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Mirror thermal noise calculation for ET

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ET – Einstein gravitational wave Telescope – Design Study * A joint European Project
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Contents

1 Introduction

Current gravitational wave detectors are limited by different noise sources. The upcoming generation of *advanced* detectors will already lead to an improvement of the sensitivity of a factor of 10 compared to the first generation detectors like LIGO, Virgo or GEO600. A further step increasing the sensitivity by another factor of 10 is planned for the 3rd generation of gravitational wave detectors. Thus, all noise sources need to be dramatically reduced. One major noise source is thermal noise of the optical components. This article gives a first estimate of what mirror dimensions might be necessary for a cryogenic ET mirror to achiev the desired sensitivity. The article is based on a standard dielectric multilayer coating and will result in a 'worst case' cenario. Improved coatings or novel techniques like (monolithic) waveguide mirrors [52] will further reduce the total thermal noise of the mirror.

The article focuses on the use of sapphire or silicon as a test mass material. As a coating material titania doped tantala and silica $\lambda/4$ layers are considered. The idea is taking the best currently available materials to scale the mirror size up to the necessary limit to reduce thermal noise below the necessary limit.

The article is based on an internal ET note "Mirror thermal noise in laser interferometer gravitational wave detectors operating at room and cryogenic temperature" [1] which summarizes the current knowledge of thermal noise calculation. This article follows exactly the calculation described in [1] focusing on the most contributing noise sources which are Brownian thermal noise and thermoelastic noise. The material properties used for the calculation are summarized seperatly.

2 Thermal properties

2.1 Thermal properties of bulk materials

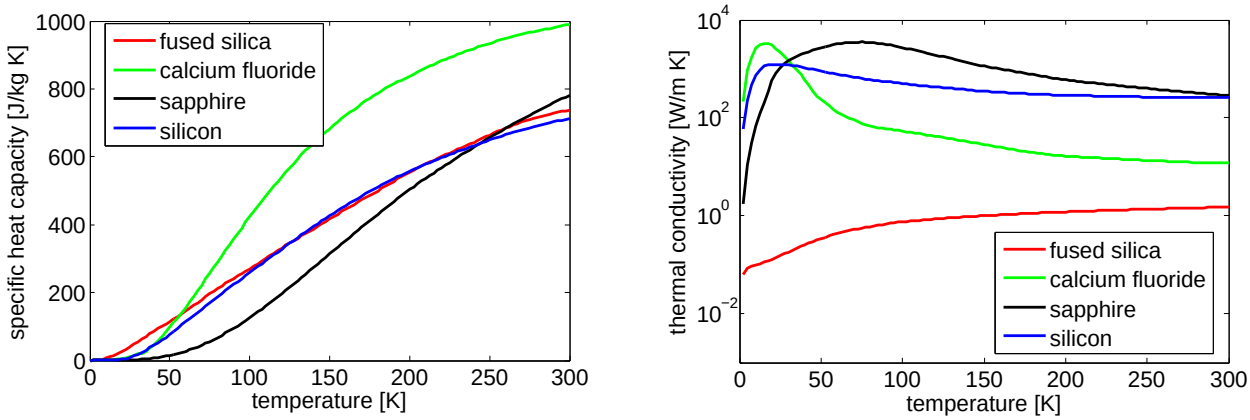
There are several thermal material properties necessary to estimate the different kinds of thermal noise. The material properties were obtained from different papers as listed below. The numerical values are accessible for each single plot in 2.5 K steps in separate data files.

The heat capacity of crystalline solids behaves similarly below their Debye temperature. There, the heat capacity obeys a Debye like $\propto T^3$ behavior. Additionally, many amorphous materials behave similarly in the temperature region of interest (5 - 300 K). Their heat capacity decreases monotonically. Bulk materials used in this article are summarized in fig. 1 (a). The values were obtained from different references: fused silica [28], calcium fluoride (below 30 K [29], above 30 K [30]), sapphire [31] and silicon [32].

The thermal conductivity of crystalline samples shows a classical behavior where a peak is visible at low temperatures. In contrast, the amorphous materials (like fused silica) show a monotonic decrease in thermal conductivity. Additionally, the thermal conductivity is smaller by orders of magnitude. The temperature dependence of the thermal conductivity for the materials of interest in this article is shown in fig. 1 (b). The values were obtained from different references: fused silica [33], calcium fluoride [34], sapphire [31] and silicon [32].

The bulk materials obey in general a decrease in the coefficient of thermal expansion (CTE) with decreasing temperature. Silicon and fused silica show a zero CTE at characteristic temperatures. Especially, the 18 K canceling of the CTE of silicon is of great interest for low thermoelastic noise samples. Theoretically, the thermoelastic noise can be fully canceled at this temperature.

The values for the CTE were obtained from different references: fused silica [35], calcium fluoride [36], sapphire [37] and silicon [32].



(a) (b)

Figure 1: Specific heat capacity (a) and thermal conductivity (b) of selected bulk materials.

