European Commission

FP7, Grant Agreement 211743

Possible holographic noise issues in Einstein Telescope

ET-0105A-10

Sergey Tarabrin

Issue: 1

Date: November 18, 2010

Max-Planck-Institut für Gravitationsphysik
Callinstrasse 38, 30167 Hannover, Germany

ET – Einstein gravitational wave Telescope – Design Study * A joint European Project
Web: http://www.et-gw.eu Email: EuropeanProjectsService@ego-gw.it

Contents

1 Holographic principle

1.1 Black hole thermodynamics

In classical general relativity field solutions are completely deterministic: there is nothing random in gravitational waves or black holes. Thus from the general relativistic point of view black holes have zero entropy. However, this comes to the contradiction with thermodynamics, since the black holes absorb matter with non-zero entropy, and the latter one cannot decrease according to the second law of thermodynamics. This fact led to the development of the black-hole thermodynamics which quasiclassically mixes general relativity and quantum mechanics. To maintain thermodynamic equilibrium black hole with mass M radiates perfect black body radiation (Hawking radiation) with temperature [1]

$$T = \frac{\hbar c^3}{8\pi k_{\rm B} G M}. \tag{1}$$

With the current status of detector technology, Hawking radiation of the astronomical-size black holes has too low temperature to be detected.

Remarkably, it turned out that the black hole entropy corresponding to the radiation process depends on the area of the event horizon and not the volume inside it [2]:

$$S = \frac{1}{4}\frac{\mathcal{A}}{l_{\rm P}^2}, \tag{2}$$

where $l_{\rm P} \approx 10^{-33}$ cm stands for the Planck length. The second law of black hole thermodynamics states that the area of event horizon never decreases, thus the entropy of the black hole also never decreases.

1.2 Entropy and information

It has been known that the thermodynamical (Boltzmann) entropy and informational (Shannon) entropy[1] are essentially the same, measured in different units. Usually for a given macroscopic system their values differ significantly. For instance, consider a computer memory chip. The amount of information depends on the state of all the transistors that make the chip, while the amount of entropy depends on the state of all the atoms that make it. Information and entropy will equal each other once each atom of the chip codes 1 bit of information.

There rises a fundamental question: what is the limiting number of degrees of freedom for a given system (transistors $\to$ atoms $\to$ nuclei $\to$ protons $\to$ quarks $\to \ldots \to$ layer X) and what is the maximal amount of information the given region of space can contain? Remarkably, the answer does not depend on the nature of fundamental layer X where subdivision of the fundamental particles stops. Consider a system bounded by the sphere of area $\mathcal{A}$. Let it collapse into the black hole. The area of the event horizon of this black hole will be less than $\mathcal{A}$ and, according to Eq. (2), the entropy will be less than $\mathcal{A}/4$ (in Planck units). Since the entropy does not decrease, the entropy of the initial system has been also less than $\mathcal{A}/4$. This is so-called *holographic limit*: the entropy of the isolated physical system bounded by the area $\mathcal{A}$ cannot exceed the value of $\mathcal{A}/4$. Therefore, maximal amount of information the system can contain is defined by the area of its boundary and not the volume inside it. Note that according to the classical field-theoretical point of view, maximal entropy/information amount depends on the spatial volume the system occupies. According to the holographic limit, black hole is the object of maximal entropy bounded by the sphere of area $\mathcal{A}$.

1.3 Holographic principle

Entropy-information relation led to the following interpretation of Eq. (2): the state of black hole is completely defined by the surface quantum fluctuations of the event horizon, i.e. all the physics of black hole is encoded on it, one bit of information (one degree of freedom) per four Planck squares.

[1] Below we shall address Boltzmann entropy as just entropy and Shannon entropy as information

This interpretation and and the holographic limit led to the conjecture of the *holographic principle* (G. 't Hooft [3], L. Susskind [4]): physics of any region of space is somehow encoded on the boudary of that region. More precisely it says that that the physical theory defined in space-time of dimensionality D is equivalent to another theory defined on the boundary of dimensionality $D-1$. The most known mathematical realization of holographic principle is the AdS/CFT correspondence by J. Maldacena [5]: string theory (which includes gravity) in anti-de Sitter space-time is equivalent to conformal field theory (without gravity) on its boundary. Currently it is widely believed that the holographic principle should be the fundamental constituent part of any quantum gravity theory and string theories, in particular.

