Black Holes Sing Their Past

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B.S. Sathyaprakash
School of Physics and Astronomy
CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD

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  - $v/c \sim 10^{-4}$ in binary pulsars

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- Perturbed black holes, on the other hand, are truly fascinating
  - They could be source of extremely luminous radiation, far exceeding the luminosity in light of all the stars in the Universe

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$$h(t) = A\,\frac{M}{r}\,\exp(-t/\tau)\,\cos(\omega t + \varphi_0)$$

† Amplitude A depends on the nature of perturbation

† r is the distance to the black hole

† ω and τ are the mode frequency and damping time

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- Absence of quasi-normal modes after merger might indicate failure of GR or existence of naked singularities

Typical Values of the Dominant Mode

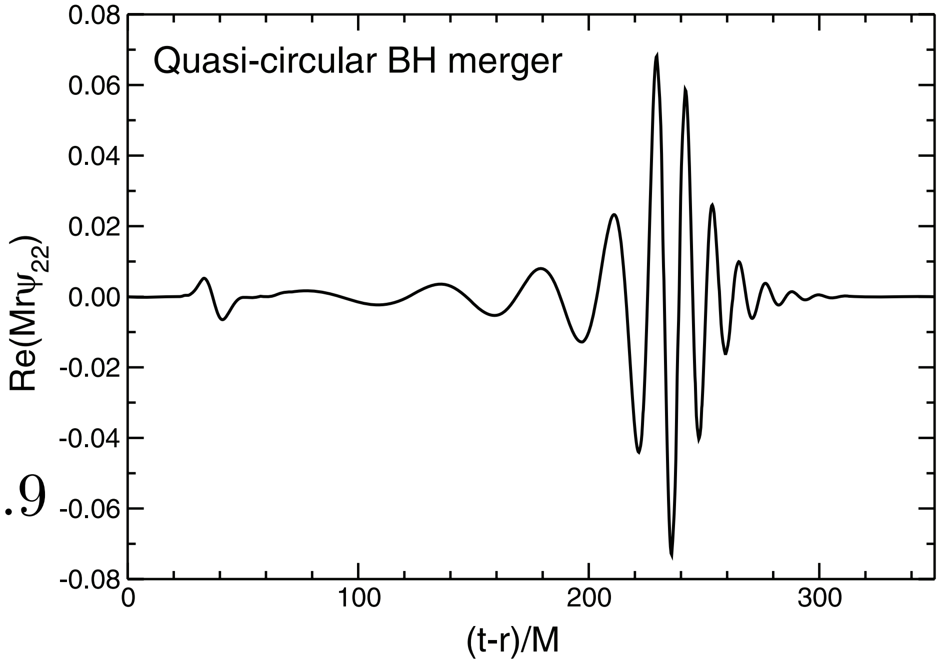
- Mode frequencies are inversely proportional to BH mass and decay times directly proportional to it
- Gravitational waves being quadrupolar the most dominant mode excited is $l = 2$
- The frequency and the decay time of the 22 mode (i.e. l=2, m=2) is

