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Considerations Regarding the Optical Layout of ET

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ET – Einstein gravitational wave Telescope – Design Study * A joint European Project
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1 Introduction

This document collects the main arguments and constraints for a baseline optical layout for ET. In particular the beam size and geometry for arm cavities with a length of $\approx 10\,\mathrm{km}$ are considered. We further compute the optical parameters such as mirror sizes and mirror positions following these considerations.

Please note that this document does not take into account other possible layout options, most notably the Sagnac interferometer and the use of suspension-point interferometers.

2 Arm cavities: beam size and mirror size

The beam size and geometry of large scale interferometers is at first determined by the long arm cavities. This section determines the size of the minimal possible size of arm cavity mirrors.

2.1 Beam waist of the arm cavity beam eigenmode

In the case of a two-mirror cavity the size of the beam can be computed conveniently from the stability parameters g_1, g_2 given as:

$$g_{1,2} = 1 - \frac{L}{R_{C\,1,2}} \tag{1}$$

with L the length of the cavity and $R_{C\,1,2}$ the radius of curvature of the input and end mirror respectively.

The waist size w_0 of the cavity eigenmode can then be computed as :

$$w_0^2 = \frac{L\lambda}{\pi}\sqrt{\frac{g_1 g_2(1-g_1 g_2)}{(g_1+g_2-2g_1 g_2)^2}} \tag{2}$$

In many cases symmetric or near symmetric cavity layout will be used (or can be used to estimate design options). In that case we set $g = g_1 = g_2$ which leads to a much simpler equation:

$$w_0^2 = \frac{L\lambda}{2\pi}\sqrt{\frac{1+g}{1-g}} = \frac{L\lambda}{2\pi}\sqrt{\frac{2R_C}{L}-1} \tag{3}$$

2.2 Beam size on the arm cavity mirrors

Typically we are interested in the size of the beam on the mirror, rather than the waist size directly. The beam size can be computed similarly as the waists; for the input mirror we get:

$$w_1^2 = \frac{L\lambda}{\pi}\sqrt{\frac{g_2}{g_1(1-g_1 g_2)}} \tag{4}$$

And in the case of a symmetric cavity we obtain:

$$w^2 = \frac{L\lambda}{\pi}\sqrt{\frac{1}{1-g^2}} = \frac{\lambda}{\pi}\sqrt{\frac{RL}{2-\frac{L}{R}}} \tag{5}$$

2.3 Arm cavity mirror size

A common method to define the mirror size is to demand the optical power loss due to clipping (light being lost because it 'falls over the edge of the mirror') to be less than 1 ppm. The computation of the scaling factors is described in [1] and results in:

mode	LG00	LG33
mirror radius to beam radius	2.63	4.31

2.4 Minimal beam size in the arm cavities

In the case of the symmetric cavity it is easy to estimate the minimal beam size. In order to have a stable resonator the magnitude of the g-parameter must smaller than 1 ($|g| < 1$).

Using equation 5 we see that the minimal beam size occurs at $L = R_C$ and is given by $w_{\mathrm{min}} = \sqrt{\frac{L\lambda}{\pi}}$.

We can compare that to the arm cavity design of advanced detectors, for example, in the design process for Advanced Virgo, the cavity geometry has been designed to use $R_C \approx L/2$. A 2% safety margin has been assumed in order to accommodate for potential mirror surface distortions so that curvature can be approximated as $R_C = L/2 \times 1.02$. We will use the same geometry in the following for computing an 'advanced' beam size. It should be noted though that this is an example and not directly relevant for the ET design. For the advanced beam size we then get $g = 49/51$ and thus:

$$w_{\mathrm{adv}} \approx 1.9\, w_{\mathrm{min}} \tag{6}$$

2.5 ET parameters

Using the currently discussed options for ET we can compute minimal and 'advanced' beam and mirror sizes for various options:

setup	min beam radius [cm]	adv. beam radius [cm]	min mirror diameter [cm]	adv. mirror diameter [cm]
LG00, 1064nm	5.8	11.1	30.6	58.2
LG33, 1064nm	5.8	11.1	50.2	95.3
LG00, 1550nm	7.0	13.3	37.0	70.2
LG33, 1550nm	7.0	13.3	60.5	115.0

In most cases the mirror sizes following the advanced detector concept are too big. Instead the minimal sizes look interesting. It should be noted that the minimal sizes are mathematical limits to the beam diffraction and cannot be reached in practice.

2.6 Cavity degeneracy

A preliminary analysis in the stability and degeneracy similar to that shown in [2] suggests that the actual cavity parameter must not be smaller than 0.1. Taking this constraint into account gives a realistic number for the smallest possible beam size, which is about 2% larger than the minimal beam size as given in the table above.

2.7 Other design elements

A further small increase of the size on the end mirror can be expected if the cavity design is chosen to be slightly asymmetric. An increase of slightly less than 10% is reported in [2].