2 Holographic uncertainty

It follows from the holographic principle that if the volumetric system can be described by the theory on the boundary, then the maximal number of volumetric degrees of freedom should not exceed the number of their "images" on the boundary. Since the "classical" field-theoretical informational content of the region of space is defined by its volume, such a description contains much more degrees of freedom than allowed by the holographic entropy bound. Therefore, 3-dimensional world must be "blurry" in order to match the number of degrees of freedom inscribed on some 2-dimensional holographic surfaces. *Holographic uncertainty* is a particular hypothesis proposed by C. Hogan about how holographic principle works in flat space-time [6, 7, 8]. Much like the holographic principle itself, holographic uncertainty is a **conjecture**, i.e. this hypothesis needs both theoretical and experimental proof.

Underlying idea is the following. Classical physics describes the interaction of matter-energy on the space-time background, either static (special relativity) or dynamical (general relativity) one. Space-time is associated with one of the classical geometries (Minkowskian, pseudo-Riemannian, etc) where space-time event is a well-defined point-like object (space-time coordinate). Point-like particles move along the geodesics and their space-time positions are identified with the point-like events.

Quantum mechanics introduces the concept of particle/wave duality which makes the space-time localization of a quantum particle ill-defined: due to Heisenberg's uncertainty principle the less uncertain momentum is the more particle is delocalized and vice versa. Although the notion of the point-like particle is inadequate in quantum mechanics, the theory itself remains defined on the classical space-time background with point-like events. In other words space-time coordinates remain commuting quantities, while particle positions do not.

Hogan's hypothesis assumes that the theory respecting the holographic principle should be defined on the space-time background with the ill-defined notion of event. This can be achieved in several ways, for instance by introducing the non-commuting operators of the transversal positions. Consider two particles positioned along the longitudinal axis z and separated by a distance L. Holographic nature of space-time, according to Hogan, exhibits itself through the new commutation relation between the transversal coordinates of these particles as measured by a light-like signal:

$$[\hat{x}_1, \hat{x}_2] = i l_{\mathrm{P}} L. \qquad (3)$$

In this commutator $\hat{x}_1$ and $\hat{x}_2$ can also stand for the transversal coordinates of a single particle measured by light at two successive instants of time. This means that the more precise is the measurement of the transversal position, the more uncertain it is made distance L away (or time L/c after):

$$\Delta x_1 \Delta x_2 \geq l_{\mathrm{P}} L. \qquad (4)$$

This uncertainty relation implies that the measurement of the transversal position of a single test mass with the optical signal will yield uncertain results with $\Delta x \geq \sqrt{l_{\mathrm{P}} L}$, where L stands for the distance the light wave travels between two measurements.

More intuitively, holographic commutation relation can be interpreted as the diffractive nature of space-time. Classical space-time structure can be defined by the rays that are the $\lambda \to 0$ limit of the waves[2]. The new

[2] The nature of these waves has no significance. The only requirement is that these wave solutions obey the wave-like equation.

assumption is that the auxiliary waves which define the space-time have finite wavelength equal to the fundamental length, the Planck length. Since $\lambda \neq 0$, diffraction of the waves takes place meaning that the transversal localization of the events in space-time becomes uncertain. One can immediately estimate the order of diffractive uncertainty relying on the well-known diffraction laws in wave optics: if the two transversal measurements, separated by a distance L, are performed then the measurement error is of the order of $\sim \sqrt{\lambda L} = \sqrt{l_{\mathrm{P}} L}$ which coincides exactly with the prediction from holographic uncertainty relation. With this restriction on the position measurement accuracy a region of space has the the number of spatial degrees of freedom equal to the one imposed by the holographic principle.