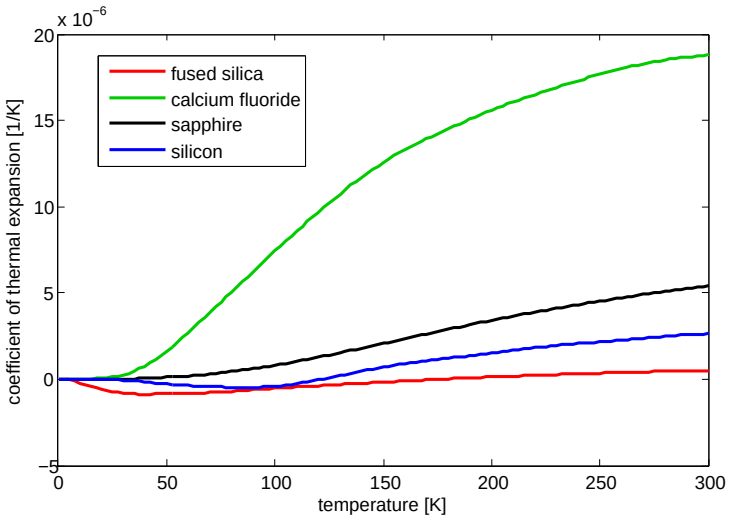


Figure 2: Coefficient of thermal expansion for the bulk materials.

2.2 Thermal properties of coatings

As already described in section 2.1 it is not easy to give a summary of the thermal properties of matter. For coating materials there is very little information about thermal properties in the temperature range from 5 to 300 K due to several factors: Firstly, transport properties like thermal conductivity are in some temperature regions geometry dependent. This always occurs when the phonon mean free path is comparable to a significant length of the sample. This length is in the case of dielectric coatings the thickness and hence only between 100 and 200 nm. Some information on thermal conductivity of coating materials is available for temperatures below 1 K. These measurements were done to investigate the propagation of thermal phonons in solids.

Most of the information about thermal properties of dielectric materials is based on assumptions in literature. Thus, in most cases the coating layer is assumed to have similar properties to the bulk material of the same substance. But even then in most cases the values are just available at room temperature.

This article focuses on tantala and silica as coating materials. Silica is assumed to have the same thermal properties as bulk fused silica (see section 2.1). In the case of tantala coatings nearly no information is available. Being also an amorphous material it was assumed that the thermal conductivity and the heat capacity have a similar temperature dependence to fused silica. The absolute values were scaled using room temperature values available in the literature (e.g. [41]). The coefficient of thermal expansion was assumed to be similar to sapphire. Fused silica has a zero CTE above 100 K which is not assumed for tantala. Sapphire shows a monotonic decrease of the CTE with decreasing temperature. This assumption is not based on measurements or experiences. It is more a guess and the results need to be taken carefully. In any case this assumption generates higher CTEs compared to other assumptions and it thus will result in an upper limit for thermoelastic noise.

parameter	density [kg/m^3]	Young's modulus Y [GPa]
fused silica	2200	72
CaF_2(100)	3183	138
Sapphire (c-axis)	3980	350
Si(100)	2330	130
Si(110)	2330	169
Si(111)	2330	188

Table 1: Density and Young's modulus of the materials investigated as test mass materials in this article.

3 Mechanical properties

3.1 Mechanical properties of bulk materials

Besides thermal properties mechanical properties are needed to estimate the thermal noise of optical materials for ET. Table 1 summarizes the density and Young's modulus for the materials investigated in this article. Both values have a small temperature dependence which is smaller than 1% in the temperature region of interest and is thus neglected.

An effective Young's modulus for selected crystalline orientations is shown for crystalline solids. This value can be calculated from the values of the compliances S_{ij} as follows for different crystal orientations of a cubic system [17]:

$$\frac{1}{Y_{100}} = S_{11}, \tag{1}$$

$$\frac{1}{Y_{110}} = S_{11} - \frac{1}{2}\left[(S_{11} - S_{12}) - \frac{1}{2}S_{44}\right], \tag{2}$$

$$\frac{1}{Y_{111}} = S_{11} - \frac{2}{3}\left[(S_{11} - S_{12}) - \frac{1}{2}S_{44})\right]. \tag{3}$$

The values for the compliances at room temperature were taken from [18]: $S_{11} = 7.68 \times 10^{-12}\,Pa$, $S_{12} = -2.14 \times 10^{-12}\,Pa$, $S_{44} = 12.6 \times 10^{-12}\,Pa$.

The mechanical loss factor Φ or its inverse the mechanical Q-factor is another important parameter in order to calculate Brownian thermal noise (see section **??**). While there is some information on loss measurements on bulk materials at room temperature available only a few papers exist on measurements of macroscopic test samples at cryogenic temperatures. Fig. 3 summarizes some selected values from the literature and talks available online.

For the Brownian thermal noise calculation the internal mechanical loss of the test mass material is needed. Loss measurements only reveal the integral mechanical loss of the test mass including suspension losses, surface dependent losses, etc. Up to now there only exists one model in literature [39] to distinguish between surface and bulk effects. This model was developed for fused silica at room temperature and is based on lots of measurements on different sample sizes and geometry. It also includes a frequency dependence of the mechanical loss of fused silica. As a result the intrinsic mechanical loss obtained from this paper reaches values as low as 4×10^{-10} at 100 Hz and 300 K. This

value is at least one order of magnitude lower than any value which was directly obtained from integral loss measurements. Thus, the mechanical loss values for other materials than fused silica are based on the direct loss measurements and therefore probably too high. In any case they will provide an upper limit and are the best values currently known.