$$f = 1.2\,\mathrm{Hz}\,\left(\frac{10M_\odot}{M}\right)$$

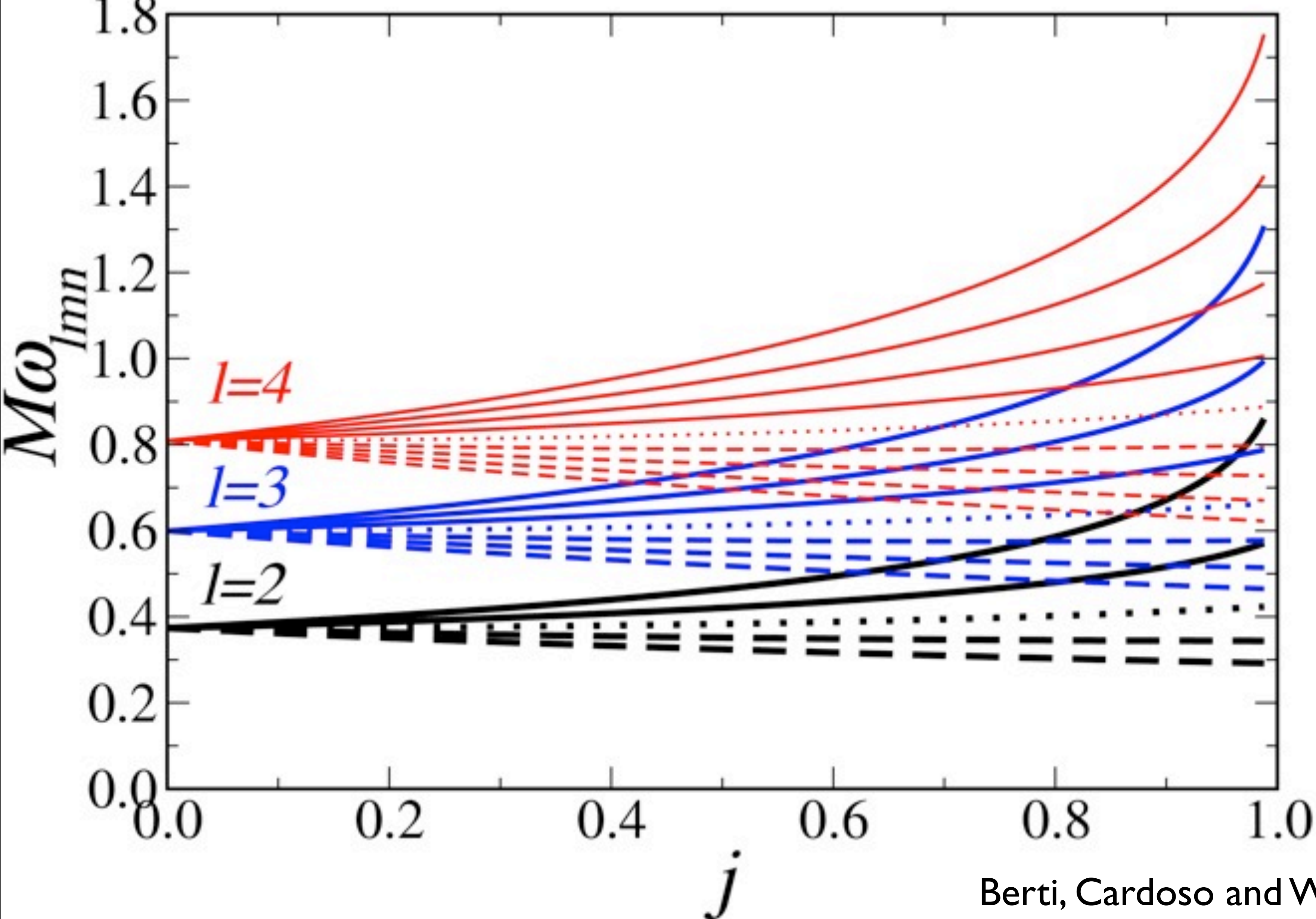
$$\tau = 0.55\,\mathrm{ms}\left(\frac{M}{10M_\odot}\right)$$

