta f)^{1/2}} \cong 1.10^{-5} \left(\frac{150Hz}{f}\right)^{1/2} \left(\frac{1\,yr}{T}\right)^{1/2}$

- And additional factor $\frac{1}{\sqrt{M(M-1)}}$ when using *M* detectors

- Note: Low frequencies are still most sensitive!

- However: Comparing SNR or $[\Omega_{GW}(f)]_{min}$ directly with model predictions lead to underestimation of detection potential due to typical broadband nature of signals!

Power-law integrated sensitivity

Bibliography:
[E.Thrane and J. D. Romano Phys. Rev. D 88, 124032 (2013)]

- Most stochastic signals will exhibit a power-law dependence

$$\Omega_{GW}(f) = \Omega_\beta \left(\frac{f}{f_{ref}}\right)^\beta$$

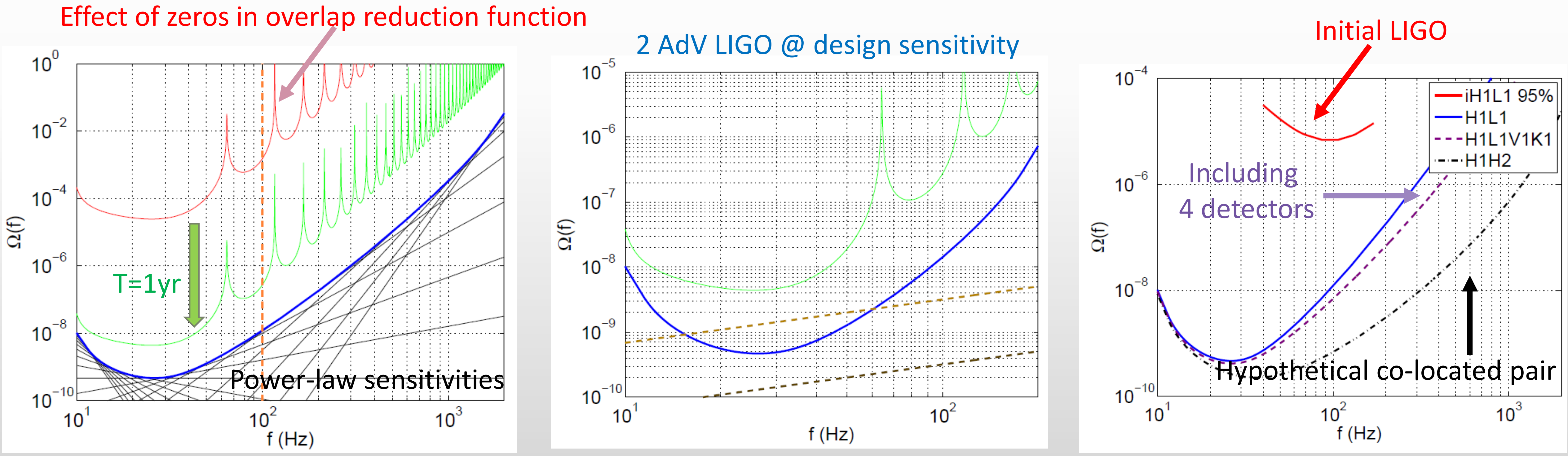
- Binary coalescences have a power-law index $\beta = 2/3$, while inflationary models have $\beta = 0$
- Note that the Wiener filter is dependent on the signal power law!
- Compute for a fixed power index the minimally detectable value (assuming a SNR treshold)

$$\Omega_{\beta,min} = \frac{(S/N)_T}{\sqrt{2T}} \left[\int_{f_{min}}^{f_{max}} df \frac{\left(f/f_{ref}\right)^{2\beta}}{\Omega_{eff}^2(f)}\right]^{-1/2} \quad \text{with} \quad \Omega_{eff}(f) = \frac{4\pi^2}{3H_0^2} f^3 S_{eff}(f)$$

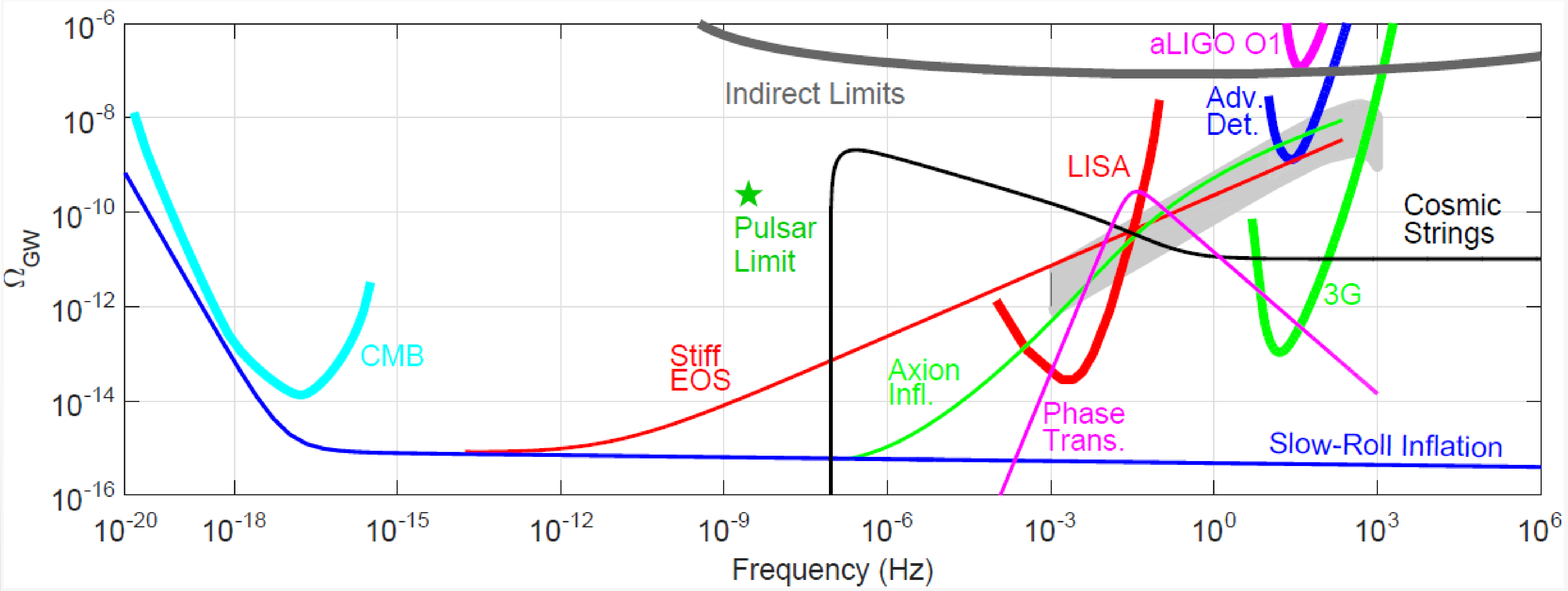
- And plot for each pair of values $\beta, \Omega_{\beta,min}$ the values of $\Omega_{gw,min}(f) = \Omega_{\beta,min} \cdot \left(f/f_{ref}\right)^\beta$

Power-law integrated sensitivity curves

- The enveloppe of the $\Omega_{gw,min}(f) = \Omega_{\beta,min} \cdot (f/f_{ref})^{\beta}$ power-law sensitivity curves is the power-law integrated sensitivity curve for the detection of a stochastic background with a cross correlation of M detectors. In all cases we put $(S/N)_T = 1$



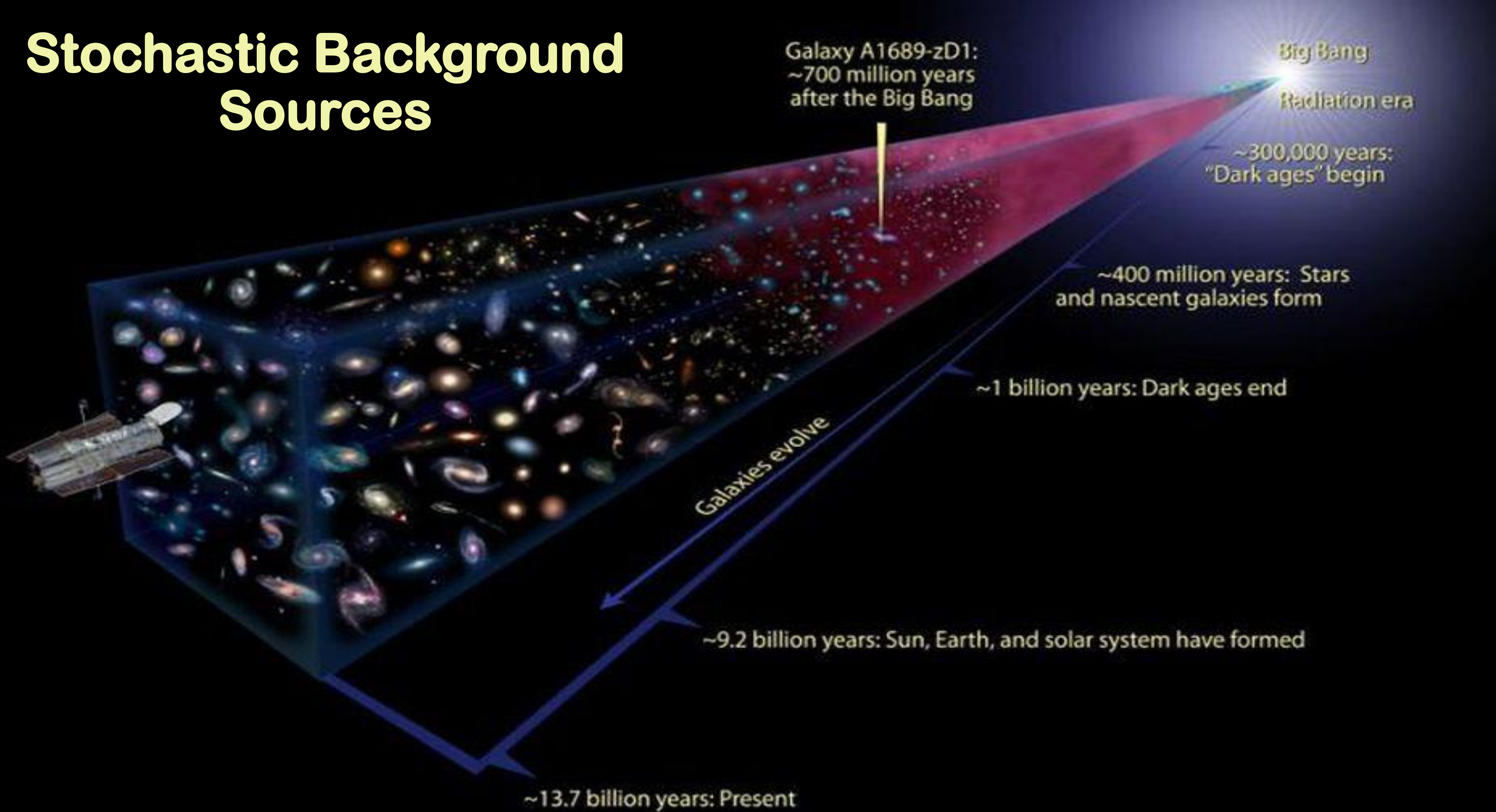
Typical frequency ranges, and sensitivities