3.2 Mechanical properties of coating materials

Tantala/silica layers were found to be the optimum selection between the optical and the mechanical parameters providing a low thermal noise component. Nevertheless, it was shown that in these layers the tantala contributes significantly more to the overall mechanical loss. Its internal mechanical loss was roughly 5 times as high as the mechanical loss of silica at room temperature [43]. In order to reduce the internal mechanical loss of the tantala a long and extensive experimental run was done at several institutions to investigate the dependence of the mechanical loss in dependence on different parameters (e.g. heat treatment, co-doping, fabrication process, etc.). It was possible to reduce the mechanical loss of tantala by about a factor of 2 by means of co-doping the tantala with 20% of titania [44]. Tab. 4 summarizes the mechanical loss values used in this article at selected temperatures for tantala and silica.

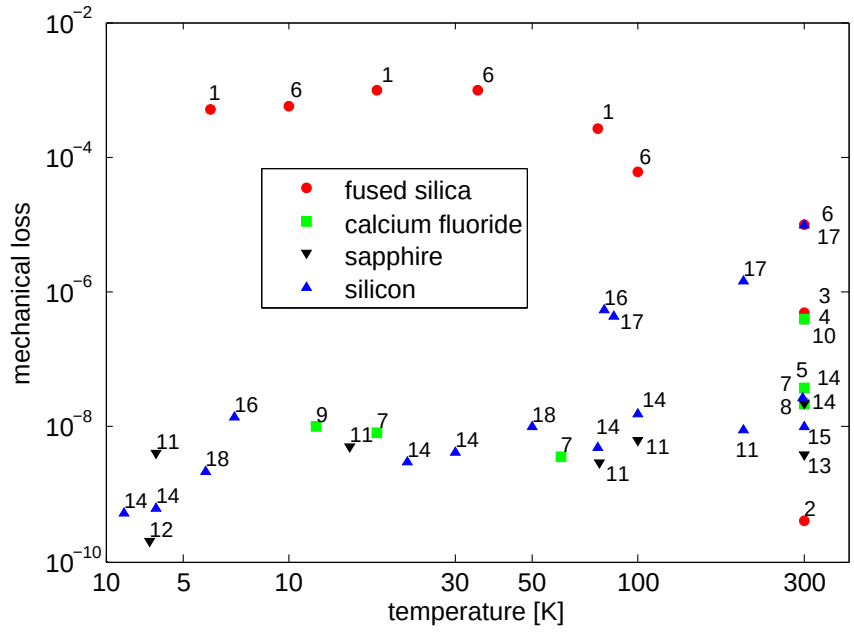


Figure 3: Mechanical loss of different test mass materials. The reference numbers are summarized in table 2.

number	reference	material	geometry	frequency
1	[4]	fused silica	dia. 75 mm × 24 mm	24 kHz
2	[39]	fused silica	modeled	100 Hz
3	[7]	fused silica	dia. 8 mm × 24-29 cm	384 Hz
4	[8]	fused silica	different fibers	
5	[10]	fused silica	dia. 70 mm × 60 mm	32 kHz
6	[20]	fused silica	5 × 45 × 0.104 mm^3	40 - 14 kHz
7	[6]	calcium fluoride	dia. 75 mm × 75 mm	41 kHz
8	[5]	calcium fluoride	dia. 85.5 mm × 50.5 mm	28.5 kHz
9	[19]	calcium fluoride	dia. 100 mm × 3 mm	4.3 kHz
10	[9]	calcium fluoride	dia. 180 mm × 38 mm	9.5 kHz
11	[23]	sapphire	dia. 100 mm × 60 mm	68 kHz
12	[21]	sapphire	dia. 137 mm × 44 mm	38 kHz
13	[22]	sapphire	dia. 30 mm × 100 mm	53.6 kHz
14	[12]	Si(111)	dia. 106 mm × 220 mm	19.6 kHz
15	[11]	Si(111)	dia. 100 mm × 60 mm	62.3 kHz
16	[13]	Si	dia. 100 mm × 0.5 mm	1.7 kHz
17	[14]	Si(110)	10 × 57 × 0.092 mm^3	670 Hz
18	[15]	Si(100)	dia. 76.2 mm × 12 mm	14.9 kHz

Table 2: Selected values for the mechanical loss of bulk materials from the literature.

4 Thermal noise calculation

As shown in our previous report [1] thermoelastic and Brownian thermal noise will dominate at 300 and 18 K in the frequency band from 10 to 500 Hz. Thus, the following description will focus on these two noise sources. The following section will summarize the euqations being used to calculate these noise forms for the bulk material and the coating.