$$Q = \tfrac{1}{2}\tau\omega \sim 2$$

$$f = 2.0\,\mathrm{kHz}\ \text{ and } Q = 5 \text{ for } j = 0.9$$

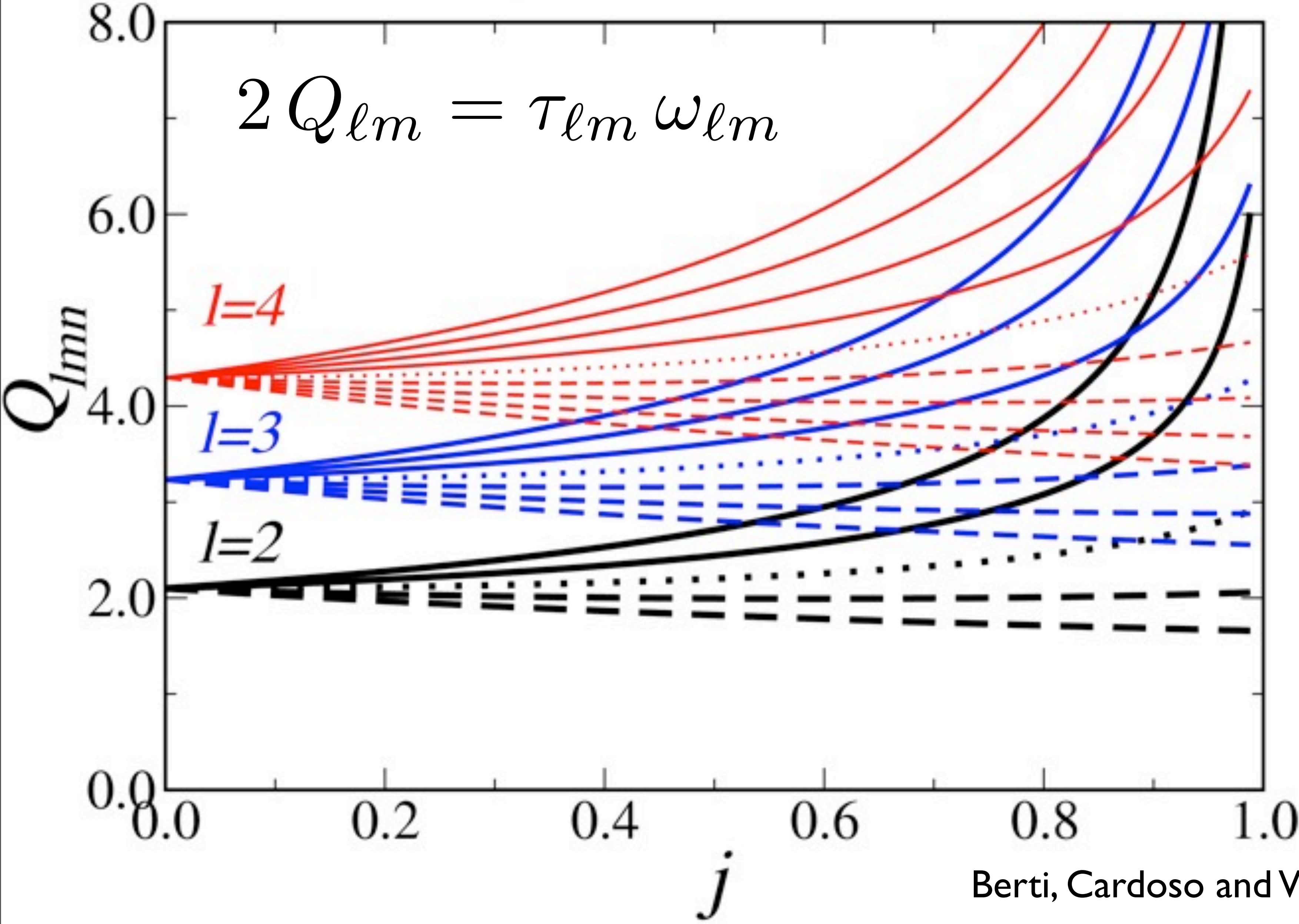


Frequency of quasi normal modes



Berti, Cardoso and Will

Quality Factor of QNMs



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Can Quasi-normal Modes Reveal Their Perturber?

- If mode frequencies depend only on the black hole's mass and spin how can they reveal what caused the perturbation?
  - No-hair theorem really doesn't apply to deformed BHs
