European Commission
FP7, Grant Agreement 211743

Substrate Thermo-Refractive Noise for future cryogenic gravitation wave detector

ET-00095A-10

J. Franc, J. Degallaix, R. Flaminio

Issue: 1
Date: September 21, 2010

Laboratoire des Matériaux Avancés-CNRS
7 rue Pierre de coubertin
69622 Villeurbanne

ET -Einstein gravitational wave Telescope -Design Study * A joint European Project
Web: http://www.et-gw.eu Email: EuropeanProjectsService@ego-gw.it

Contents

Abstract

This work reports about the substrate thermo-refractive noise contribution for the future ET interferometer. In this note, the results for sapphire and silicon substrates at low temperatures are discussed.

1. Substrate Thermo-Refractive Noise: literatures

Two references can be mentioned for the substrate thermo-refractive noise formula. The first one was the formula given by S. Rao in his thesis [1]. Unfortunately, this formula is neither demonstrated nor referenced. An other formula was given by Braginsky et al. in 2004 [2]. This second formula is well detailed in the paper and thus, has been chosen for simulations of this note. The expression is:

$$\varphi_n^2 = (kl\beta)^2 \frac{4k_b T^2 \kappa}{(\rho C)^2 l_c} \int_0^\infty \frac{k_\perp dk_\perp}{2\pi} e^{-\frac{w^2 k_\perp^2}{2}} \frac{k_\perp^2}{(2\pi f)^2 + a^4 k^4} \qquad (1.1)$$

With $k = \frac{2\pi}{\lambda}$, $a^2 = \frac{\kappa}{\rho C}$ and $k_\perp^2 = k_x^2 + k_y^2$

l_c is the thickness of the input mirror

β is the thermo-optic coefficient of the substrate material

T is the temperature

k_b is the Boltzmann's constant

κ is the thermal conductivity

C is the specific heat

ρ is the density of the substrate material

w is the size of the beam

This formula can be simplified by the expression:

$$\varphi_n{}^2 = \frac{4\beta^2 k^2 l_c T^2 k_b \kappa}{(\rho C)^2 \pi w^4 (2\pi f)^2} \text{ for adiabatic case, i.e., for } 2\pi f >> \frac{\kappa\omega^2}{\rho C} \tag{1.2}$$

In the paper of Braginsky, the thermo refractive noise is demonstrated for a type of cavity based on 2 prisms. In the following section, we propose to discuss the formula in order to be adapted for the cavities of the future ET telescope.

2. Expression for the Substrate Thermo-Refractive Noise

The expression of the thermo refractive noise for this note considers the interferometer as a whole. In this calculation we assume that the future ET telescope will use Fabry-Perot cavities in the arms; we will refer to the two cavity mirrors as the input mirror and the end mirror. The light is trapped between the two mirrors and makes several round trips inside the cavity. The input mirror has a finite transmission for light to get in and out. The thermal noise must be, therefore, multiplied by a factor 2 corresponding to its backward and forward propagation in the input mirror.

Then, because the interferometer is composed of two independent arms, the formula must be multiply by $\sqrt{2}$ as: $\sqrt{(\varphi_1)^2 + (\varphi_2)^2}$. The two arms having the same length and configuration: $\varphi_1 = \varphi_2$

The refractive thermal noise can be expressed as follow:

$$\Delta\varphi_{TR} = 2\sqrt{2}\varphi_n \tag{2.1}$$

The phase due to the gravitational waves signal is expressed as:

$\Delta\varphi_{signal} = TF(f)hL$ with h the strain sensitivy in $\sqrt{Hz}$ and L is the length of the interferometer arms. The factor TF(f) is dependent on the finesse of the cavity F, the wavelength of the light λ and the cavity cutoff frequency fc.

$$TF(f) = \frac{4\pi}{\lambda}\frac{2F}{\pi}\frac{1}{\sqrt{1+(f/fc)^2}} \tag{2.2}$$

With fc:

$$fc = \frac{c}{2L}\frac{1}{2F} \tag{2.3}$$

Finally, the signal recycling is not taken into account in the formula because it takes place in the both phase fluctuations $\Delta\varphi_{GW}$ and $\Delta\varphi_{TR}$ and is thus cancelled.