The thermal noise calculation follows the description given in [1]. The bulk Brownian thermal noise is given for a homogeneously distributed loss by [25, 24, 26]:

material	T [K]	Φ	reference
fused silica	300 K	4×10^{-10}	[27]
fused silica	18 K	1×10^{-3}	[4]
calcium fluoride	300 K	2.2×10^{-8}	[5]
calcium fluoride	12 K	1.0×10^{-8}	[19]
sapphire	300 K	3.8×10^{-9}	[22]
sapphire	4 K	2×10^{-10}	[21]
Si(111)	300 K	1×10^{-8}	[11]
Si(100)	5.8 K	2.2×10^{-9}	[15]
Si(111)	3.5 K	5×10^{-10}	[12]

Table 3: Overview of mechanical loss values used in this article for Brownian thermal noise calculation. In many cases 18 and 300 K were taken as reference temperatures. The closest in literature available value was used as the mechanical loss.

material	temperature [K]	mechanical loss	reference
SiO_2	300	4×10^{-5}	[43]
	300	1×10^{-4}	[46]
	20	6×10^{-4}	[16]
Ta_2O_5	300	4.4×10^{-4}	[43]
(undoped)	300	3.8×10^{-4}	[46]
Ta_2O_5	300	2.6×10^{-4}	6% TiO_2, [44]
(TiO_2 doped)	300	2.0×10^{-4}	14.5% TiO_2, [47]
	18	8.0×10^{-4}	14.5% TiO_2, [47]

Table 4: Summary of the mechanical loss of coating materials used in this article.

$$S_x(f,T) = \frac{2k_BT}{\pi^{3/2}f}\frac{1-\sigma^2}{wY}\Phi_{substrate} \tag{4}$$

where S_x is the spectral power density in m^2/Hz, $k_B = 1.38 \times 10^{-23} J/K$, T the temperature, f the frequency, w the beam diameter ($1/e^2$ drop of the laser power), σ the Poisson's ratio, Y the Young's modulus and $\Phi_{substrate}$ is the mechanical loss of the substrate material. The mechanical loss is used in the structural damping approximation which assumes a frequency independent mechanical loss. Thus, the Brownian thermal noise spectrum is proportional to $1/f$. Eq. (4) assumes a semi-infinite test mass.

Braginsky pointed out the relevance of thermoelastic damping to gravitational wave detectors in 1999. The thermoelastic noise for the infinite half space is given by [38]:

$$S_{TE}(f,T) = \frac{4k_BT^2\alpha^2(1+\sigma)^2\kappa}{\sqrt{\pi^5}\rho^2C^2w^3f^2}. \tag{5}$$

with the coefficient of thermal expansion α, mass density ρ, heat capacity per volume C, the beam radius w and the frequency f. The parameter a is given by:

$$a^2 = \frac{\kappa}{\rho C} \tag{6}$$

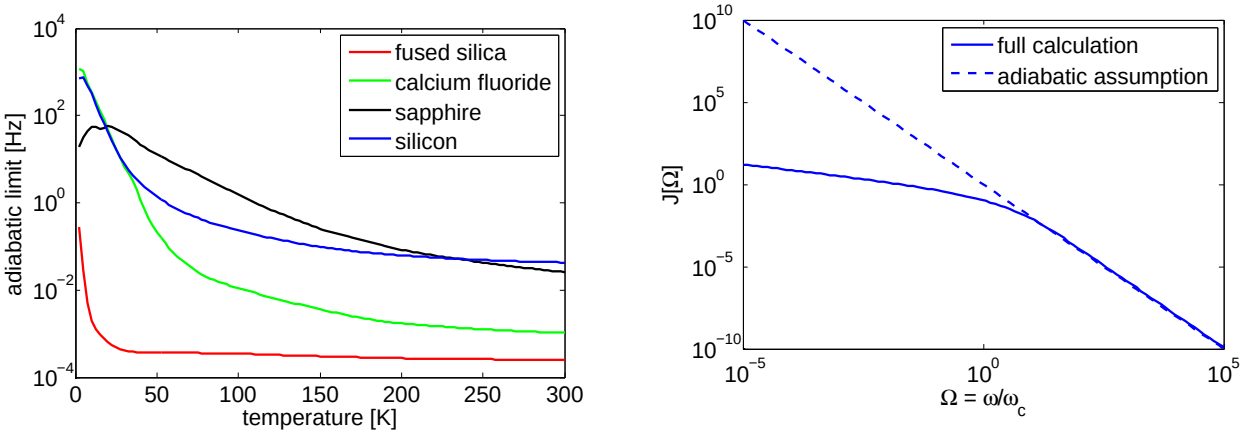
with the thermal conductivity κ.

Eq. (5) is only valid in the so-called *adiabatic assumption*. The calculation is based on the assumption that all thermal fluctuations are "faster" than the thermal conduction of heat out of the beam diameter. Thus, all thermal fluctuations within the beam diameter will fully contribute to the thermoelastic noise.

The thermal diffusion length at the frequency f is given by [40]:

$$l_T = \sqrt{\frac{\kappa}{\rho C f}}. \tag{7}$$

If $l_T < w$ then the thermal fluctuation just affects the region of the beam diameter and is fully contributing. In other words for all angular frequencies ω_c greater than



(a) (b)

Figure 4: (a) – Adiabatic limit f_c in dependence on the sample temperature (w=60 mm). (b) – Comparission of the corrected frequency dependence of the thermoelastic noise with the adiabatic assumption.

$$\omega_c = \frac{\kappa}{\rho C w^2} \tag{8}$$

the adiabatic assumption is fulfilled and thus eq. (5) is valid.

Fig. 4 (a) shows the adiabatic limit $f_c = \omega_c/2\pi$ in dependence of the temperature.

Assuming a lower frequency of 1 Hz the adiabatic limit is only fulfilled for fused silica between 5 and 300 K. Crystalline materials - due to their high thermal conductivity - violate this limit already at higher temperatures (around 50 K for CaF_2 or 120 K for sapphire).