Note the other components of Ω
(source: Wikipedia)

Baryon density parameter[b]	Ω_b	0.0486 ±0.0010[e]
Dark matter density parameter[b]	Ω_c	0.2589 ±0.0057[f]
Matter density parameter[b]	Ω_m	0.3089 ±0.0062
Dark energy density parameter[b]	Ω_Λ	0.6911 ±0.0062

Stochastic Background Sources



Main components

1. Cosmological, primordial, fossil:
   Rich phenomenology, inspired by elementary particle physics QFT, but also thermodynamics, topology, etc.

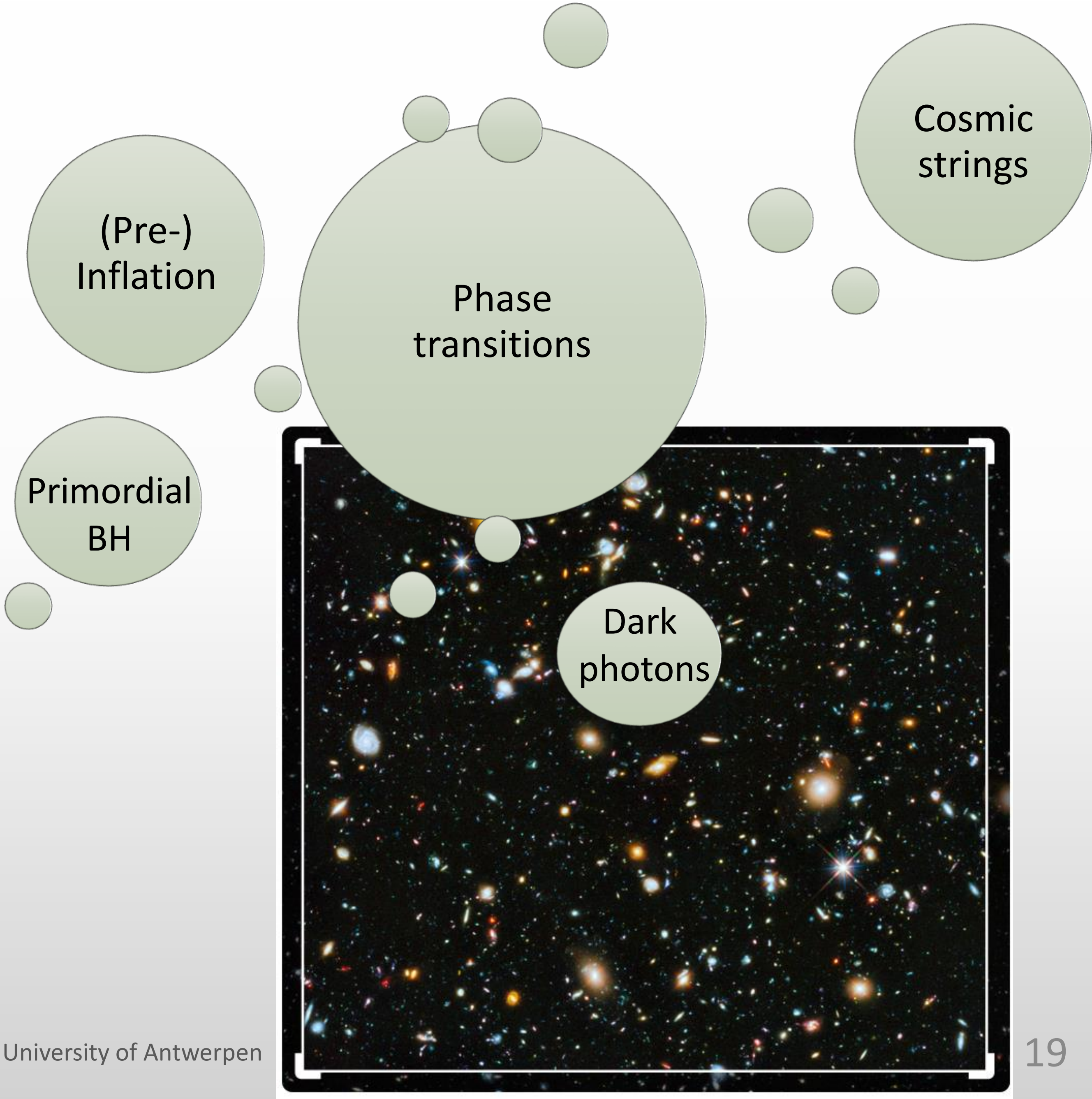
   Often speculative, but enticing GW allow

   To probe time and energy scales unaccesible by

   Earth based accelerators

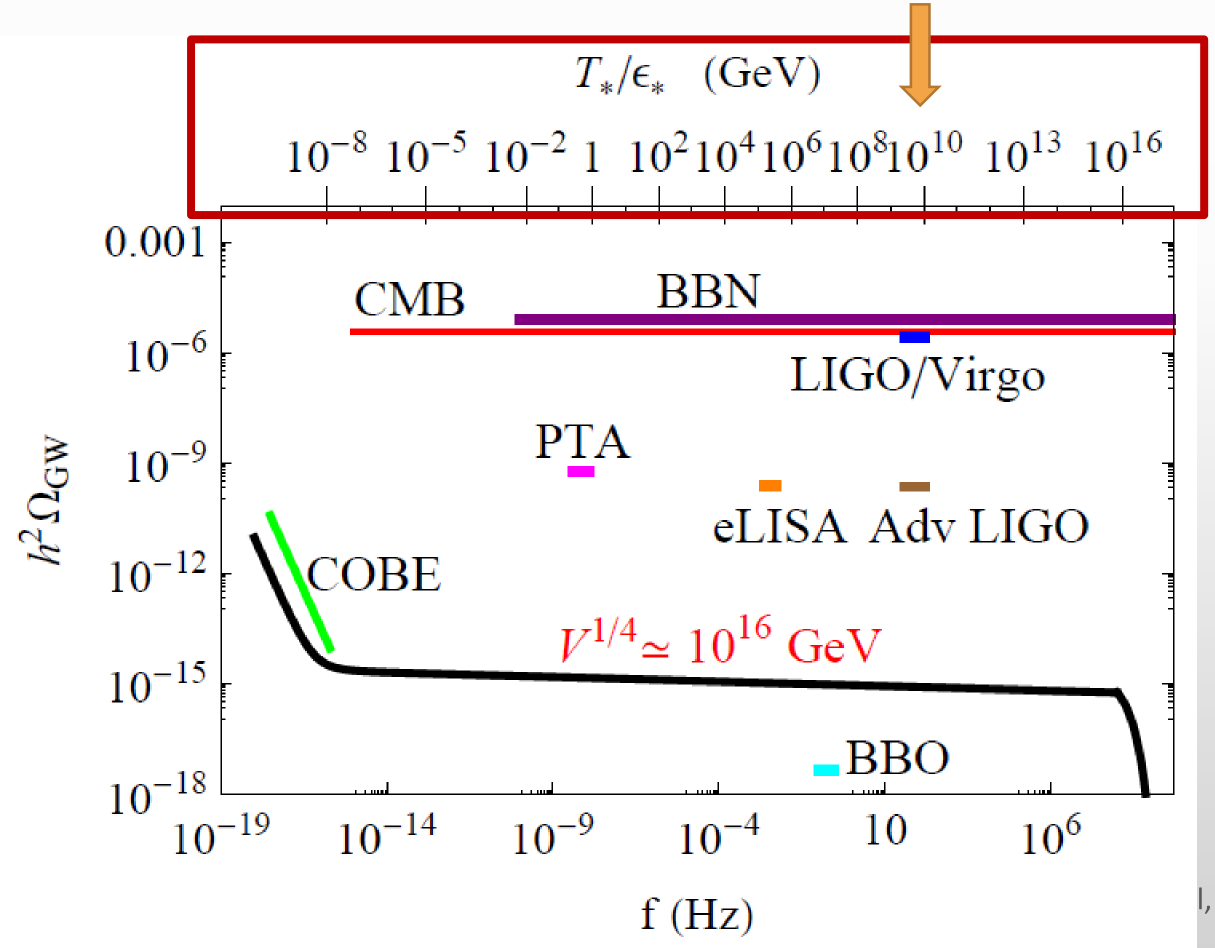
2. Astrophysical:
   Diverse spectrum of various known astrophysical objects

   Most likely to be dominant contribution
   Could shed light on populations, historical evolution



Cosmological

Bibliography:
[M. Maggiore, Phys.Rept. 331 (2000) 283-367
C. Caprini, D. G. Figueroa, Classical and Quantum Gravity, Vol 35, Nr 16 (2018)]