The above mentioned auxiliary waves, by their nature, are the quantum mechanical wavefunctions that define the distribution of the transversal position of any mass-energy contained within the given region of space-time. It is posited that the governing wave equation is the effective parabolic equation derived from Matrix theory [9]:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{2\pi i}{l_{\mathrm{P}}}\frac{\partial u}{\partial z^+} = 0. \tag{5}$$

Here $z^+ \equiv (z+ct)/2$ is one of the light-like directions and $u = u(x,y,z,t)$ is the space-time wavefunction. Note that this equation closely resembles the paraxial equation in wave optics and Schrodinger equation in quantum mechanics. The difference from the latter one is the evolution parameter $z^+ \equiv (z+ct)/2$ (instead of time t) which reflects the fact that the theory is formulated on the two-dimensional sheets moving at the speed of light. Holographic wave theory is normalized by matching the degrees of freedom to black hole physics, yielding the wavelength to be equal twice the Planck length. As usually, the solutions can be represented by the linear combination of plane waves (omitting the 2nd transversal coordinate y):

$$u(z^+, x) = \int u(k^+, k^\perp) e^{-ik^+z^+ + ik^\perp x} \frac{dk^+}{2\pi}\frac{dk^\perp}{2\pi}, \tag{6}$$

where transversal and longitudinal wavenumbers are related by

$$k^\perp = \sqrt{\frac{2\pi k^+}{l_{\mathrm{P}}}}. \tag{7}$$

It is worth mentioning that the effect of holographic uncertainty (and holographic noise, see below) doest not correspond to any quantum fluctuations of space-time predicted in various quantum gravitational theories, but rather reflects the constraints on the informational content in the space-time respecting the holographic principle. It is the reason for the predicted effect to be highly non-local and thus unavailable for test in purely local experiments.

3 Holographic noise in interferometers

3.1 Spectral density

Holographic fuzziness with associated uncertainty should be seen in precise interferometry, otherwise it would be possible to distinguish more test masses configurations than is allowed by the holographic entropy/information bound.

In a Michelson interferometer optical waves measure the transversal position of the beamsplitter relative to the direction of the incident laser beam. The first measurement event occurs when the beam is splitted and half of it redirected into the perpendicular arm. The second measurement event occurs after time $2L/c$, where L is the arm length, when the beam is recombined at the beamsplitter and is redirected towards photodetector. The uncertainties of these two measurements obey the uncertainty relation (4). Since the measurement in an actual interferometer is a continuous process, uncertain measurement results yield a fluctuating time series, i.e.

noise. The noise associated with the uncertain transversal position of the beamsplitter due to the holographic nature of space-time is called *holographic noise* (in a Michelson interferometer).

Obviously, one can estimate the uncertainty in beamsplitter transversal position as $\Delta x \sim \sqrt{l_{\rm P} L}$. Since the time interval between the measurements $\tau \sim L/c$ corresponds to the characteristic frequency interval $\Delta f \approx 1/\tau$, one can estimate the corresponding power spectral density as

$$S(f) \sim \frac{(\Delta x)^2}{\Delta f} \sim (\Delta x)^2 \tau = L^2 t_{\rm P}, \tag{8}$$

where $t_{\rm P} \approx 10^{-43}$ s stands for the Planck time.

In a more accurate way calculation of the holographic noise power spectral density should be done using the solutions of Eq. (5). It is well-known from quantum mechanics that the system in the gaussian state possesses the least uncertainty allowed by the Heisenberg uncertainty principle. It is also well-known that the gaussian state is the one mostly "preferred by nature". Therefore, in full analogy we may assume that the natural state of the transversal position of the beamsplitter in the holographic space-time governed by Eq. (5) is also gaussian [8]:

$$u(r,z) = \frac{w_0}{w(z)} \exp\left[-i\,\frac{\pi}{l_{\rm P}}\,z - i\phi - r^2\left(\frac{1}{w(z)^2} + \frac{i\pi}{2l_{\rm P}R(z)}\right)\right]. \tag{9}$$

Here $r^2 = x^2 + y^2$, $\phi = \arctan(z/z_d)$, $z_d = \pi w_0^2/2l_{\rm P}$ is the diffraction length, $w(z) = w_0\sqrt{1+z^2/z_d^2}$ is the width function and $R(z) = z(1 + z_d^2/z^2)$ is the radius of wavefront curvature. Diffraction length z_d and the waist of the gaussian "beam" w_0 (width at $z = 0$ corresponding to plane wavefront) are related to each other as $w_0 = \sqrt{2l_{\rm P} z_d/\pi}$.