Several authors pointed out that thermoelastic noise will be lower than expected from eq. (5) if the adiabatic assumption is not fulfilled. In this case, the thermal conduction is faster than the thermal fluctuations. Thus, a non-negligible amount of the thermal fluctuation is already moved out of the beam diameter before it is read out. Only a smaller fraction of the fluctuation contributes to the thermoelastic noise and thus it is further reduced. A detailed explanation can be found in [1] and [40].

The final equation incorporating the non-adiabatic case is [40]:

$$S_{TE}(f,T) = \frac{8}{\sqrt{2\pi}}\alpha^2(1+\sigma)^2\frac{k_B T^2 r_0}{\rho C a^2} J[\Omega] \tag{9}$$

with

$$J[\Omega] = \sqrt{\frac{2}{\pi^3}}\int_0^\infty du \int_{-\infty}^{+\infty} dv \frac{u^3 e^{-u^2/2}}{(u^2+v^2)[(u^2+v^2)^2+\Omega^2]}. \tag{10}$$

Ω is the relative frequency compared to the adiabatic limit: $\Omega = \omega/\omega_c$. This equation deviates

from eq. (21) of [40] by the exponent 3 in the first root which is a typo in the paper. The following calculations after eq. (21) include the exponent again and are correct.

The Brownian thermal noise of dielectric coatings can be calculated using [42]:

$$S_x(f,T) = \frac{2k_BT}{\pi^2 f}\frac{1-\sigma^2}{w^2 Y}\frac{d}{YY'(1-\sigma'^2)(1-\sigma^2)} \times \left[Y'^2(1+\sigma)^2(1-2\sigma)^2\Phi_{||} + YY'\sigma'(1+\sigma)(1+\sigma')(1-2\sigma)(\Phi_{||}-\Phi_\perp) + Y^2(1+\sigma')^2(1-2\sigma')\Phi_\perp\right]. \tag{11}$$

Y and σ are the Young's modulus and Possion's ratio of the substrate, Y' and σ' are the parameters for the coating. $\Phi_{||}$ is the loss angle associated with the energy density of parallel coating strains and $\Phi_\perp$ is the mechanical loss for the perpendicular case. d is the coating thickness. Eq. (11) assumes an infinite half space.

Thermoelastic noise in thin dielectric layers was investigated by Fejer et al. [41]. They obtain the following expression for a multilayer coating:

$$S_{TE}(f,T) = \frac{8k_BT^2}{\pi^2 f}\frac{L}{w^2}\frac{\alpha_s C_F}{C_s^2}(1+\sigma_s)^2\Delta^2 g(\omega) \tag{12}$$

with the index s for substrate and f for the dielectric film. L is the thickness of the dielectric coating. The parameter Δ is given by:

$$\Delta = \frac{C_s}{2\alpha_s C_F}\left(\frac{\alpha}{1-\sigma}\left[\frac{1+\sigma}{1+\sigma_s} + (1-2\sigma_s)\frac{Y}{Y_s}\right]\right)_{AVG} - 1. \tag{13}$$

$(...)_{AVG}$ indicates the averaging operator given by:

$$(X)_{AVG} = \frac{d_A}{d_A+d_B}X_a + \frac{d_B}{d_A+d_B}X_a. \tag{14}$$

The frequency dependence can be described as:

$$g(\omega) = Im\left[-\frac{1}{\sqrt{i\omega\tau_F}}\frac{\sinh\sqrt{i\omega\tau_F}}{\cosh\sqrt{i\omega\tau_F} + R\sinh\sqrt{i\omega\tau_F}}\right]. \tag{15}$$

with $\tau_F = L^2/k_F$. k_F is the thermal diffusivity of the multilayer. The thermal properties are volume averaged as follows:

$$C_F = (C)_{AVG}, \tag{16}$$
$$k_F = \frac{\kappa_F}{C_F}, \tag{17}$$
$$\kappa_F^{-1} = (\kappa^{-1})_{AVG}. \tag{18}$$

κ is the thermal conductivity C the specific heat capacity per unit volume of the dielectric material.

Eq. (12) again assumes that the adiabatic assumption is fulfilled. Due to the fact that the dielectric coatings have a low thermal conductivity within the temperature range from 5 to 300 K the adiabatic assumption is always fulfilled (see fig. 4).

5 What mirror size is necessary?

This section will combine all results from the previous calculations and give an estimate of a potential mirror consisting of a bulk material and an optical coating. The total thermal noise is evaluated and compared to the desired sensitivity for ET.

5.1 Summary of the results so far

The bulk material selection is driven by several aspects: The first and important issue is low thermal noise - especially at the temperatures where the detector will be operated. Thus, fused silica is not a candidate for a cryogenic detector. Crystalline materials have the advantage of low mechanical losses at cryogenic temperatures. The second important issue is the availability in sufficient large pieces. It is desirable to manufacture the test mass from one piece. Otherwise, other noise increasing techniques like bonding have to be used to manufacture a sufficiently large piece. Thus, from the current point of view, silicon and sapphire might be candidates. Silicon has two advantages over sapphire: The demand of the semiconductor industry drives the development of large and high purity samples. The second is the vanishing coefficient of thermal expansion at 18 K making silicon a very interesting candidate for a cryogenic gravitational wave detector.