Main difference between GWCB and CMB:

Photons decoupled at temp T=0.3eV, but GW are out of thermal equilibrium since Planck scale

Any cosmological source that gives rise to non-zero anisotropic stress tensor in the early iniverse can seed GW:

- EM Fields
- Scalar field with spatial gradient in distribution
- Velocity perturbations in early universe fluid
- Amplification of vacuum fluctuations

Characteristic frequency observed today for a causal early universe process. Wave vector $k_* = H_*/\epsilon_*$ must always be larger than Hubble rate at that time

$$f_c = H_*/(2\pi\epsilon_*)(a_*/a_0)$$

Relationship between characteristic GW frequency and epoch when source was operating, characterized by temperature T_*

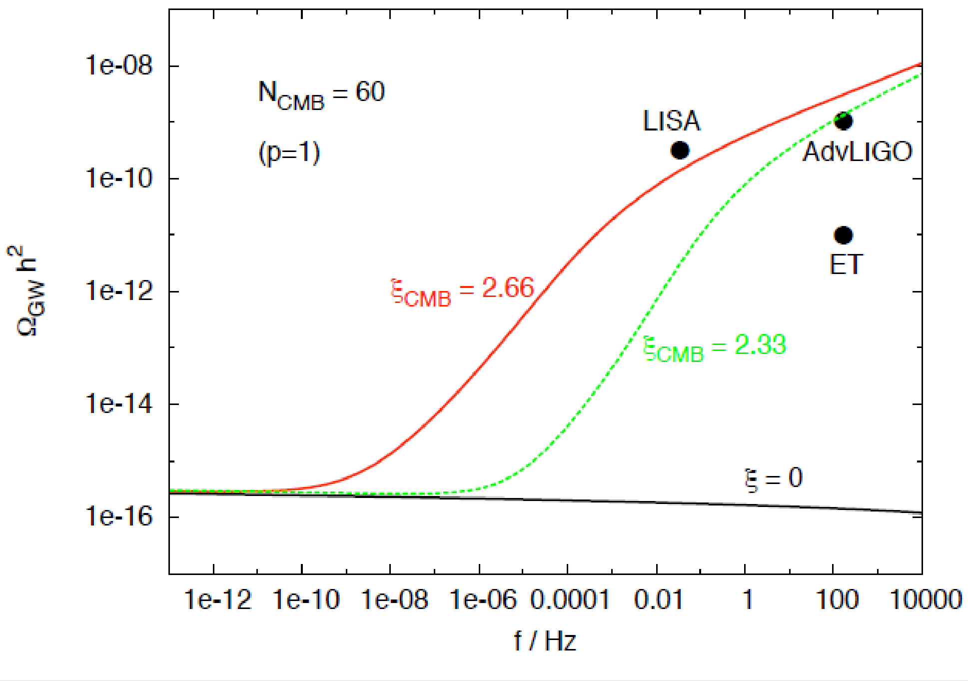
$$f_c \cong 2.6.10^{-5}Hz/(T_*/\epsilon_*/1TeV)(g_*/100)^{1/6}$$

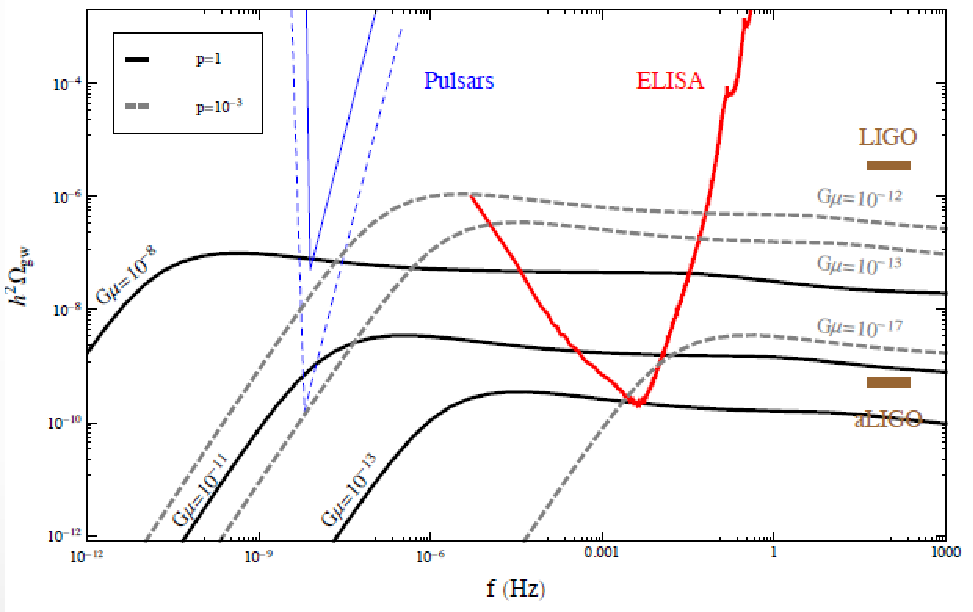
Example: First order phase trasition at EW scale (T*=100GeV), with $\epsilon_* \cong 0.001 - 1$

$$f_c \cong 10^{-5} - 1\ Hz$$

Observable cosmological scenarios

Bibliography:
[C. Caprini, 2015 J.. Phys.: Conf. Ser **610** 012004]

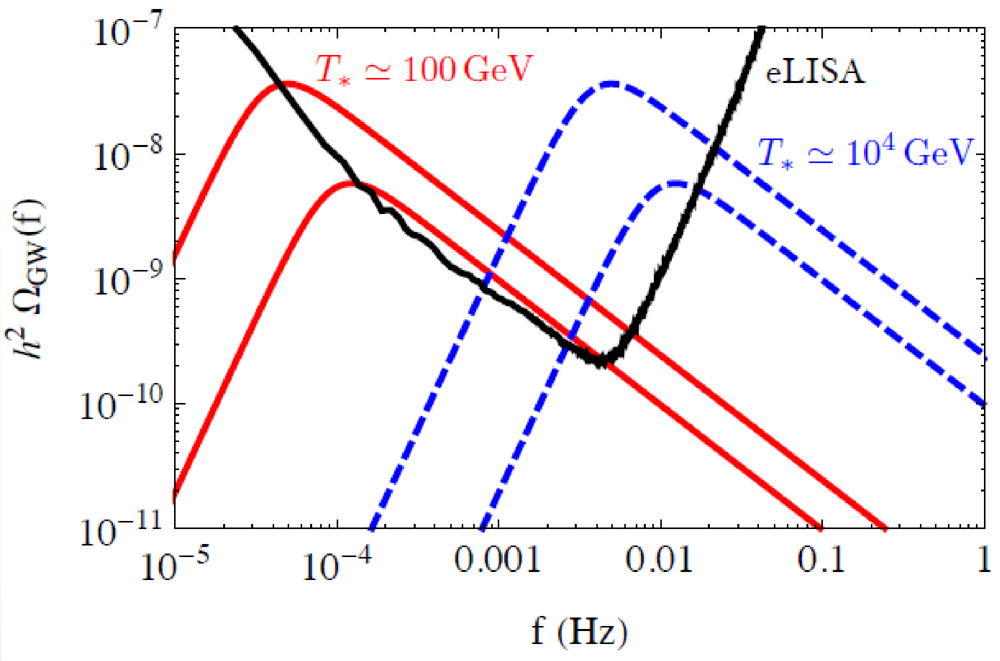




- Inflation with pre-heating: Instead of just producing particles, convert potential energy during slow-roll directly into GW
- Inflationary energu scale < 10^{11} GeV to fall in 1Hz-1kHz frequency and

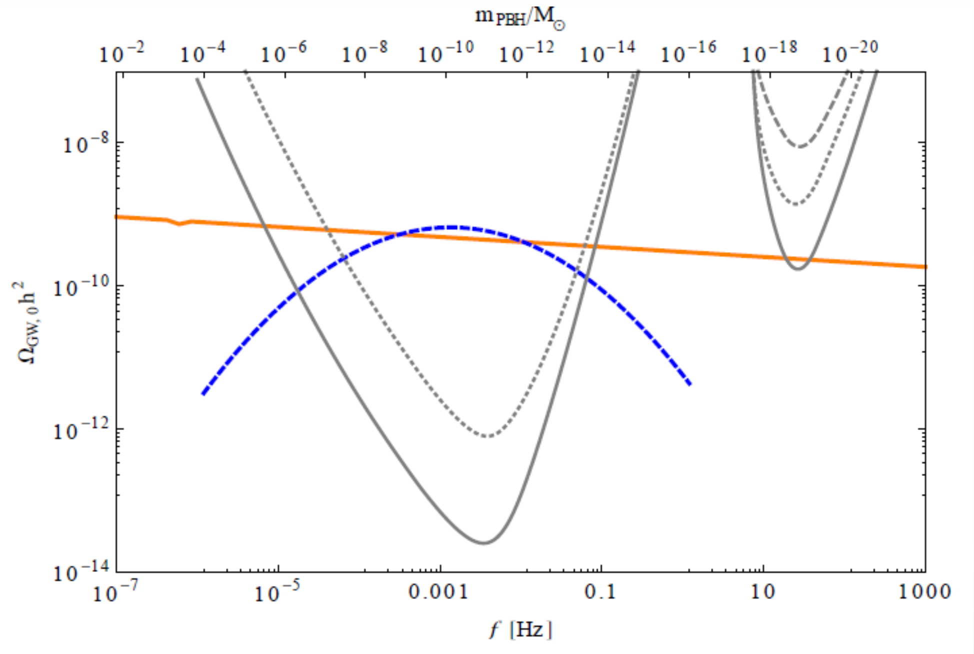
- Cosmic strings: topological structures as following phase transition at end of inflation or during thermal evolution of Universe
- Predicted by several BSM theories (GUT, SUSY, ..) and in some string theories
- Long string segments can reconnect and form loops that oscillate and generate GWs
- Few theory parameters: String tension, loop size, ,,,
- Typically very broad frequency range
- Also capable of producing bursts via cusps and kinks

Observable cosmological scenarios ctd.