In order to get a signal:

$$\Delta\varphi_{GW} > \Delta\varphi_{TR} \tag{2.4}$$

Therefore,

$$h > 2\sqrt{2}\varphi_n\sqrt{1+(f/fc)^2}\,\frac{1}{L}\frac{\lambda}{8F} \tag{2.5}$$

Using the general expression of φ_n, we finally show that strain sensitivity of the thermo refractive noise is equal to :

$$h > 2\sqrt{2}\sqrt{1+(f/fc)^2}\,\frac{1}{L}\frac{\lambda}{8F}\sqrt{(kl\beta)^2\frac{4k_bT^2\kappa}{(\rho C)^2 l_c}\int_0^\infty \frac{k_\perp dk_\perp}{2\pi}e^{-\frac{R_b^2k_\perp^2}{2}}\frac{k_\perp^2}{\omega^2+a^4k^4}} \tag{2.6}$$

In the next section, at low temperature, the heat diffusion length increases and becomes larger than the laser beam size, hence the adiabatic approximation is no longer valid ($2\pi f << \frac{\kappa\omega^2}{\rho C}$). Therefore, to evaluate the non-adiabatic case, the precise formula 2.6 is taken into account.

3. Substrate's mirror properties

In this note, two materials have been considered for the mirror substrates: sapphire and silicon. These two materials have been first selected for a future cryogenic interferometer [3]. Sapphire offers a perfect transparency at 1064 nm and a very low dn/dT. Moreover, cooled sapphire payloads have been already studied in Japan as part of a proposal to construct the large-scale cryogenic gravitation wave telescope (LCGT) [4].

Silicon substrate is an alternative to sapphire at low temperature and has by now demonstrated very good properties in a cryogenic environment. A thermal expansion that crosses zero around 18K, low mechanical losses, and other performances comparable to the ones of the sapphire substrates. However, the silicon wavelength needs to be changed from 1064 nm to 1550 nm to get a very good transparency. It leads to a thicker coating and a most important source of noise [4]. The coating for Sapphire is a multilayer $(HL)_{17}$ HLL coating made of Ti:Ta_2O_5 (defined by H for High refractive index material) and SiO_2 (defined by L for Low refractive index material) quarter wavelength layers. It corresponds to a transmission of 6 ppm. In the case of Silicon the multilayer is a $(HL)_{19}$HLL to get the same transmission. The mirror, studied in this note at 10K, has a diameter of 45 cm and a thickness of 30 cm. These dimensions have been proposed by Stefan Hild in [5] as a plausible configuration for the future Einstein Telescope. For a first proposition, the shape of the beam is a Gaussian beam with a width of 8.65 cm that involve 1 ppm diffraction losses on the mirror. Because the formula of the thermo-refractive noise has not been studied, at our knowledge, for a finite case, all the simulations in this note have been done for an infinite case. The table 1 below considers the configuration of mirror (dimension, environment, etc).

Substrates	Silicon @ 1550 nm or Sapphire @ 1064 nm
Mirror's dimensions	Diameter : 45 cm Thickness : 30cm
Shape and size of the beam	LG00 8.65 cm for 1 ppm diffraction losses
Temperature	10K
Coating	SiO_2-TiTa_2O_5 $(HL)_{19}$HLL for Silicon $(HL)_{17}$HLL for Sapphire

Table 1.Mirror configuration

The table 2 below summarizes the present knowledge of all the relevant parameter's values at 10K and at the working wavelength (see [4] for more details) of the silicon and sapphire substrate.