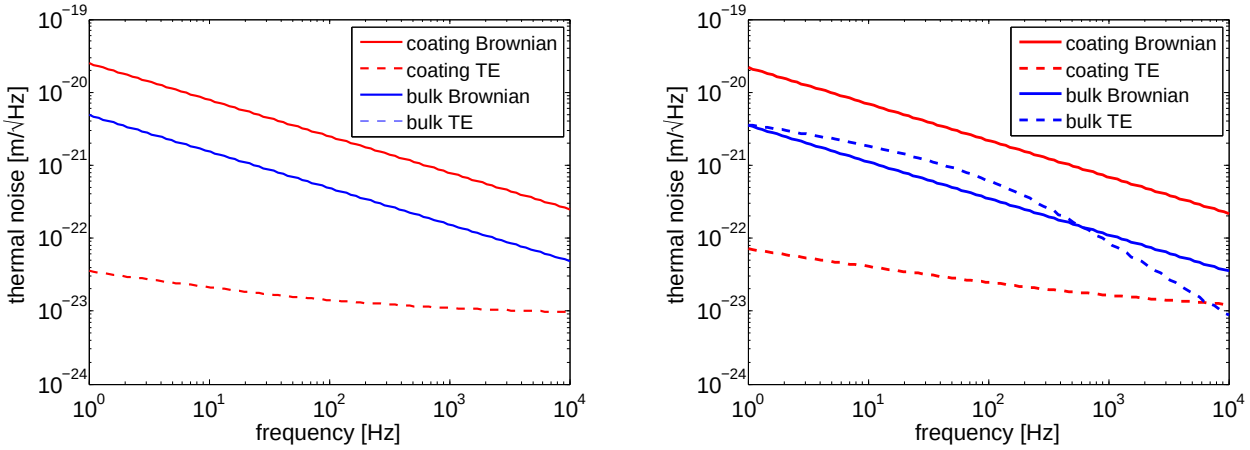
Up to now titania doped tantala seems to be the best material as a high refraction index dielectric coating layer. In conjunction with silica it is assumed to produce the lowest currently available coating thermal noise.

Fig. 5 compares silicon and sapphire test masses at 18 K coated with 15 $\lambda/4$ double layers of titania doped tantala and silica.

It is obvious from both plots in fig. 5 that the overall thermal noise of the mirror will be dominated by the coating Brownian noise. The calculation for silicon includes the vanishing coefficient of thermal expansion at 18 K and thus the vanishing contribution of bulk thermoelastic noise. The contribution of the thermoelastic noise to the sapphire total noise is reduced by the inclusion of the non-adiabatic case [40]. Nevertheless, the thermoelastic noise of sapphire will dominate over the Brownian thermal noise between 1 and 300 Hz at around 18 K. At 10 K operation temperature the thermoelastic contribution is as high as the Brownian contribution. Below 10 K Brownian thermal noise dominates again for sapphire.

The reason why the use of sapphire might also be restricted to temperatures around 20 K is of a technical nature. Initial cooling estimates provided a stable operation temperature of about 20 K assuming reasonable values for the cooling power and the optical absorption.

Silicon would provide the lower total thermal noise contribution from the bulk material due to the vanishing thermoelastic contribution at 18 K. Thus, the following estimates are focused on silicon although the coating contribution will be dominant in both cases.



(a) Si(111). (b) Sapphire

Figure 5: Comparison of the thermal noise of a silicon and a sapphire test mass coated with 15 $\lambda/4$ double layers of titania doped tantala and silica at 18 K (beam radius w=60 mm).

In order to compare the achieved thermal noise level with existing, advanced detectors and the desired sensitivity curve for ET it is necessary to transform this position thermal noise into strain thermal noise. Assuming a Michelson interferometer like detector the strain noise is given by (e.g. [38]):

$$h = \frac{\sqrt{NS_x}}{L} \tag{19}$$

where N represents the number of mirrors, S_x thermal noise and L the arm lengths of the interferometer.

Let's assume 4 identical mirrors (2 end mirrors and 2 cavity coupler assumed to be identical in their noise behavior) and a 10 km arm length. This leads to a comparison given in figure 6. The graph shows the calculated thermal noise for an advanced detector mirror (fused silica, Ta_2O_5:TiO_2 multilayer, 300 K operation temperature, 4 km arm length) and the cryogenic mirror (Si(111), Ta_2O_5:TiO_2 multilayer, 18 K operation temperature, 10 km arm length) in comparison to existing detectors, advanced detectors and the ET sensitivity curve obtained from [48]. The last curve is an initial design curve. Several crucial noise contributions are missing (see [49]). Nevertheless, this curve was chosen to be a good reference for a noise limit regarding thermal noise.

The change of the bulk material, the longer arms and the reduction of the operation temperature to 18 K reduce the strain noise by a factor of about 9. This noise level is still too high and would limit the sensitivity of the detector in a frequency band from 30 to 300 Hz.

5.2 Further reducing thermal noise

There are several ways to further reduce the thermal noise of the optics:

- further reduction of the operation temperature,
- use of better coatings,

- increase of the beam diameter.