For a given separation $z = z_0$ between two measurements there is an optimum gaussian state which minimizes the uncertainty. Note that the gaussian solution (9) can be interpreted as the family of solutions parameterized by diffraction length z_d (or, equivalently, waist w_0). The width function $w(z)$ explicitly depends on diffraction length z_d as a parameter: $w(z) = w(z; z_d)$. Obviously, condition of the optimum state is $dw(z_0; z_d)/dz_d = 0$, which yields $z_d = z_0$ or $w_0 = \sqrt{2l_{\rm P} z_0/\pi}$. This means that the minimum transverse width at distance z_0 occurs for the state with $z_d = z_0$. And conversely, at a distance z_d in either direction from the waist, there is a minimum transverse width $w(z_0) = w_0\sqrt{2}$.

The optimum gaussian state for the beamsplitter in a Michelson interferometer yields the displacement variance $\sigma = l_{\rm P}L/\pi$ which also equals the value of autocorrelation function of displacement $B(\tau)$ at time lag $\tau = 0$. The distribution of the transversal position is uncorrelated for times separated by more than $2L/c$. Since the variance increases linearly with the scale, one can suggest that the autocorrelation should linearly decrease with time argument [8]:

$$\begin{cases} B(\tau) = l_{\rm P}(2L - c\tau)/2\pi, \ 0 < \tau < 2L/c; \\ B(\tau) = 0, \ \tau > 2L/c \end{cases} \tag{10}$$

According to the Wiener theorem, this autocorrelation function yields the following power spectral density:

$$S(f) = \frac{c^2 t_P}{\pi(2\pi f)^2}\left[1 - \cos\left(\frac{f}{f_c}\right)\right], \tag{11}$$

where $f_c = c/4\pi L$. In the low-frequency limit $f \ll c/2L$ holographic noise is white with the spectral density equal to

$$S(f) = \frac{2}{\pi}\, t_{\rm P} L^2, \tag{12}$$

which coincides with rough estimation in Eq. (8) up to numerical multiplier.

One can immediately calculate the effective metric strain corresponding to spectral density (12):

$$h(f) = \sqrt{\frac{S(f)}{L^2}} = \sqrt{\frac{2t_{\rm P}}{\pi}} = 1.84 \times 10^{-22}/\sqrt{\rm Hz}. \tag{13}$$

Holographic noise prediction is thus fixed with no free parameters, therefore the hypothesis can be either confirmed or ruled out experimentally. Holographic noise signatures are currently being looked for in the noise spectrum of GEO-600 interferometer[3]. However, the available sensitivity does not allow to make unambiguous conclusions.

3.2 Correlation and multiple-pass issues

Since the space-time wavefunction universally defines the transversal distribution of mass-energy, holographic noise should exhibit particular cross-correlation features. Namely, the two closely positioned interferometers without any physical connection between them should produce correlated measurements of the holographic displacement, because they occupy nearly the same space-time volume and thus holographic motion of their test masses (beamsplitter, in particular) is defined by nearly the same wavefunction. If the two interferometers are aligned along their arms and are displaced by $\Delta L \ll L$ along one of them, then the cross-correlation function should be [8]:

$$\begin{cases} B_\times(\tau) = l_\mathrm{P}(2L - 2\Delta L - c\tau)/2\pi, \ 0 < c\tau < 2L - 2\Delta L; \\ B_\times(\tau) = 0, \ c\tau > 2L - 2\Delta L \end{cases} \tag{14}$$

Corresponding spectral density equals to

$$S_\times(f) = \frac{2t_\mathrm{P} L^2}{\pi}\left(1 - \frac{\Delta L}{L}\right). \tag{15}$$

This expected feature of the holographic noise is to be tested in the Fermilab holometer which is under construction currently [10].

If a Michelson interferometer has folded arms, delay lines or Fabry-Perot arm cavities then the influence of the additional mirrors should be considered. The reason for this is the correlation feature of the holographic noise (see above): the mirrors positioned in the vicinity of the beamsplitter share nearly the same holographic wavefunction and thus should exhibit correlated position fluctuations. However, from Hogan's hypothesis it seems that the holographic fluctuations of the additional mirrors should not affect the measurement, since corresponding motions are transverse to the laser beam and thus do not influence the optical phase. Under these reasonings, the effective metric strain in a Michelson interferometer with the multiple-pass arms equals to [8]:

$$h(f) = \frac{1}{N}\sqrt{\frac{S(f)}{L^2}} = \frac{1}{N}\sqrt{\frac{2t_\mathrm{P}}{\pi}} = \frac{1}{N} 1.84 \times 10^{-22}/\sqrt{\mathrm{Hz}}, \tag{16}$$

where N is the average number of photon round trips in the arms. The reason for the N^{-1} factor is that the multiple-pass arms effectively lengthen them for the gravitational waves (this holds true for the frequencies $f < 1/\tau^* \approx c/2LN$, where τ^* is the relaxation or ring-down time), thus amplifying the response to the gravitational waves, but do not change the beamsplitter holographic displacement spectrum [8]. However, this statement is not absolutely clear and requires detailed study, since the influence of cavities and delay lines on light are different, in general.