  - Should be possible to measure not just BH mass and spin but also, for instance, the mass ratio of the progenitor binary from the QNMs produced in the aftermath of merger
- They key is that the amplitude of the modes carry additional information
  - They depend on the nature of the perturber

$$h^{+}_{lm} - i h^{\times}_{lm} = \frac{A_{lm} M}{r} \, e^{i\omega_{lm}} e^{-t/\tau_{lm}}$$

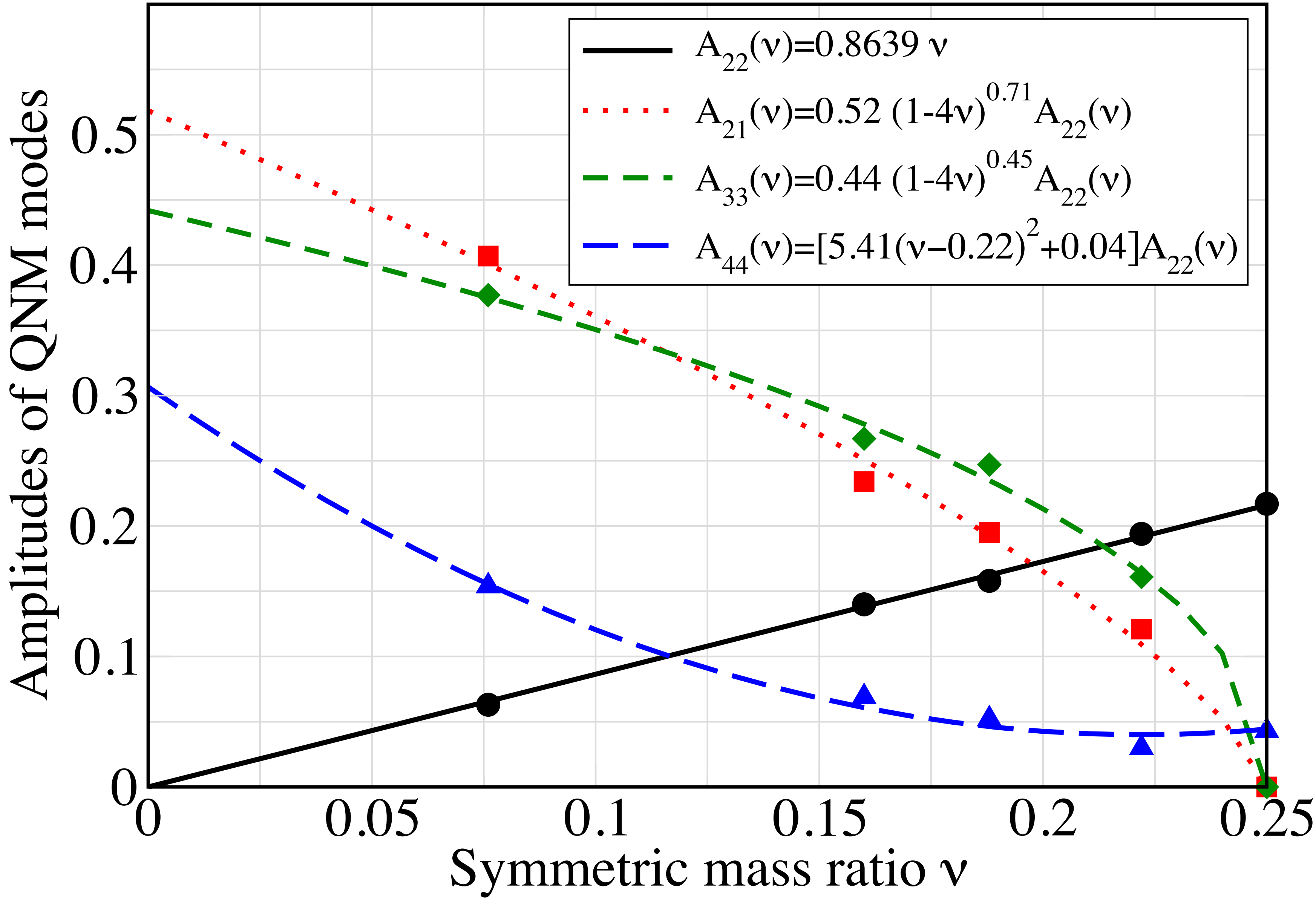
- If we have observe only one mode then the amplitude would be degenerate with other parameters - distance to the black hole, its location on the sky, etc.
  - Observing higher order modes should help break the degeneracy