C. Caprini, 2015 J. Phys.: Conf. Ser **610** 012004

- First-order phase transition: Nucleation and collisions of broken phase bubles
- Characteristic frequency spectrun is broken power-law where peak frequency relates to maximal bubble size towards end of PT and maximum Ω by amount of tensor-type stress that is available (strength of FOPT)



J. Garcia-Bellido, M. Peloso, C. Unal, JCAP 09 (2017) 013

S.Clesse, J. García-Bellido, S. Orani, e-Print: 1812.11011 [astro-ph.CO]

- Quantum fluctuations caus epeaks in curvature power spectrum
- These collapse into Primordial Black Holes (PBH), and generate GWs
- After re-entry, PBH can aggregate and form binaries that coalesce

Astrophysical

Bibliography:
[T. Regimbau, 2011 Res. Astron. Astrophys. 11 369
T. Regimbau, M. Evans, N. Christensen, E. Katsavounidis, B. Sathyaprakash, S. Vitale, Phys. Rev. Lett. 118, 151105 (2017)]

$$\Omega_{Gw,astr}(f_0) = \frac{f_0 \cdot F_{f0}}{\rho_c c^2}$$

With flux, as function of source parameters, θ

$$F_{f_0} = \int d\theta\, dz\, p(\theta) f(\theta, z, f_0) \frac{dR_0(\theta, z)}{dz}$$

Fluence of source at redshift, z

$$f(\theta, z, f_0) = \frac{1}{4\pi r^2(z)} \frac{dE_{GW}}{df}\big(\theta, f_0(1+z)\big)$$

$r(z)$: Proper distance (cosmology dependent)
$\frac{dE_{GW}}{df}$: emitted gravitational spectrum
$f_0(1+z)$: frequency in source frame

- Stochastic GW are expected to result from incoherent superposition of many unresolved sources since the beginning of stellar activity
- Current binary detections suggests existence of population of BH with relatively large masses that migt have formed in old (low metallicity) stellar environments:
  - Evolution of isolated binaries in galaxies
  - Mass seggregation and dynamical evolution in globular clusters
- This background is within reach of current 2G Earth-based interferometers
  - Contain wealth of information about history and evolution of populations of point sources
  - Detection can reduce unceratinties is cosmic star formation models at large redshifts
- Forms a confusion noise (foreground) that is detrimental to observe primordial signals

Astrophysical

- Number of sources per interval $\theta - \theta + d\theta$, per unit time and per redshift interval

$$\frac{dR_0(\theta,z)}{dz} = \dot{\rho}_0(\theta,z)\frac{dV}{dz}(z)$$

With $\dot{\rho}_0(\theta,z)$: event rate in $Mpc^{-1}yr^{-1}$

$\frac{dV}{dz}(z)$: the comoving volume element

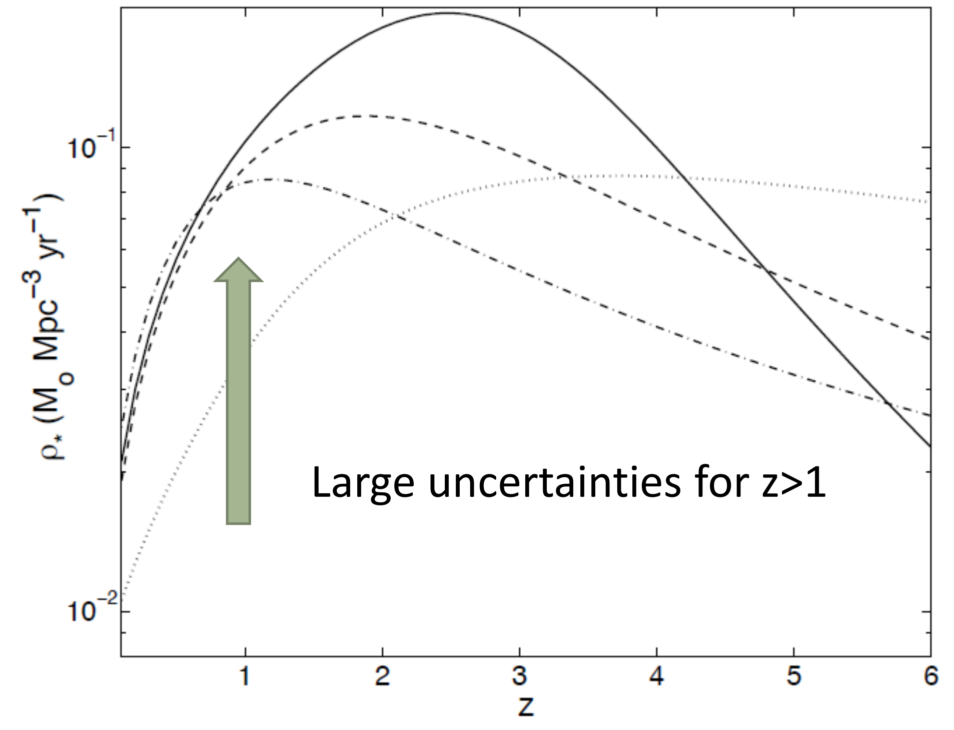
- Results in a predicted energy density

$$\Omega_{GW,ast}(f) = \frac{8\pi G}{3c^2H_0^2} f \int d\theta p(\theta) \int_{z_{inf}}^{z_{sup}} dz \frac{\dot{\rho}_0(\theta,z)}{E(\Omega,z)} \frac{dE_{GW}}{df}(\theta, f_0(1+z))$$

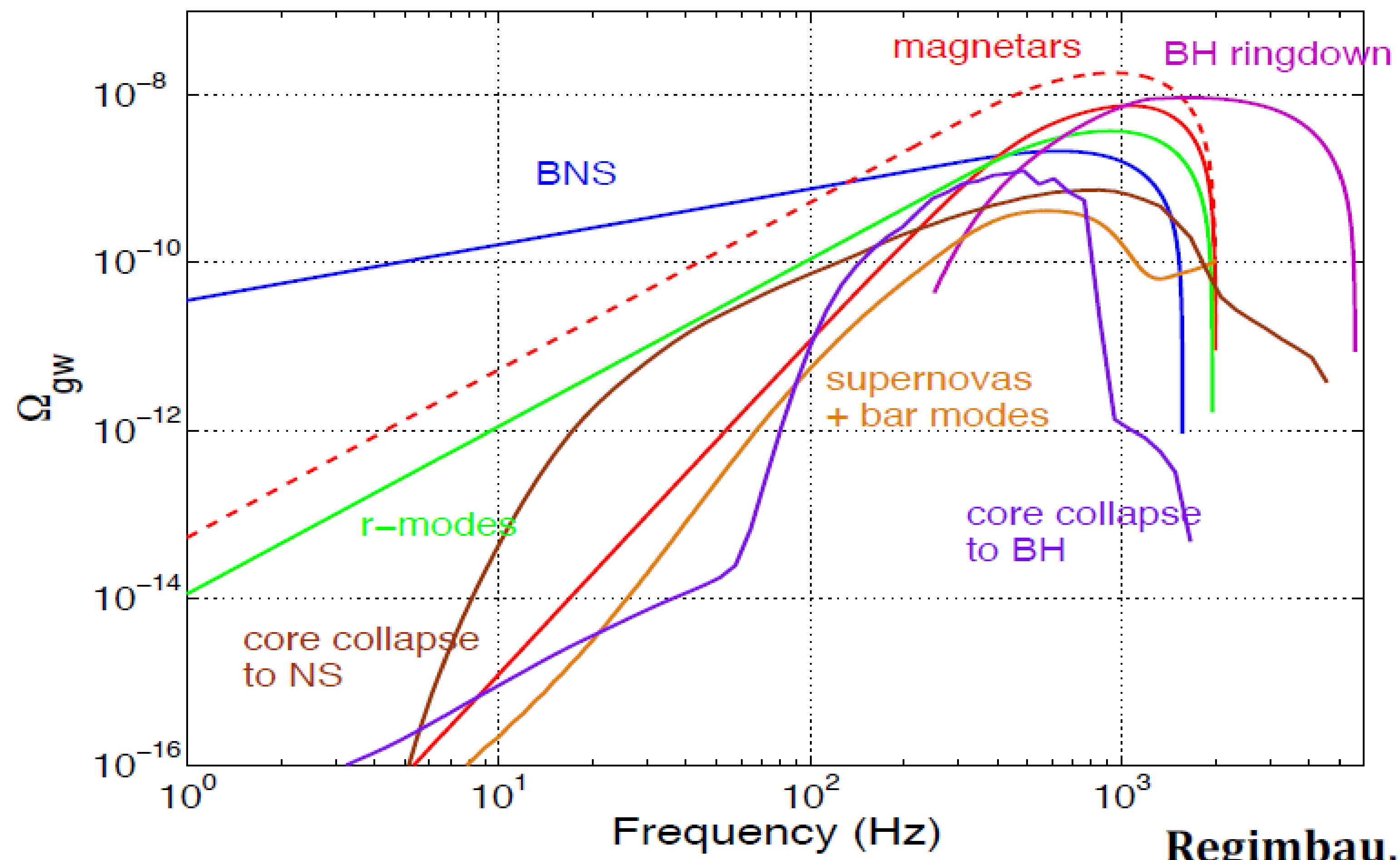
- Event rate per unit redshift can be derived from cosmic star formation rate $\dot{\rho}_*(z)$ and mass fraction $\lambda(\theta,z)$

$$\dot{\rho}_0(\theta,z) = \lambda(\theta,z)\frac{\dot{\rho}_*(z)}{1+z}$$

[T. Regimbau, 2011 Res. Astron. Astrophys. 11 369]