With planned ITM and ETM transmittances of 7000 and 10 ppm respectively, the number of photon round-trips inside the ET cavities equals to $N \approx 277$, thus lowering the holographic metric strain to $h_\mathrm{ET}(f) \approx 0.66 \times 10^{-24}/\sqrt{\mathrm{Hz}}$.

4 Conclusion

We have reviewed the hypothesis of holographic uncertainty, proposed by C. Hogan, about how holographic principle inspired by black hole physics and quantum gravitational theories should work in flat space-time.

[3] Actually, Hogan's prediction for GEO is twice smaller, $h_\mathrm{GEO}(f) = 0.92 \times 10^{-22}/\sqrt{\mathrm{Hz}}$, because of the folded arms, see Sec. 3.2)

Holographic uncertainty is a restriction to the accuracy of the measurements of the transversal positions which allows the number of spatial degrees of freedom to match the holographic entropy bound. It is predicted that the new uncertainty should be seen in precise interferometry in the form of the so-called holographic noise. The power spectral density of the noise in a Michelson interferometer has been derived from the phenomenological theory based on the picture of diffractive space-time defined by the auxiliary waves obeying parabolic equation. In the low-frequency region holographic noise in a Michelson interferometer is white and is characterized by the parameter-free effective metric strain $h(f) = 1.84 \times 10^{-22}/\sqrt{\mathrm{Hz}}$. We have discussed correlation feature of the holographic noise which allows physically disconnected but closely located interferometers to have common noise. We have also considered the interferometers with multiple-pass arms, such as the Michelson/Fabry-Perot one, where the effective metric strain is decreased by the factor of number of round trips in the arms. We have estimated the value of effective metric strain for the planned ET interferometers as $h(f) \approx 0.66 \times 10^{-24}/\sqrt{\mathrm{Hz}}$.

It should be underlined that the holographic uncertainty hypothesis has no solid foundation at the moment, either theoretical or experimental one. It remains a vague, although beautiful, idea supplemented with some incomplete phenomenological principles and equations. Derivation from some fundamental theory or proof that such a derivation cannot be performed is needed. From the other side, experimental confirmation or disproof of the predicted value of the holographic noise is also needed. Although holographic noise might show up in GEO-600 interferometer when the latter achieves higher sensitivity, the probability this noise exists seems minimal at the moment. More experiments with better sensitivity are required to make unambiguous conclusions. We consider possible issues of the holographic noise in ET for the sake of completeness rather than the expected impact on the interferometer sensitivity.

...

References

[1] S.W. Hawking, "Particle Creation by Black Holes", Comm. Math. Phys. **43**, 199, 1975. 2

[2] J.D. Bekenstein, "Black Holes and Entropy", Phys. Rev. D **7**, 2333, 1973. 2

[3] G.'t Hooft, "Dimensional Reduction in Quantum Gravity", arXiv:gr-qc/9310026v2. 3

[4] L. Susskind, "The World as a Hologram", J. Math. Phys. **36**, 6377, 1995. 3

[5] J.M. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity", Adv. Theor. Math. Phys. **2**, 231, 1998. 3

[6] C.J. Hogan, "Measurement of Quantum Fluctuations in Geometry", Phys. Rev. D **77**, 104031, 2008. 3

[7] C.J. Hogan, "Indeterminacy of Quantum Geometry", Phys. Rev. D **78**, 087501, 2008. 3

[8] C.J. Hogan, "Holographic Noise in Interferometers", arXiv:0905.4803v8. 3, 5, 6

[9] C.J. Hogan and M.G. Jackson, "Holographic geometry and noise in matrix theory", Phys. Rev. D **79**, 124009, 2009. 4

[10] C.J. Hogan *et al.*, "The Fermilab Holometer. A program to measure Planck scale indeterminacy", http://holometer.fnal.gov/presentations/holometer-proposal-2009.pdf 6