No analytical approach is known to compute the relative amplitudes of modes excited during a merger

A complete model of the ringdown signal would require high-accuracy merger simulations

We carried out a large number of numerical simulations to understand the relation between progenitor parameters and ringdown amplitudes

Dependence of Amplitudes on Mass Ratio



Waveform as seen by a detector

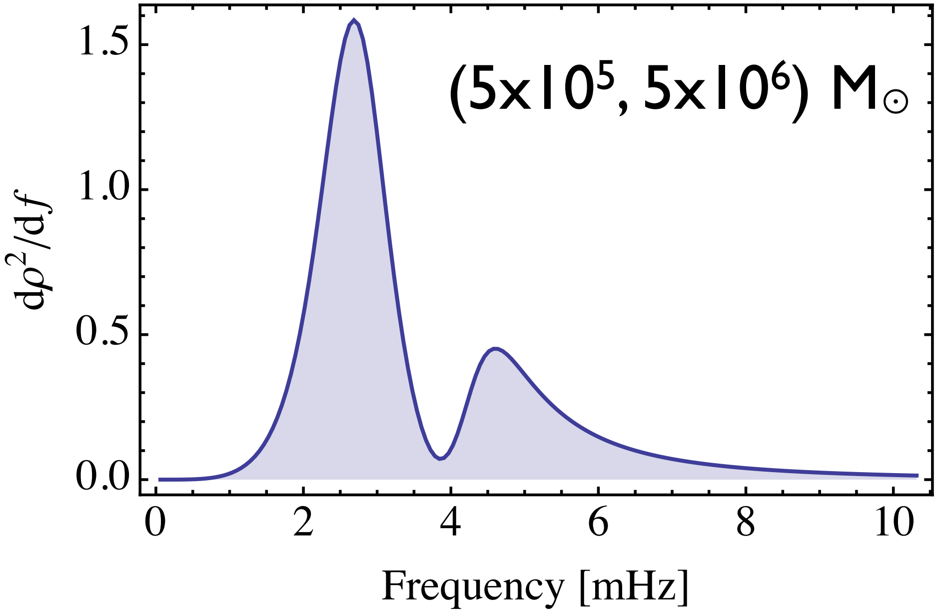
$$h_+(t) = \sum_{\ell,m>0} \frac{\alpha_{\ell m} M}{D_\mathrm{L}} Y_+^{\ell m}(\iota) e^{-t/\tau_{\ell m}} \cos(\omega_{\ell m} t - m\phi),$$

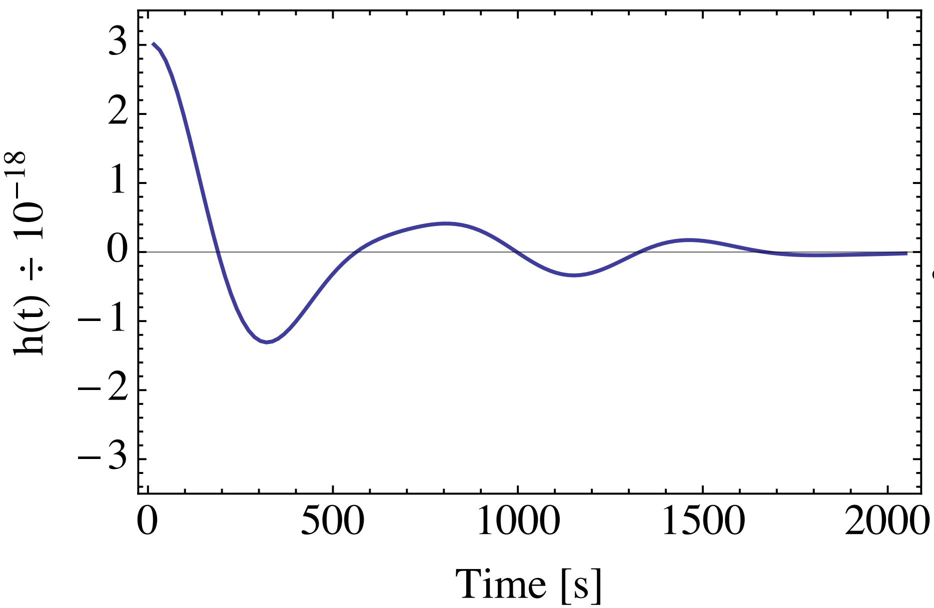
$$h_\times(t) = \sum_{\ell,m>0} \frac{\alpha_{\ell m} M}{D_\mathrm{L}} Y_\times^{\ell m}(\iota) e^{-t/\tau_{\ell m}} \sin(\omega_{\ell m} t - m\phi).$$