Composition of astrophysical Stoch bg

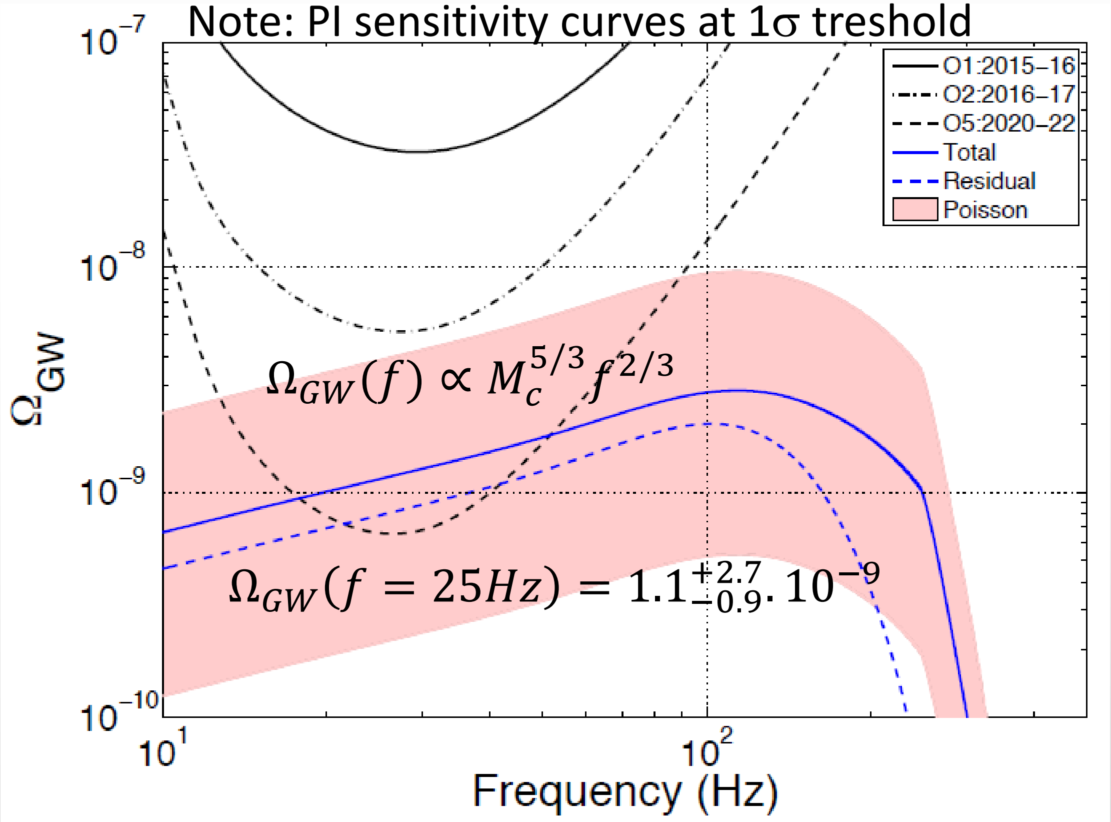


Regimbau, arXiv:1101.2762

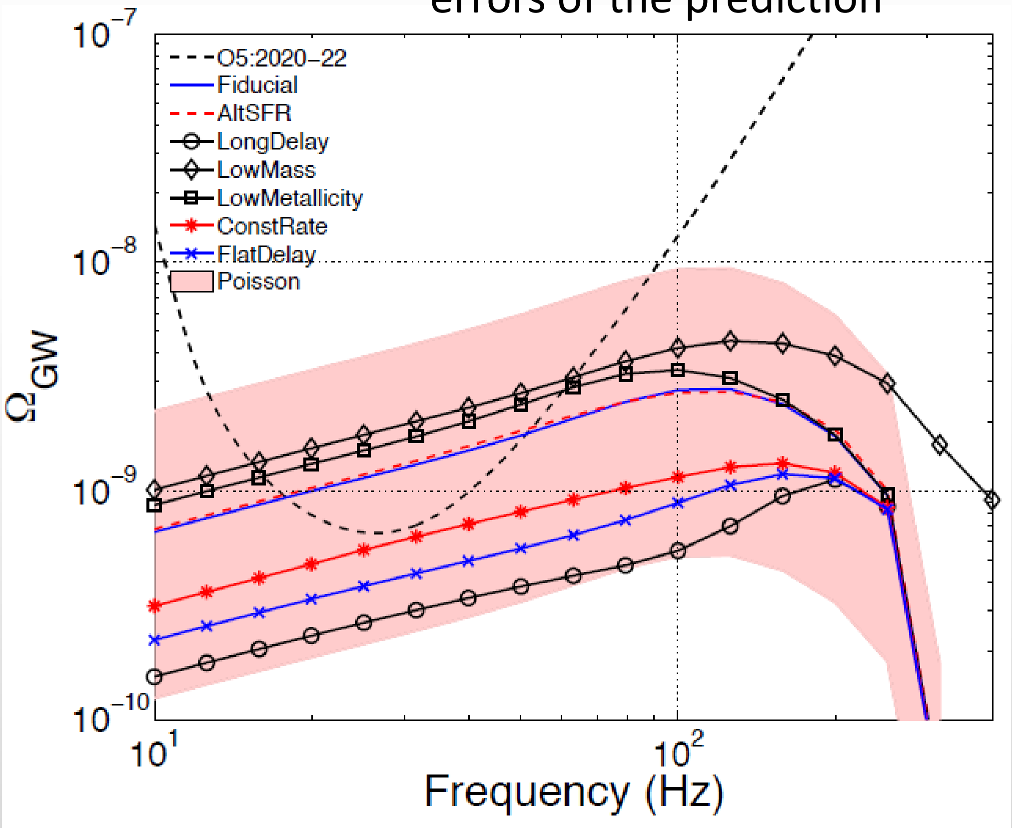
- Very rich landscape of astrophysical point sources
- Predictions based on models, tuned to large diversity of EM observations
- Some predictions directly from first CBC GW observations (see next slide)
- Also possible to infer on high z CBC rates once stochastic bg is detected

Predictions from GW150914: Fiducial Model

Bibliography:
[B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) Phys. Rev. Lett. 116, 131102 (2016)



Model uncertainties fall within Poisson errors of the prediction



Estimates after first BNS: GW170817 (+5 BBH)

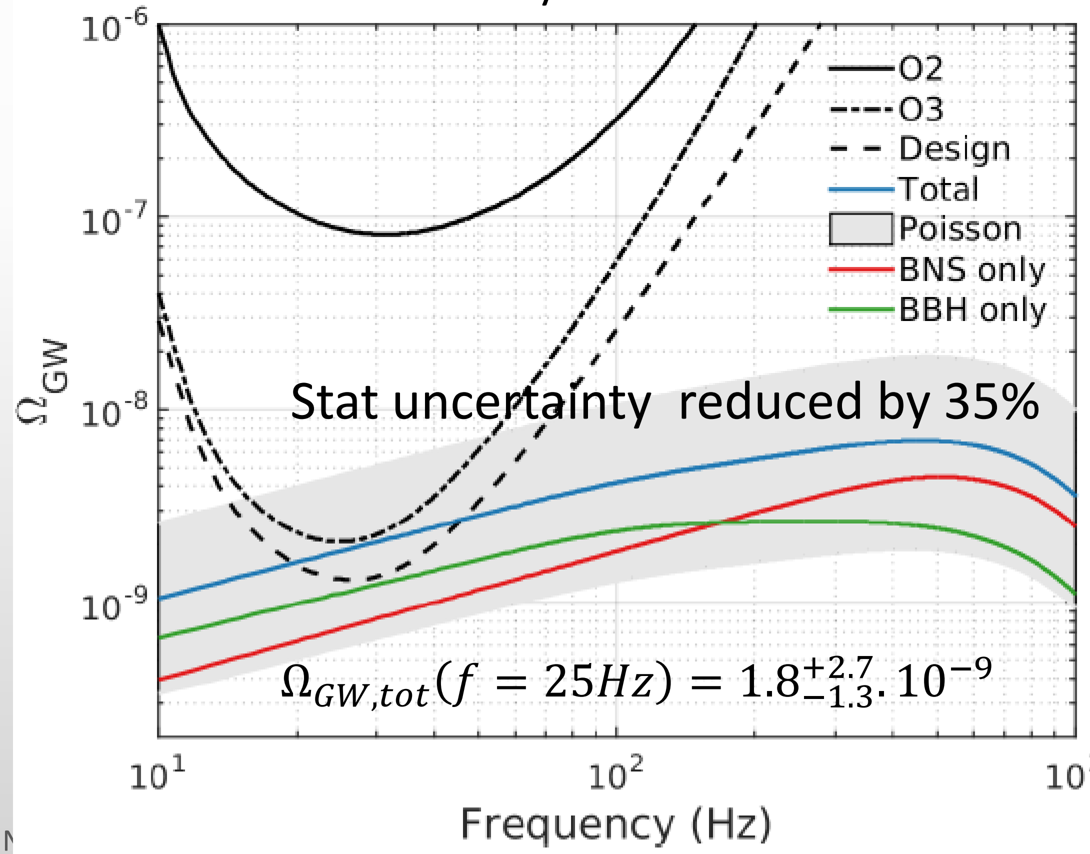
Bibliography:
[B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration)
Phys. Rev. Lett. 120, 091101 (2018)