It should also be noted that the effects of radiation pressure on the alignment control change with the cavity geometry. No analysis for the geometries presented here has been done for ET.

2.8 Remarks on the size of beam splitters or ring-cavity mirrors

The incidence angle on the main beam splitter might be 60 (triangle) or 45 degrees. In both case the effective beam size in the horizontal plane would be larger by $1/\cos(\alpha)$, which means that with the BS close to the arm cavity input mirrors we can expect the following size requirements for the beam splitter:

setup	beam splitter diam. (45 deg) [cm]	beam splitter diam. (60 deg) [cm]
LG00, 1064nm	50	70
LG33, 1064nm	80	115
LG00, 1550nm	60	84
LG33, 1550nm	97	136

The same argument can be done for the input mirrors of ring-type arm-cavities (such as in a Sagnac interferometer). In that case the mirror sizes would be equivalent to those given for the 45 deg BS above.

This makes it likely that linear arm cavities are to be preferred and that a lens or telescope is used to reduce the beam size on the beam splitter. The latter require a large distance (>100 m) between the arm cavity and the beam splitter, see below.

2.9 Summary of arm cavity mirror sizes

Using all the above we conclude that realistic lower limits for mirror sizes are given by:

setup	mirror diameter [cm]
LG00, 1064nm	35
LG33, 1064nm	57
LG00, 1550nm	42
LG33, 1550nm	68

(These numbers basically represent a size equivalent to 112% of the minimal size given above.)

Throughout the rest of the document we will make use of the mirror and beam dimensions as they are defined for ET-D sensitivity [4] and summarised in the table below:

Parameter	low frequency IFO	high frequency IFO
wavelngth [nm]	1550	1064
beam profile	LG00	LG33
mirror diameter [cm]	42 (or larger)	62
beam radius [cm]	8.0	7.25

3 Arm cavity design

Taking the beam parameters computed in section 2 we can compute the arm cavity parameters. For the moment we restrict our design to a symmetric cavity. An optimised design might lead to a slight deviation from the symmetry as in the case of the Advanced detectors, however, this does not affect the design significantly and can be ignored for the time being.

For reasons described further below we compute the beam parameters for potential arm cavity length of $L = 10\,\mathrm{km}$, $L = 9.3\,\mathrm{km}$ and $L = 8.6\,\mathrm{km}$[1].

3.1 Interferometer with a wavelength of 1064 nm

A mirror size of 62 cm results in a beam radius of 7.25 cm for a LG33 mode. Thus we get the following cavity and beam parameters:

$L = 10\,\mathrm{km}$				
R_C	w_0	z_0	w	z_R
5643 m	2.46 cm	5000 m	7.25 cm	1793 m
$L = 9.3\,\mathrm{km}$				
R_C	w_0	z_0	w	z_R
5148 m	2.27 cm	4650 m	7.25 cm	1521 m
$L = 8.6\,\mathrm{km}$				
R_C	w_0	z_0	w	z_R
4680 m	2.08 cm	4300 m	7.25 cm	1279 m

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[1] Note that the minimal beam sizes and corresponding mirror size for smaller cavity length should be re-computed. However, pending a final optimisation it is prudent to use mirror sizes which can be used in a 10 km scenario.

3.2 Interferometer with a wavelength of 1550 nm

In case of the 1550 nm wavelength and a beam size for a LG00 mode of 8 cm, the same computations as above then give:

$L = 10\,\mathrm{km}$				
R_C	w_0	z_0	w	z_R
6109 m	3.41 cm	5000 m	8.0 cm	2354 m
$L = 9.3\,\mathrm{km}$				
R_C	w_0	z_0	w	z_R
5489 m	3.11 cm	4650 m	8.0 cm	1964,m
$L = 8.6\,\mathrm{km}$				
R_C	w_0	z_0	w	z_R
4918 m	2.84 cm	4300 m	8.0 cm	1630 m

3.3 Recycling cavity design

In order to simplify the design of the central interferometer (main beam splitter and recycling mirrors) we would like to have small beam sizes and a stable, non-degenerate cavity[2] preferably without the need of extra optics. This can be facilitated by moving the input mirrors several hundred meters away from the beam splitter and using a lens directly outside the ITM (or using the substrate of the ITM as a lens by having a curved AR surface).

3.4 Astigmatism due to the beam splitter

From the design of the advanced detectors it is known that the astigmatism introduced by the main beam splitter can significantly decrease the contrast of the interferometer. A simple calculation to estimate the order of magnitude of this effect can be performed as follows: the arm cavity eigenmode is propagated to and then through the beam splitter. This produces a mode with different beam parameters for the sagittal (q_s) and tangential (q_t) plane. The astigmatism for a LG00 mode can then be estimated as:

	1064 nm	1550 nm
c_astigm	4.6%	3%

The LG33 mode is expected to be much more fragile with respect to astigmatism than the LG00.