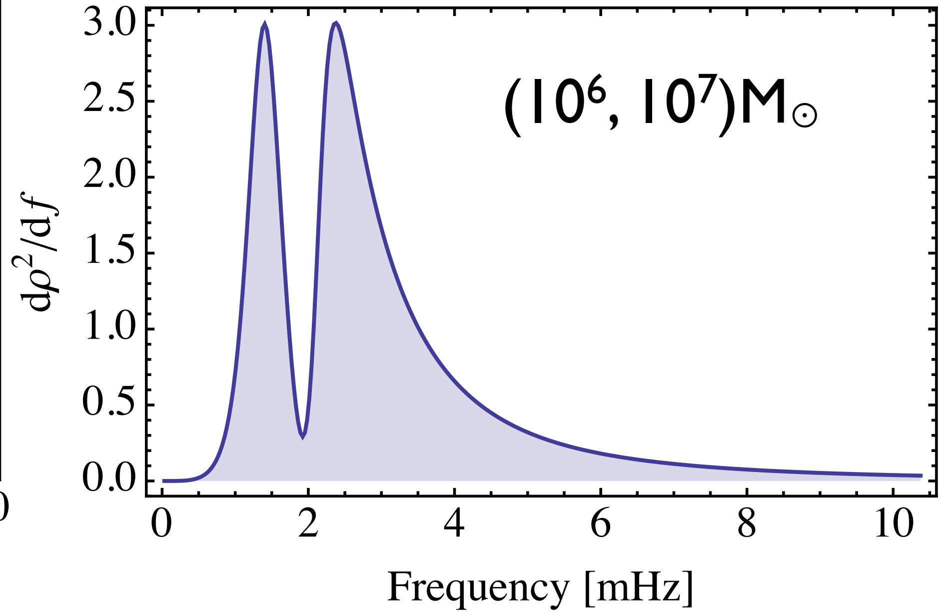
$$h^A(t) = F_+^A(\theta, \varphi, \psi) h_+(t) + F_\times^A(\theta, \varphi, \psi) h_\times(t).$$

Quasi-Normal Modes in LISA

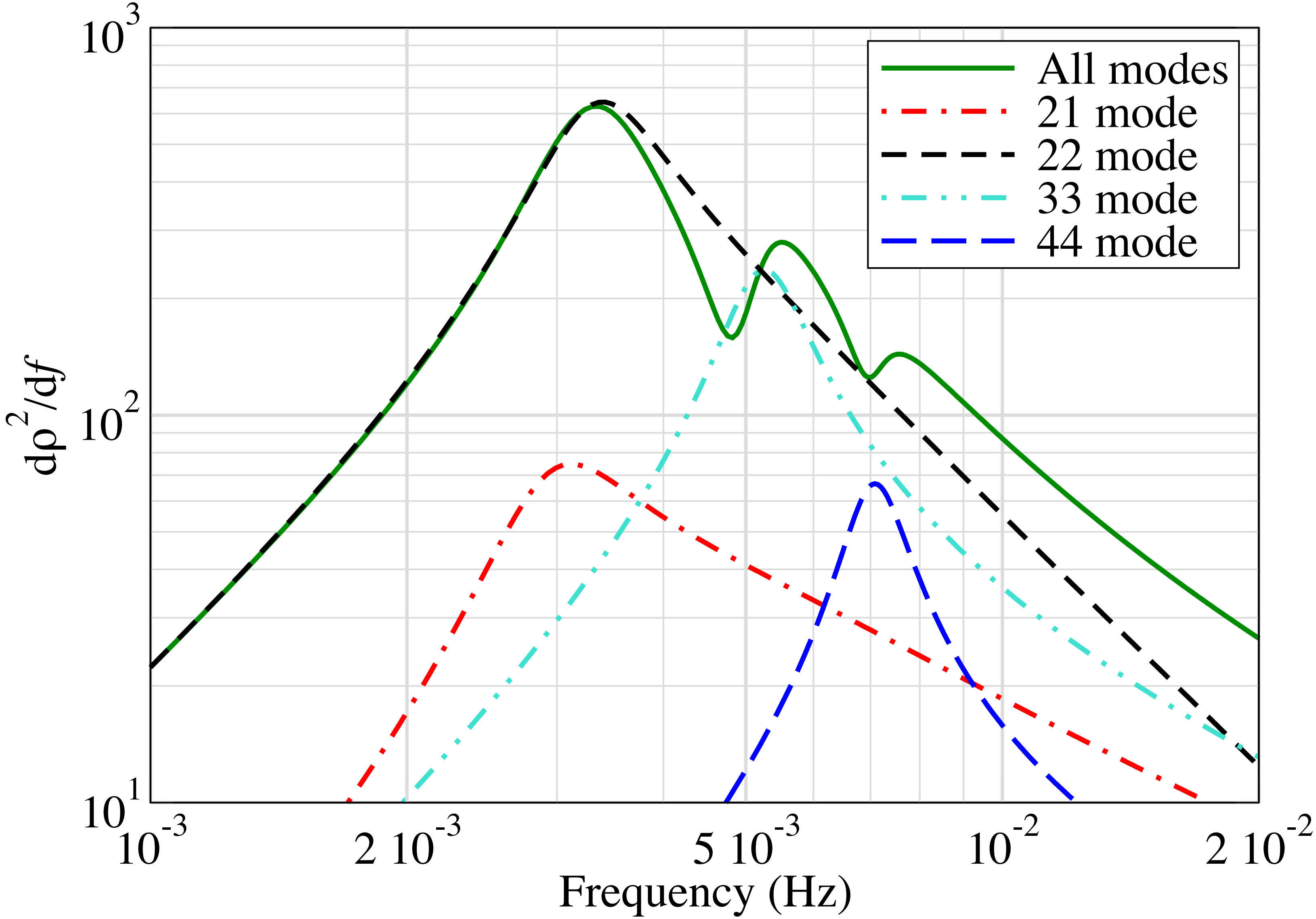
- Depending on the mass of the black hole and mass ratio of the progenitor one or more modes could be visible