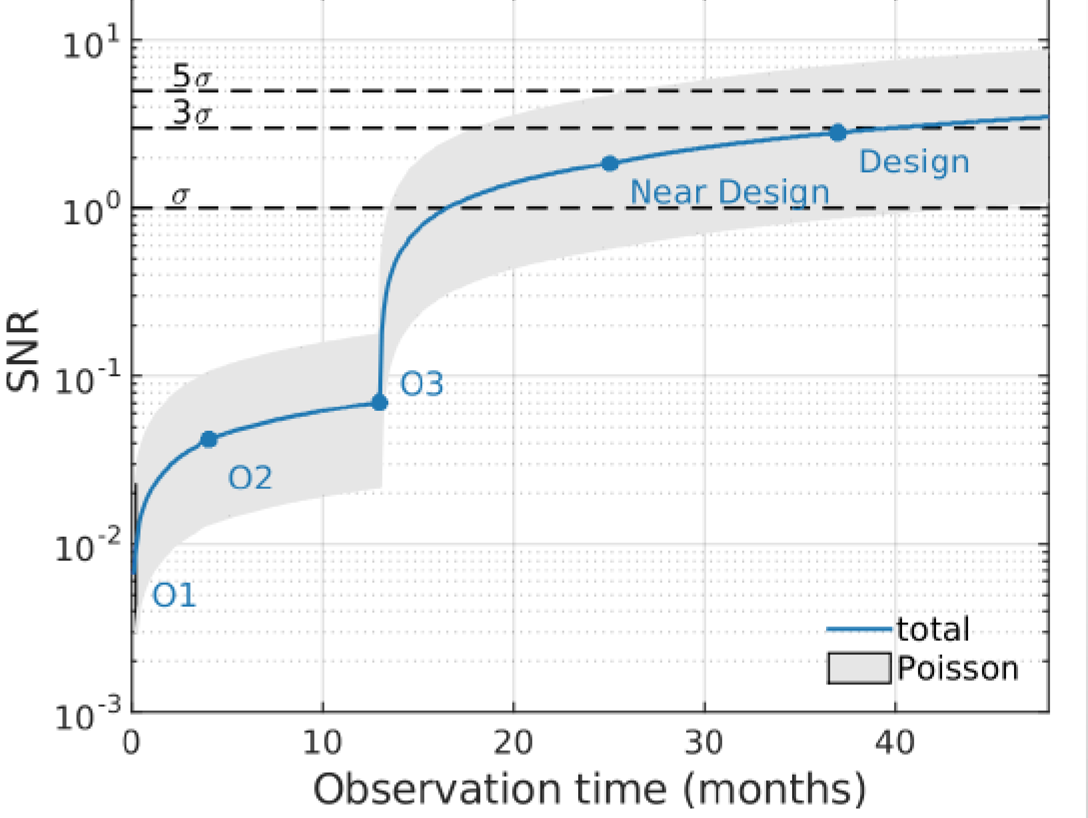
Time between events

Nr of overlapping signals
At any given time

	$\Omega_{GW}(25\ \mathrm{Hz})$	τ [s]	λ
BNS	$0.7^{+1.5}_{-0.6} \times 10^{-9}$	13^{+49}_{-9}	15^{+30}_{-12}
BBH	$1.1^{+1.2}_{-0.7} \times 10^{-9}$	223^{+352}_{-115}	$0.06^{+0.06}_{-0.04}$
Total	$1.8^{+2.7}_{-1.3} \times 10^{-9}$	12^{44}_{-8}	15^{+31}_{-12}

Note: PI sensitivity curves at 1σ treshold





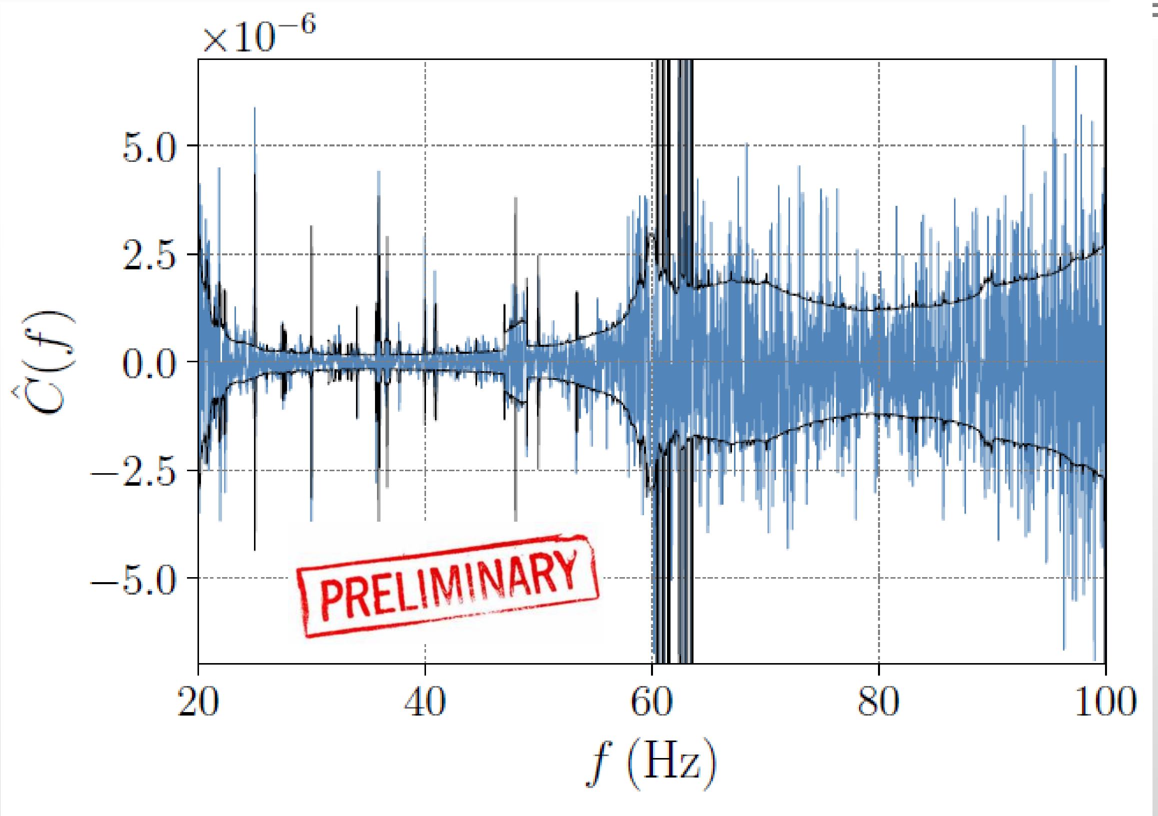
Results from O1+O2+O3!

To be submitted on Monday!
DCC link: https://dcc.ligo.org/P2000314

Upper limits on $\Omega_{ref}(25Hz)$ at 95% confidence level for various spectral indices:

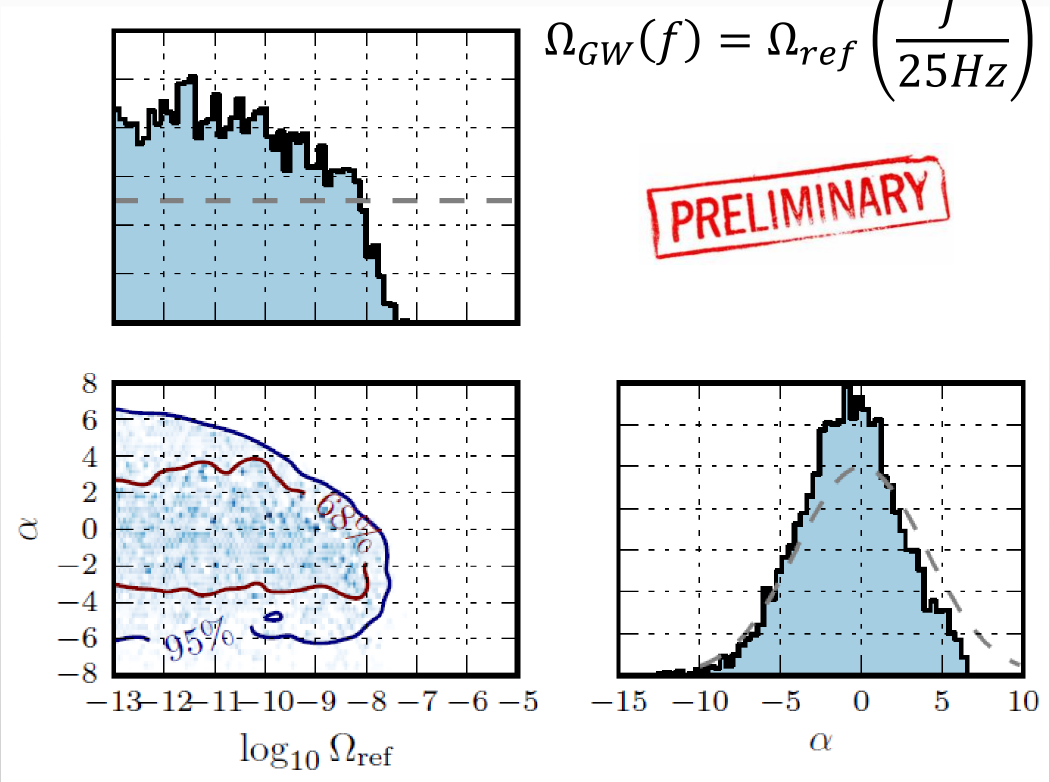
	Uniform prior			Log-uniform prior		
α	O3	O2 [43]	Improvement	O3	O2 [43]	Improvement
0	1.5×10^{-8}	6.0×10^{-8}	3.9	5.8×10^{-9}	3.5×10^{-8}	6.0
2/3	1.1×10^{-8}	4.8×10^{-8}	4.5	3.4×10^{-9}	3.0×10^{-8}	8.8
3	1.2×10^{-9}	7.9×10^{-9}	6.4	3.9×10^{-10}	5.1×10^{-9}	13.1
Marg.	2.6×10^{-8}	1.1×10^{-7}	4.3	6.7×10^{-9}	3.4×10^{-8}	5.1

Combined cross correlation spectrum, in absence of correlated noise : $\hat{C}(f) = \Omega_{GW}(f)$



Posterior probabilities for magnitude and spectral index

$$\Omega_{GW}(f) = \Omega_{ref}\left(\frac{f}{25Hz}\right)^{\alpha}$$



Results from O3!