	Silicon	Sapphire
Loss angle	$1\ 10^{-9}$	$4\ 10^{-9}$
Density (kg.m-3)	2331	3997 @ 20K
Thermal conductivity (W.m-1.K-1)	2330	1500 @ 12.5 K
Specific heat (J. K-1. Kg-1)	0.276	0.0934
Thermal expansion coef. (K-1)	$4.85\text{x}10^{-10}$	$5.3\text{x}10^{-10}$
Thermo optic coef.	$5.8\text{x}10^{-6}$ @ 30K	$9\text{x}10^{-8}$
Young's modulus (GPa)	162.4	464
Poisson's ratio	0.2205 @ 30K	0.23 (estimated)
Refractive index	3.45 @ 30K	1.75

Table 2. List of the values of materials substrates parameters at low temperature and good working wavelength

In the table 2, we can remark that some parameters are unknown at 10K. In this case, the temperature is specified. It is notably the case for the thermo optic coefficient. This parameter plays an important role in the thermo refractive formula, unfortunately, at our knowledge, it exists no literature with the thermo optic coefficient at 10K. Nevertheless, an extrapolation can be performed thanks to the formula:

$$n^2(\lambda,T)-1=\sum_{i=1}^{3}\frac{S_i(T)\lambda^2}{\lambda^2-\lambda_i^2(T)} \qquad (2.1)$$

given in [6]. The formula being correct until 20K, figure 1 shows the thermo-optic coefficient from 20K to 290K at 1.5 µm. For the sapphire, different values have been given in the literature. The Japan group [7] found a very low thermo-refractive coefficient of $9\ 10^{-8}$. This value has been used to do calculation in this note. Other literature [8] gives also some point for the dn/dT of the sapphire substrate. Thanks to an interpolation of these 6 points it is possible to get the data from 25K to 300K. At 25K the dn/dT of the sapphire is around 10^{-7}. It confirms the value mentioned by [7].

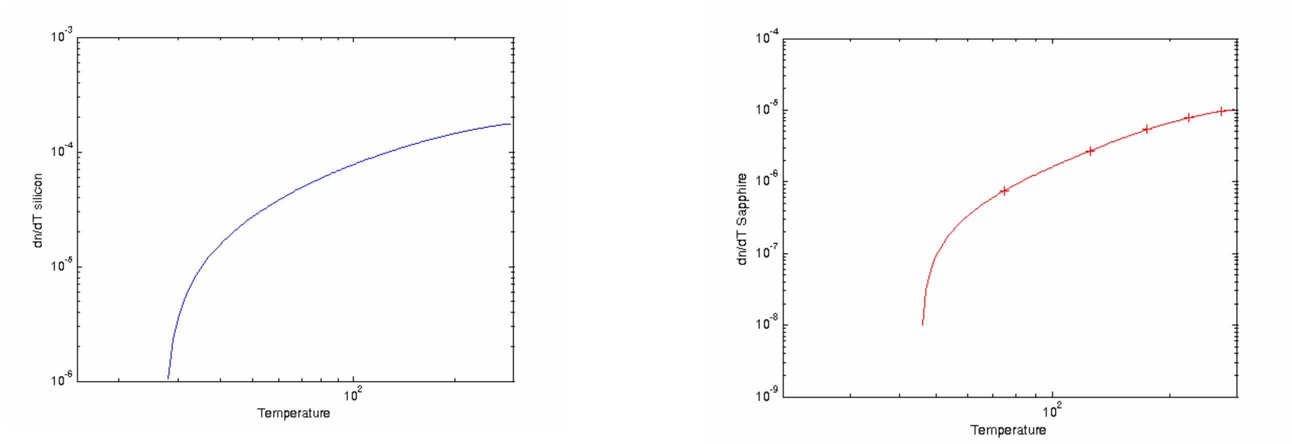

Figure 1. Thermo-optic coefficient of Silicon (a) [6] @ 1.5 µm and Sapphire (b) [8] @ 1.064 µm from 30 to 290K