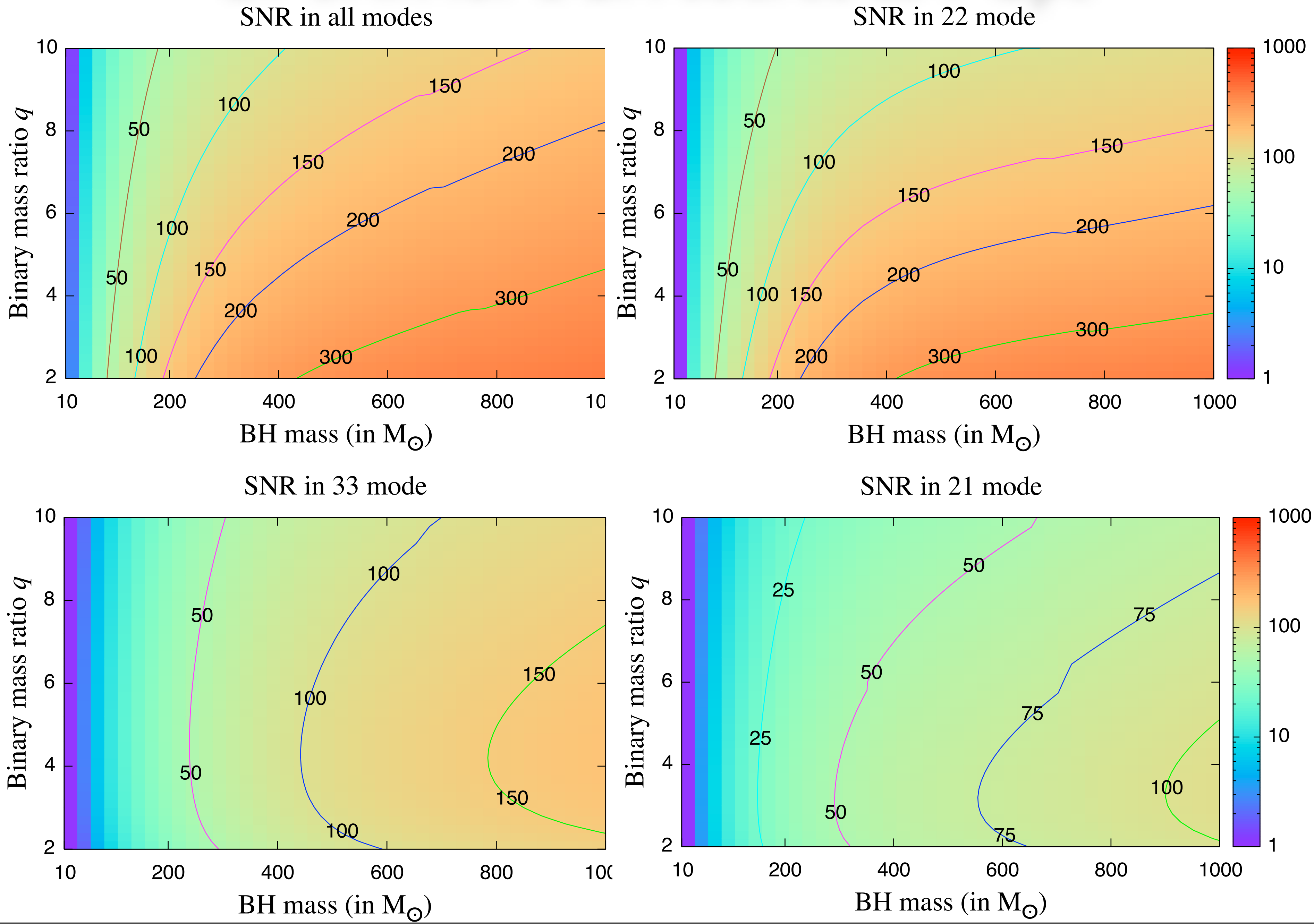




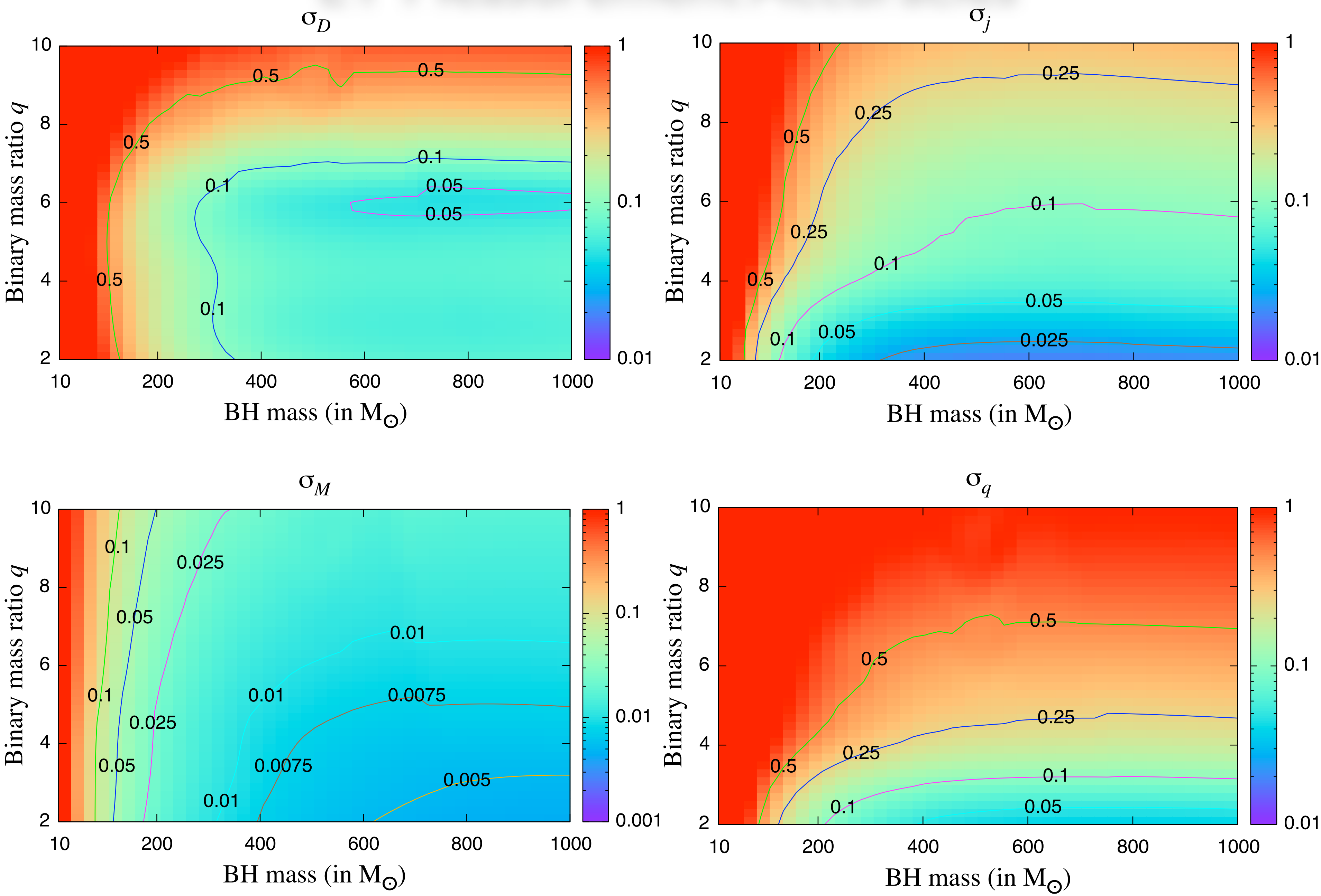
Spectrum with Four Modes



ET SNR for a BH source at 1 Gpc



ET Measurement Accuracies



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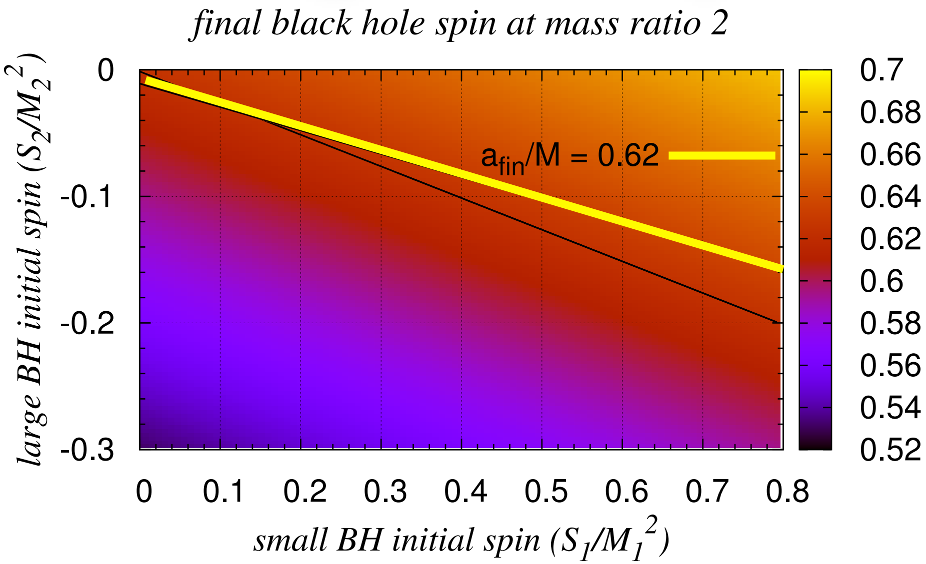
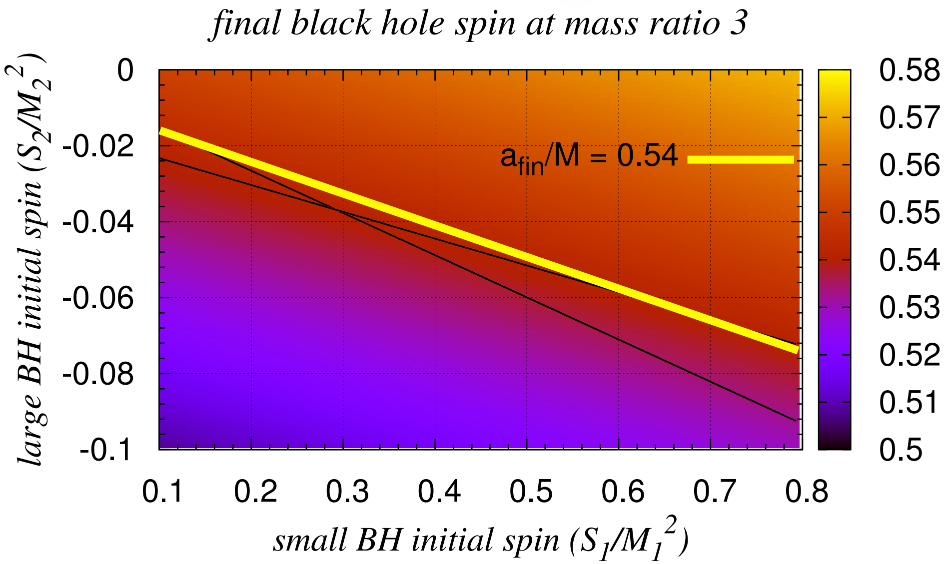