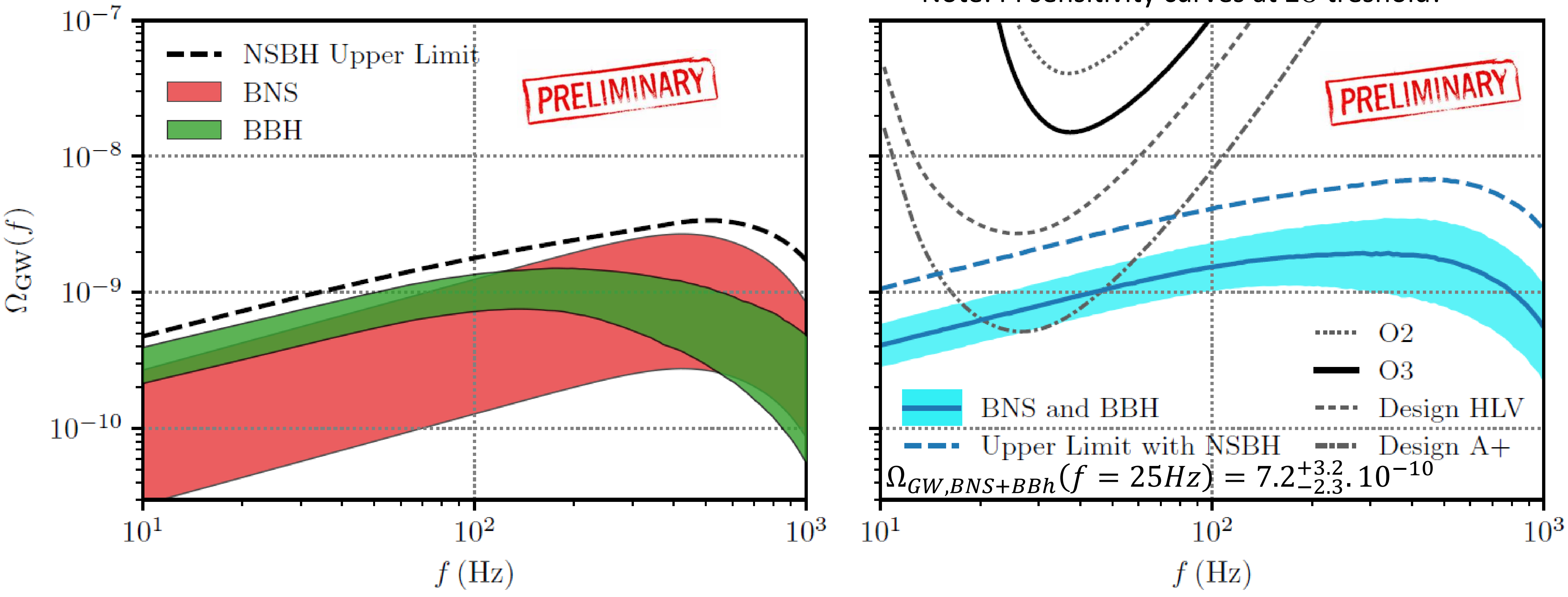
To be submitted on Monday!
DCC link: https://dcc.ligo.org/P2000314

Fiducial model predictions updated with GWTC-2 catalog!
BBH:

- Metallicity weighted star formation rate used,
- mass ans spin distributions included in model
- Broken power law for mass distibution

BNS: reduced poisson uncertainties due to 2nd detection

Note: PI sensitivity curves at 2σ treshold!



Future Avenues (selection of)

- Bayesian optimal search:

Bibliography:

[R. Smith and E. Thrane, “Optimal Search for an Astrophysical Gravitational-Wave Background”, Phys. Rev. X 8, 021019 (2018)]

- Subtracting (large statistics samples of) single detections from data:

Bibliography:

[T. Regimbau, et al., “Digging deeper: Observing primordial gravitational waves below the binary black hole produced stochastic background”, Phys.Rev.Lett. 118 (2017) 15, 151105]

- Optimal combination of signals from co-located detectors, null stream:

Bibliography:

[J. Harms et al., “Subtraction-noise projection in gravitational-wave detector networks”, Phys.Rev.D 77 (2008) 123010

A. Lazzarini et al., “Optimal combination of signals from co-located gravitational wave interferometers for use in searches for a stochastic background”, Phys.Rev.D 70 (2004) 062001

T. Regimbau, et al, “A Mock Data Challenge for the Einstein Gravitational-Wave Telescope”, Phys. Rev. D 86, 122001 (2012)]

Bayesian Search

[R. Smith and E. Thrane, Phys. Rev. X 8, 021019 (2018)]

- Ideally for non-Gaussian stochastic background (i.e. low duty cycle events, with no overlap)
- Split data in small time segments where you expect small probability $\xi \ll 1$ that a segment contains one (small significance) BBH event
- Construct segment-by segment likelihood for signal and null (or bg) hypothesis

$\mathcal{L}(\vec{s}_i|\xi) = \xi \mathcal{Z}_s^i + (1-\xi)\mathcal{Z}_N^i$ with $\mathcal{Z}_N^i = \mathcal{L}(\vec{s}_i|0)$ and $\mathcal{Z}_S^i = \int d\theta\, \mathcal{L}(\vec{s}_i|\theta)\pi(\theta)$

Model depenent and computationally intensive

Construct test statistic

$$\mathcal{Z}_{stoch} = \int d\xi \prod_{i=1}^{n_{seg}} \mathcal{L}(\vec{s}_i|\xi)\pi(\xi)$$

$$\mathcal{Z}_0 = \prod_{i=1}^{n_{seg}} \mathcal{L}(\vec{s}_i|\xi = 0)$$

And bayes factor

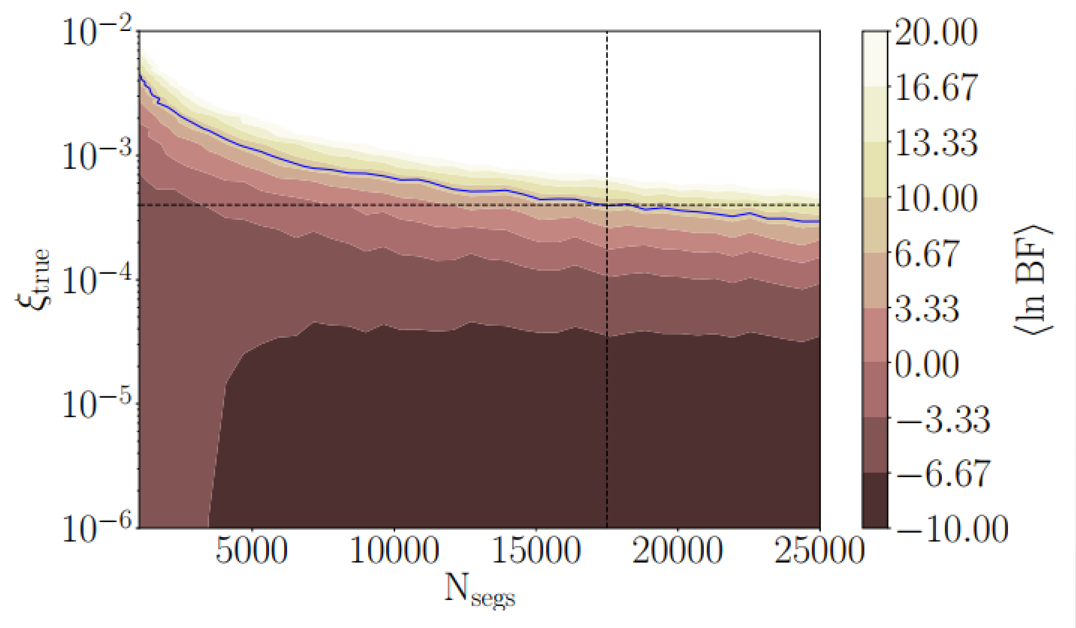
$$BF = \mathcal{Z}_{stoch}/\mathcal{Z}_0$$

For segments of 4 sec, expected $\xi = 4.10^{-4}$

$\log(BF) > 8$ attainable for ~20hrs of data with A+-sensitivity

BUT, assumes

- stationary and gaussian noise
- Relatively simple signal model(s) (computer intensive)



[skippable]

Subtraction method

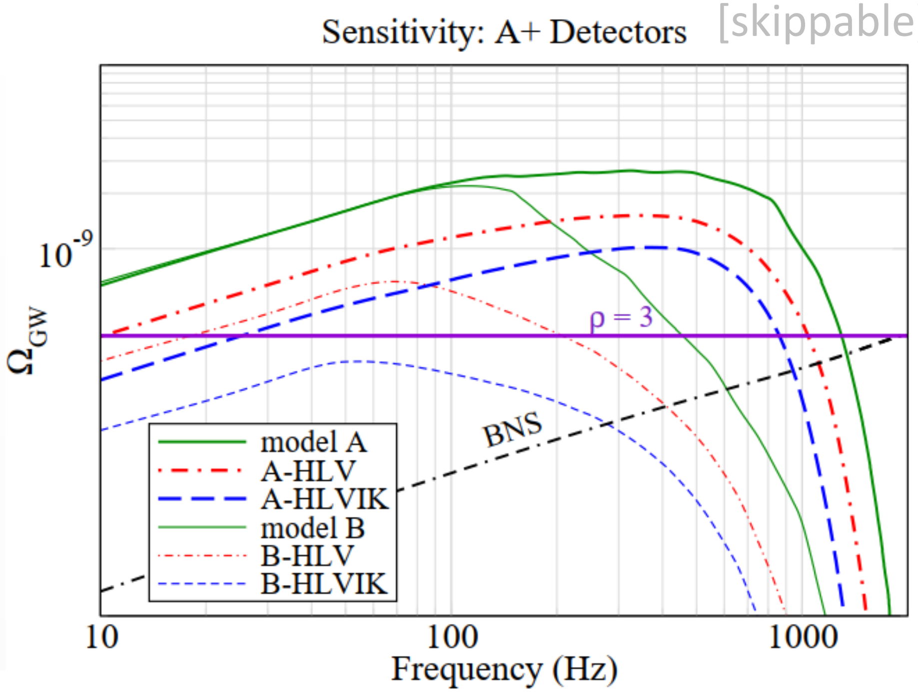
[T. Regimbau, et al.Phys.Rev.Lett. 118 (2017) 15, 151105]