4. Simulations

Figure 2 shows the evolution of total thermal noises on silicon and sapphire without and with the adiabatic assumption according to the formula of φ_n 1.1 and 1.2 respectively explained in section I of this note. The continuous lines use the standard equation referred by [2] for the interferometer, it correspond to the adiabatic approximation. The dashed lines, on contrary, refer to the non-adiabatic case. Without adiabatic assumption the thermo-refractive noise tends to decrease at low frequencies. At higher frequency, the two spectral densities become approximately the same. The results in figure 2 show, finally, that from the point of view of substrate thermo-refractive noise, sapphire is clearly better at cryogenic temperatures. Moreover, for silicon substrate, the thermo-refractive noise is above the ET sensitivity target.

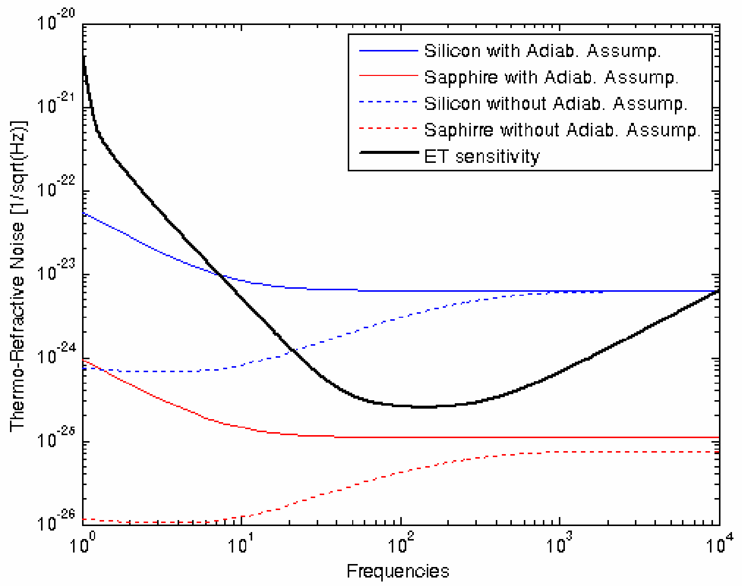

Figure 2. Evaluation of substrate thermo-refractive noise with and without adiabatic assumption

Figure 3 shows the evolution of substrate thermo-refractive for silicon for different temperatures 10K, 20K, 30K, 40K. The thermo-optic coefficient has not been changed. Only the specific heat and the thermal conductivity, which are two parameters that exhibit important variation at low temperatures, can modify the formula. The substrate thermo refractive noise decreases when the temperature increases. At 40K, this latter is even below the sensitivity target.

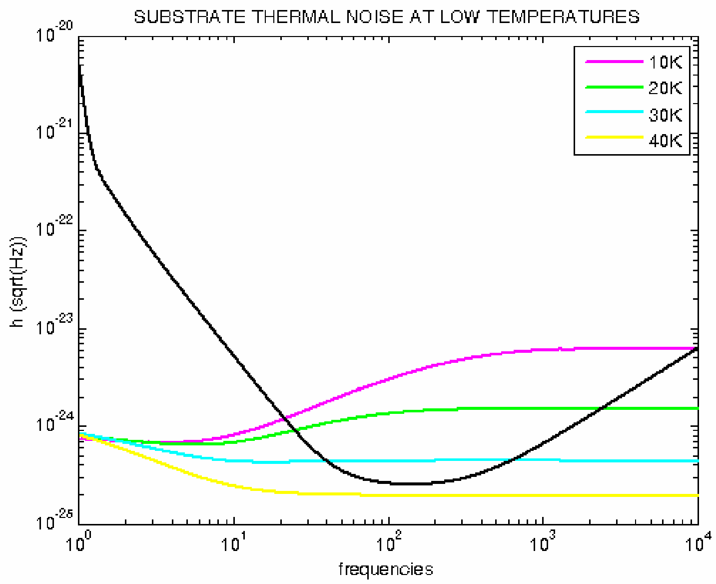

Figure 3. Silicon substrate thermo refractive noise at different cryogenic temperatures: 10K, 20K, 30K, 40K