Each single contribution to the overall thermal noise is temperature dependent. Using crystalline bulk materials a reduction of the operation temperature will result in lower bulk thermal noise. The mechanical loss of the dielectric coating materials will be constant or even decrease with temperature below 18 K (see e.g. [47]). Nevertheless, cooling an optical mirror with roughly 1 ppm optical absorption will be critical well below 18 K. The laser power planned to be used in ET will be in the region of megawatts thus resulting in at least 1 W heating power in the mirror. This heat needs to be extracted through the suspension system of the detector. A stable operation temperature around 20 K seems to be achievable from the current point of view. Lower temperatures are an extreme technical challenge and might not be achievable within the next years if at all. In addition, lower optical absorption coatings would be interesting to further reduce the temperature. 1 ppm seems currently to be the limit for coatings being available on the market from the desired materials and with sufficiently large size.

The calculations presented above use the best values currently available in literature. They are based on the lowest mechanical and optical losses. Nevertheless, improved coatings with even lower mechanical loss would reduce the coating Brownian noise and thus the main contributing noise source. The current research on coatings is ongoing. There are several investigations on different coating types and fabrication techniques. Up to now they resulted in an improvement of about a factor of 2. An estimate of the coating mechanical loss based on a possible realistic mirror geometry is given below.

All noise contributions scale with the beam diameter (see table 5). Increasing the beam will result in a bigger surface area which the thermal noise is averaged over during read-out. This results in a smaller detected thermal noise.

Fig. 7 summarizes the strain noise for different beam radii w of the detector. An operation temperature of 18 K was assumed. Although the potential sensitivity curve for ET does not yet contain all

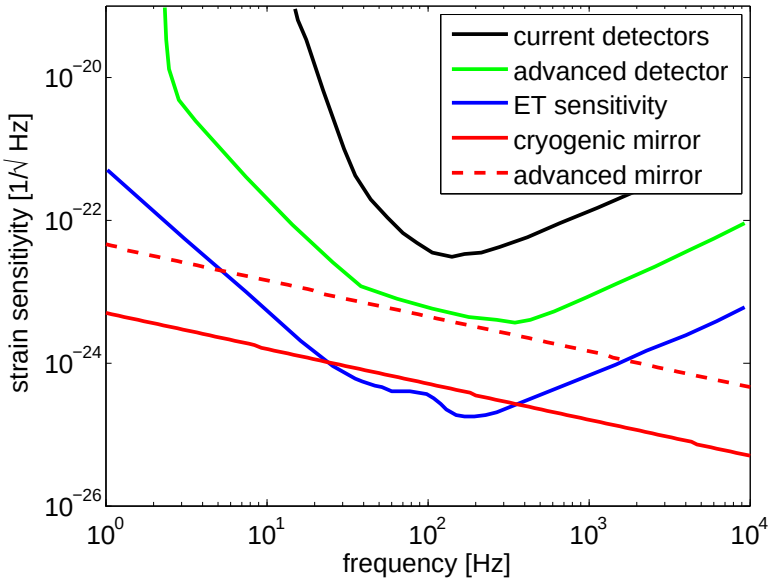


Figure 6: Strain sensitivity calculated for an advanced detector mirror and the cryogenic mirror discussed in this article. The reference curves for the detectors were taken from [48].

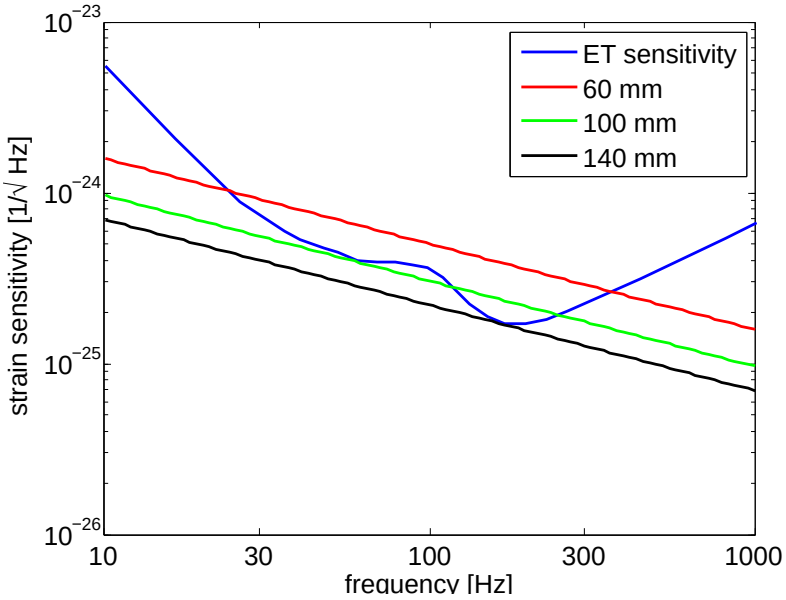


Figure 7: Reduction of the total thermal noise by increasing the beam radius of the laser (18 K operation temperature).

contributing noise sources (e.g. gravity gradient noise) it is a valid curve to compare thermal noise in a frequency range from 30 to 300 Hz to. An upscaling of the beam radius to 140 mm would result in a thermal noise level which is always below the quantum noise level given by the ET sensitivity curve. This beam radius would result in a substrate diameter (assuming the same ratio of 2.8 between the laser beam and the substrate diameter as given in table **??**) of about 780 mm (30 inch) which is far away from currently available high purity silicon bulk samples. The semiconductor industry currently has an interest in silicon wafers up to 20 inches due to handling reasons. Therefore, the supply of a 30 inch bulk sample with extraordinary high purity requirements is questionable. Furthermore, assuming a similar diameter to thickness ratio as used for advanced detectors would result in a 460 mm thick test mass. This sample would have a mass of 512 kg which is surely too high for a realistic design. Especially cooling and suspension requirements are enormous for such a high mass. Further, a beam radius of 140 mm cannot be achieved in a 10 km cavity. The radius of curvature of the mirrors needed for such a large beam diameter exceeds the possibilities of current optical companies. It is not possible to fabricate such a large radius of curvature today.