These numbers can be improved for example by reducing the beam size on the BS or by adding additional lenses. A full optimisation needs to take into account astigmatism, thermal noise (especially thermo-refractive noise) and optical aberration of the focusing elements at the same time. This type of optimisation will be done at a later stage. The setup detailed here provides a good starting point for such an investigation.

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[2] Note that the argument for a non-degenerate cavity will be made elsewhere.

4 Optical layout summary

4.1 Layout option for LG00, 1550nm

The optical parameters of a possible solution based on a arm cavity length of $L = 9.3\,\mathrm{km}$ and a LG00 mode at 1550 nm are provided below:

- focussing element in or near the ITM with a focal length of $f = 685\,\mathrm{m}$
- distance ITM-BS: 700 m
- distance BS-MPR: 10 m
- beam size on BS: 0.95 cm
- beam size on MPR: 0.86 cm
- Rayleigh range in central interferometer: 47.0 m

The recycling cavity formed by MPR and ITM has a length of 710 m and a FSR of 211 kHz. The round-trip Gouy phase is given by ≈ 9.7 deg which corresponds to a mode separation frequency of 11 kHz.

4.2 Layout option for LG33, 1064nm

Using the same distances and focussing elements for the interferometer with a LG33, 1064 nm beam, we also obtain reasonable numbers:

- beam size on BS: 0.89 cm
- beam size on MPR: 0.81 cm
- Rayleigh range in central interferometer: 40 m
- Gouy phase: 7.6 deg
- mode separation frequency: 9 kHz

These layout options are not yet optimised in any way but they show that a separation between beam splitter and input optics in the order of 700 m represents a useful baseline. The numbers for the beam sizes at the beam splitter and recycling mirrors in both cases need to be checked against a detailed thermal noise computation.

5 Optics Parameters

Figure 1 shows a simplified overview of the core interferometer optical layout for the low-frequency and high-frequency interferometers of ET-D. Table 1 gives an overview of the optical parameters of the two main interferometers used to compute the ET-D sensitivity curves.

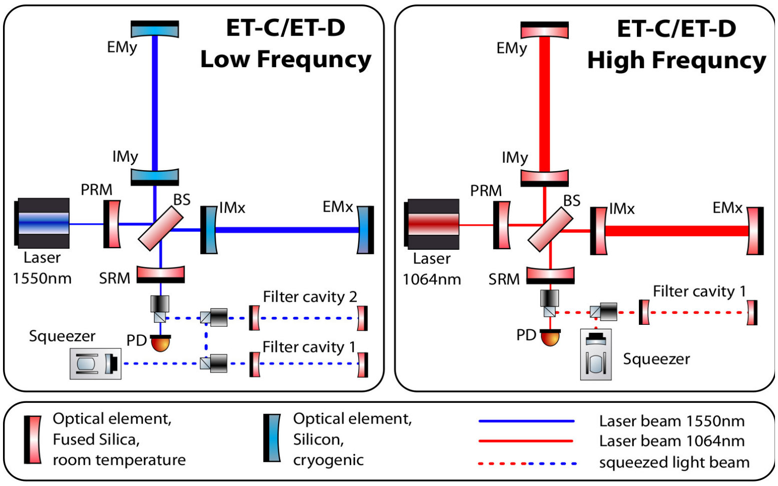

Figure 1: Simplified overview of the core interferometer optical layout for the low-frequency and high-frequency interferometers of ET-D.

Parameter	ET-HF	ET-LF
Input power (after IMC)	500 W	3 W
Arm power	3 MW	18 kW
Mirror material	Fused Silica	Silicon
Mirror diameter / thickness	62 cm / 30 cm	62 cm / 30 cm (or 42/43)
Mirror masses	200 kg	211 kg
Laser wavelength	1064 nm	1550 nm
ITM transmittance	7000 ppm	7000 ppm
ETM transmittance	10 ppm	10 ppm
PR transmittance	0.046	0.046
arm round-trip loss	75ppm	75ppm
SR-phase	tuned (0.0)	detuned (0.6)
SR transmittance	10 %	20 %
Beam shape	LG_{33}	TEM_{00}
Beam radius	7.25 cm	8 cm

Table 1: Summary of the most important parameters optical components of the main interferometers, excluding the filter cavities.

References

[1] S. Chelkowski, S. Hild and A. Freise: 'Prospects of higher-order Laguerre-Gauss modes in future gravitational wave detectors' *Physical Review D* **79** 122002 (2009). 2

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[3] S.Hild, S.Chelkowski, A.Freise, J.Franc, N.Morgado, R.Flaminio and R.DeSalvo: "A Xylophone Configuration for a third Generation Gravitational Wave Detector" *CQG* **vol. 27**, pp015003 (2010).

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