However, if the silicon thermo-refractive noise tends to decrease when the temperature increases, it is not the case for all the thermal noises considered. Other parameters values vary with temperature as loss angles, thermal expansion coefficients, etc. These modifications are not necessarily favourable for the final total thermal noise. For example, the figure 4 shows the evolution of the mechanical losses angles of the two coating materials: $TiTa_2O_5$ and SiO_2 studied from 0 to 300K [9]. For $TiTa_2O_5$, we can observe that loss angles attain a peak around 20K and then, decreases from 20 to 300K. For SiO_2 layer, the value is stable from 5 to 30 K and then, tends to decreases from 5 to 0.5 10^{-4} at 300K.

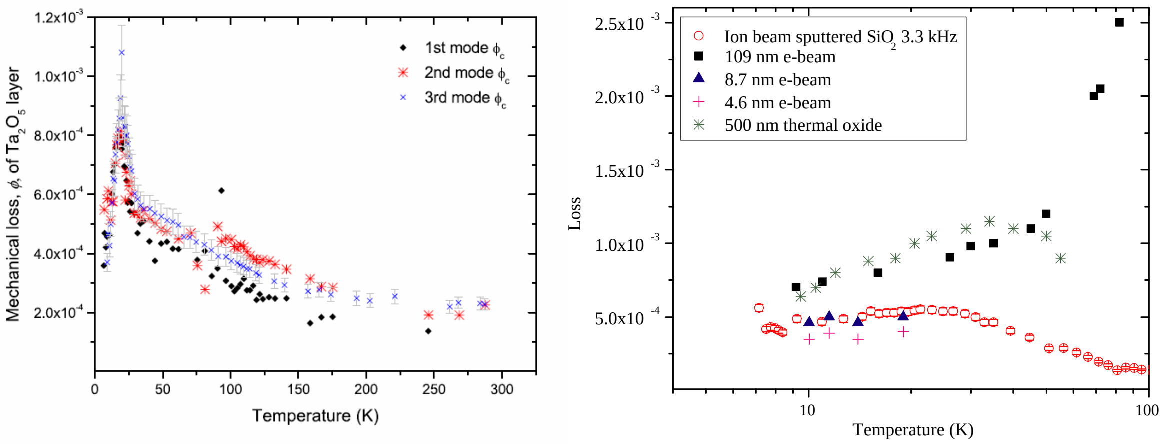

Figure 4. Temperature dependence of the measured mechanical loss of $TiTa_2O_5$ and SiO_2 thin films

We propose to take into account the realistic data of the mechanical loss angles of each coating material for all the coating thermal noise and check the evolution of the total thermal noise. Figure 5 represents the total thermal noise at different cryogenic temperatures (10K, 18K, 30K, 40K).