In contrast, a lightweight test mass cannot be used either. Calculations [50] have shown that a minimum mass of at least 120 kg is necessary to suppress the radiation pressure noise beyond the

type of noise	frequency scale	beam radius scale
bulk Brownian	$\propto f$	$\propto w$
bulk thermoelastic	$\propto f^2$	$\propto w^3$
coating Brownian	$\propto f$	$\propto w^2$
coating thermoelastic	see eq. (12)	$\propto w^2$

Table 5: Scaling of different thermal noise contributions with frequency and beam radius. The scaling is always given for the power spectral density (e.g. S_x) of the value.

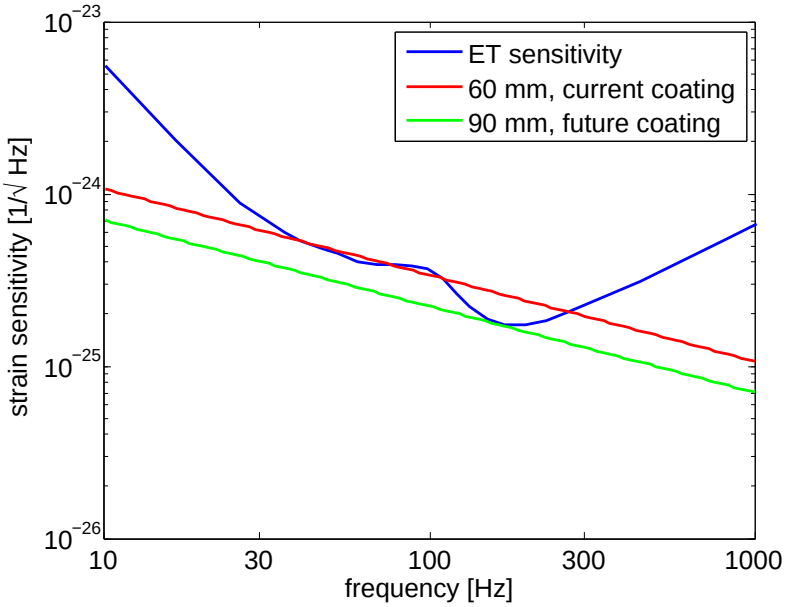


Figure 8: Strain sensitivity achievable using a Si(111) test mass (dia. 510 mm × 300 mm) at 18 K with a further reduced coating mechanical loss of 4×10^{-4} for tantala and 2×10^{-4} for silica.

desired sensitivity limit.

Assuming a mass of 150 kg and the aspect ratio of the advanced detector test mass given in table **??** would result in a test mass diameter of about 510 mm, a thickness of 300 mm and a beam radius w of roughly 90 mm. This beam diameter is not sufficient to reduce the mirror thermal noise below the desired sensitivity as fig. 7 indicates. If it would be possible to achieve lower mechanical loss values of the coating materials at 18 K a 90 mm beam radius and a 150 kg test mass of silicon would fulfill the requirements as shown in fig. 8.

6 Summary

This report summarizes initial calculations which have been done in Glasgow in order to give a first estimate of a potential mirror size for ET. The calculation is based on the currently available values for the thermal and mechanical properties. The calculation shows that Brownian coating noise will dominate the overall thermal noise. It is not possible to overcome this issue only by cooling. Larger beam diameters would further reduce the thermal noise of the detector. Assuming the aspect ratio of an advanced detector mirror the necessary test mass would have a total mass of over 500 kg. Besides the fact that this would be a big technical challenge the calculated beam diameter is not achievable right now. The radius of curvature necessary to produce a beam radius of 140 mm on the end mirror is not available. Furthermore, it is not likely that silicon will be available in such large sizes with the desired quality. Therefore, a rather realistic assumption of the test mass (150 kg to overcome radiation pressure noise) was made and the limits for the coating materials calculated. The maximum values for a silica and tantala coating at 18 K are around 4×10^{-4} for tantala and 2×10^{-4} for silica. Currently, these values are not achievable and further work needs to be done.

A possible further reduction of the requirements can be done by including considerations of optimized dielectric mirrors [51], reduced dielectric layer thicknesses in waveguide mirrors [52] or even dielectric layer free coatings [53]. Thus, the influence of the coating thermal noise is reduced and the bulk contribution will appear. A further reduction is desirable because in a realistic detector more noise sources will be present than just Brownian and thermoelastic noise. This increased noise level will demand even bigger test masses. Thus a reduction of the coating contribution is needed to keep the bulk material geometry manageable.

Considering these novel techniques as well as a fully finite size calculation are currently under investigation and will be the content of a future report.

In any case, the first results on an estimate of a potential test mass size of ET will contribute to the design study of the suspension system giving a first initial guess what geometries need to be handled.

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