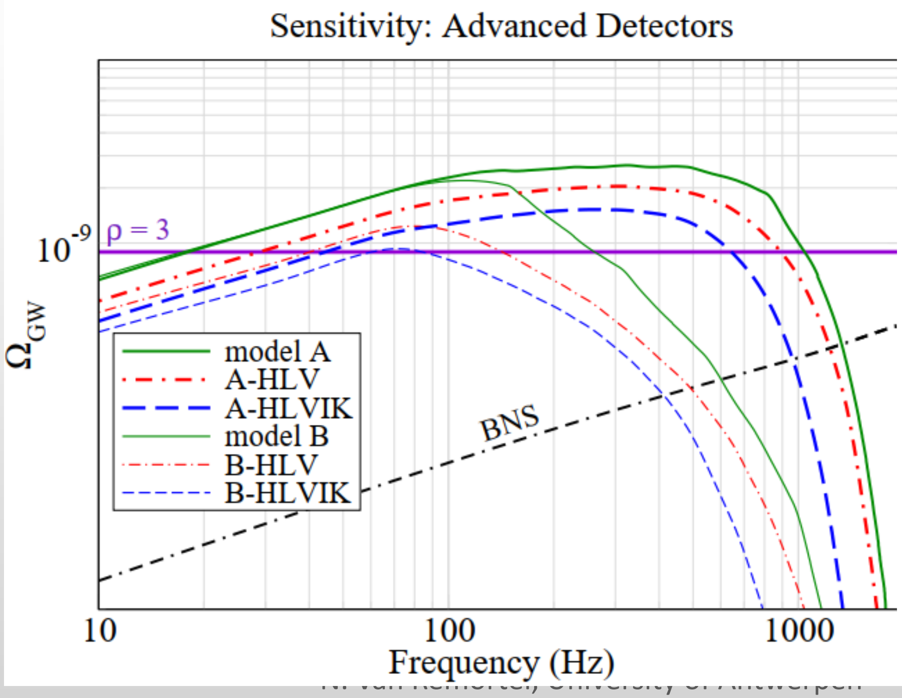
- Assuming BBH mergers will dominate the stochastic Bg
- BBH signals do not overlap (BNS will overlap)
- Similar fiducial models as before (A and B use different mass distributions for the primaries or component masses
- Remove all CBC detectable sources with SNR>12 from data
- Increased sensitivities allow for more efficient detection, ie. Removal
- No subtraction:
  $[\Omega_{GW}(10Hz)]_{BBH} = 6.10^{-10}$
- Subtract all SNR>12 events

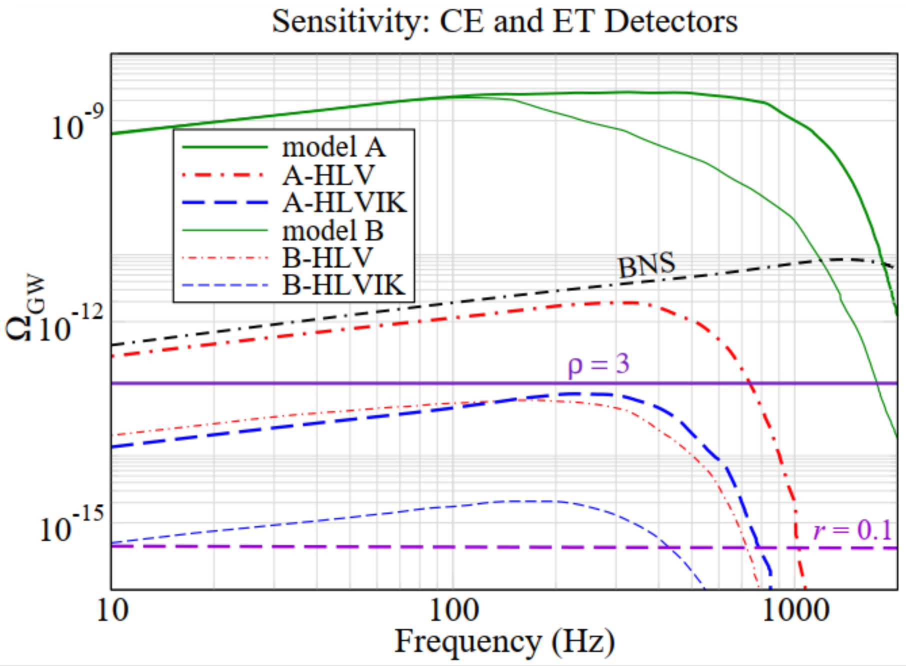
$[\Omega_{GW}(f)]^{A+}_{min} > 10^{-10}$

$[\Omega_{GW}(f)]^{3x3G}_{min} > 10^{-14} - 10^{-13}$

$[\Omega_{GW}(f)]^{5x5G}_{min} > 10^{-16} - 10^{-14}$

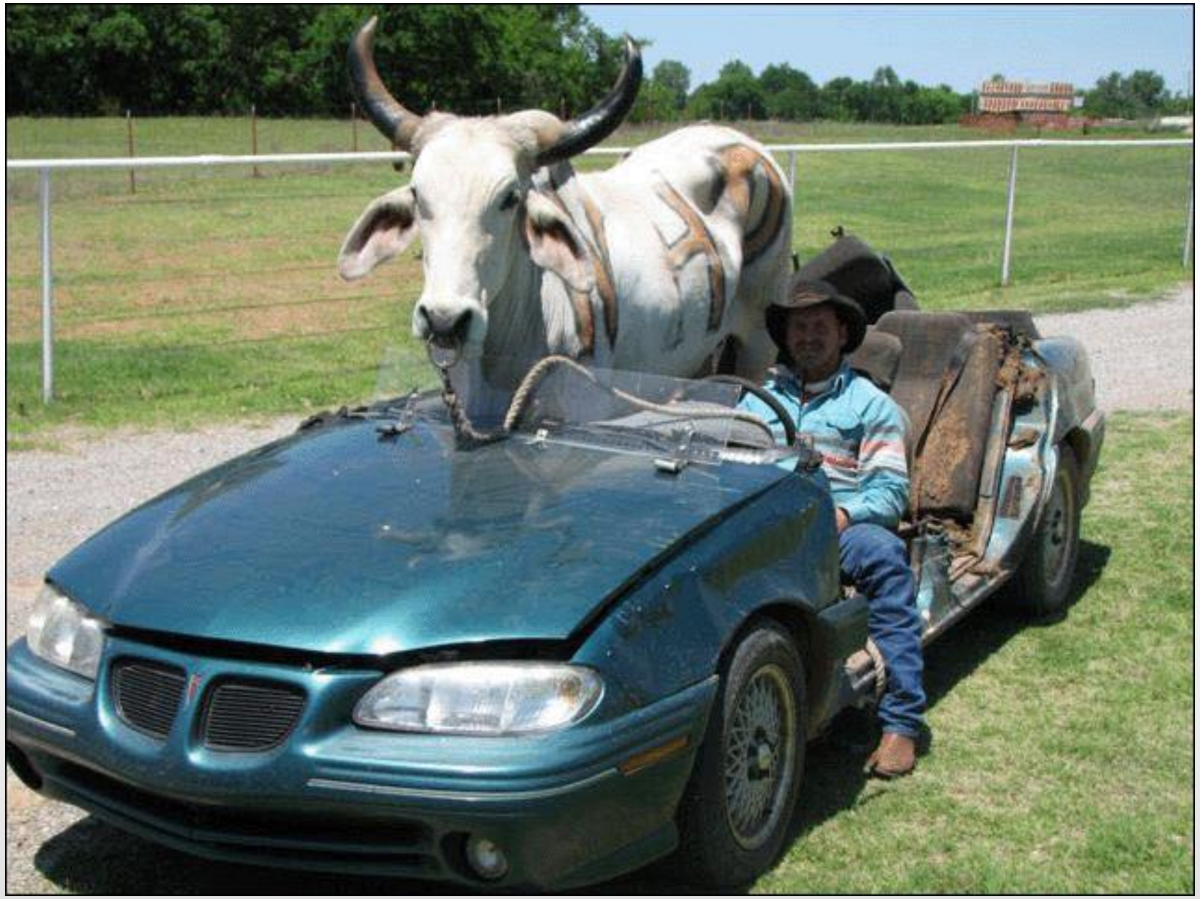






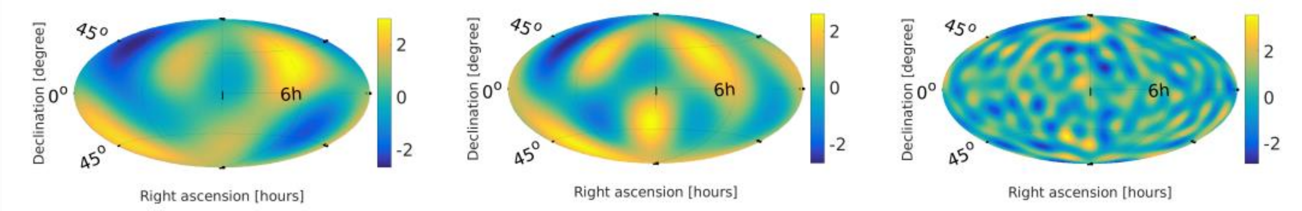
[skippable]

NO TIME TO EXPLAIN!



- DIRECTIONAL STOCHASTIC BG SEARCHES

[B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. D 100, 062001 (2019)



- PULSAR TIMING ARRAYS

[Z. Arzoumanian et al. (The NANOGrav Collaboration), The Astrophysical Journal Letters, Volume 905, Number 2 (2020)]