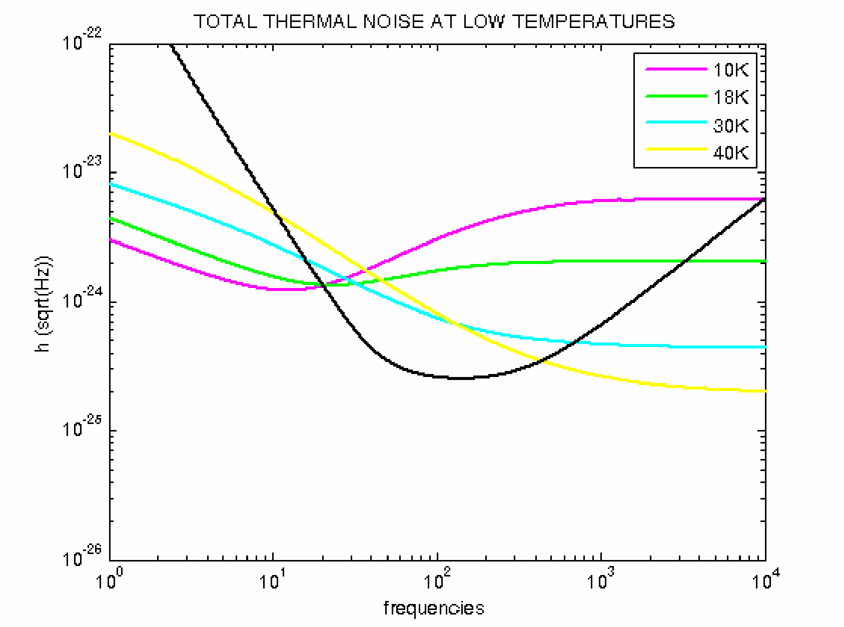

Figure 5. Total thermal noise at different cryogenic temperature for an interferometer using silicon substrates

We remark that from 10K to 40K the total thermal noise is always above the ET sensitivity target. Indeed different thermal noise can limit the total thermal noise because of the parameters values changing with temperature. In fact, at 10K and 18K, the substrate thermo refractive noise and the coating Brownian noise limit the total thermal noise. At 30K and 40K, the noise is limited by the Substrate thermo elastic and thermo refractive noise. The contribution of the different noises at different temperatures is given in the appendix.

5. Conclusion

Silicon and Sapphire substrates are two promising solutions proposed for future cryogenic interferometers based on Fabry-Perot cavities. However, this note has demonstrated that Silicon can show some disadvantages considering all its parameters at low temperature. Notably, the thermo-optic coefficient is, in the literature, more important than the one of Sapphire. The silicon thermo refractive noise appears as a limiting thermal noise from 10 to 20K. If the simulation questions silicon about its competences, a doubt can be still discussed concerning the reality of the value of dn/dT at 10K. In the literature, this value is only given at 30K and in this note, we have considered that this latter was the same at 10K. But this latter could be also lower than the one given at 30K if we go by the last dn/dT study. Henceforth it is necessary to confirm this value which casts doubt on Silicon substrate.

References

1. Rao, S., *Mirror thermal noise in interferometric gravitational wave detectors*. 2003, Caltech.

2. V.B. Braginsky and S.V. Vyatchanin, *Corner reflectors and quantum-non-demolition measurements in gravitational wave antennae.* Physics Letters A, 2004. **324**: p. 345-360.

3. M. Punturo, M. Abernathy, F. Acernese, B. Allen, et al., *Third generation of gravitational wave observatories and their science reach.* Class. Quantum Grav., 2010. **27**(8).

4. J. Franc, N. Morgado, R. Flaminio, R. Nawrodt, et al., *Mirror thermal noise in laser interferometer gravitational wave detectors operating at room and cryogenic temperature*, in *ET-021-09*, E. Note, Editor. 2009.

5. S. Hild, A. Freise, S. Chelkowski, J. Franc, et al., *A xylophone configuration for a third-generation gravitational wave detector.* Class. Quantum Grav. , 2010. **27**.

6. B.J. Frey, D.B. Leviton, and T. J. Madison, *Temperature-dependent refractive index of silicon and germanium.* SPIE, 2006. **6273**.

7. T. Tomaru, T. Uchiyama, C.T. Taylor, S. Miyoki, et al., *Temperature coefficient of refractive index of sapphire substrate at cryogenic temperature for interferometric gravitational wave detectors.* 2000.

8. S.G. Kaplan and M.E. Thomas, *Measurement of the o-ray and e-ray infrared refractive index and absorption coefficient of sapphire from 10K and 295K.* SPIE, 2002. **4822**.

9. I. Martin, H. Armandula, c. comtet, M. Fejer, et al., *Measurements of a low-temperature mechanical dissipation peak in a single layer of Ta2O5 doped with TiO2.* Class. Quantum Grav., 2008. **26**(5).