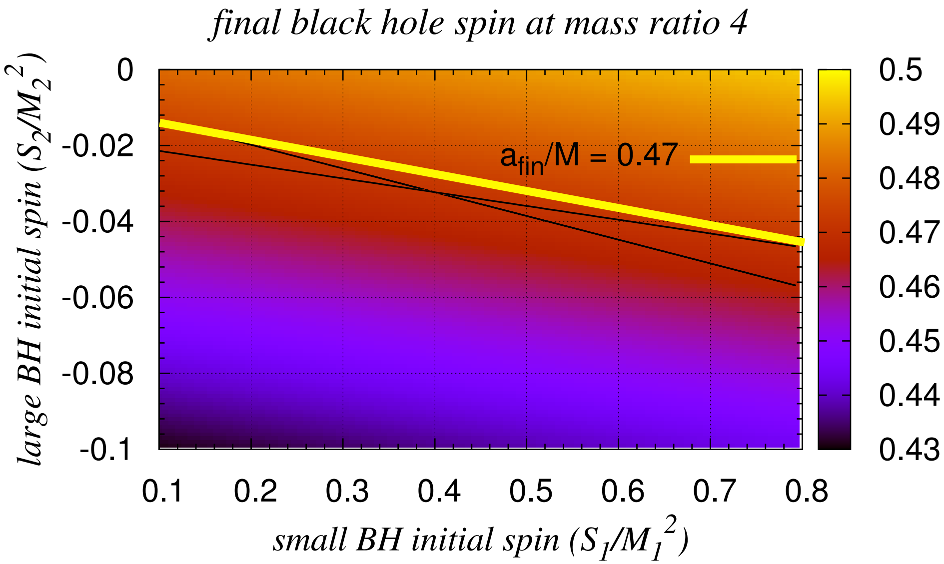
  - In the case of spinning black hole binaries our model of ringdown waveforms will not be correct
  - So, our measurement of mass ratio will not give the true mass ratio

Degeneracy in Parameter Space







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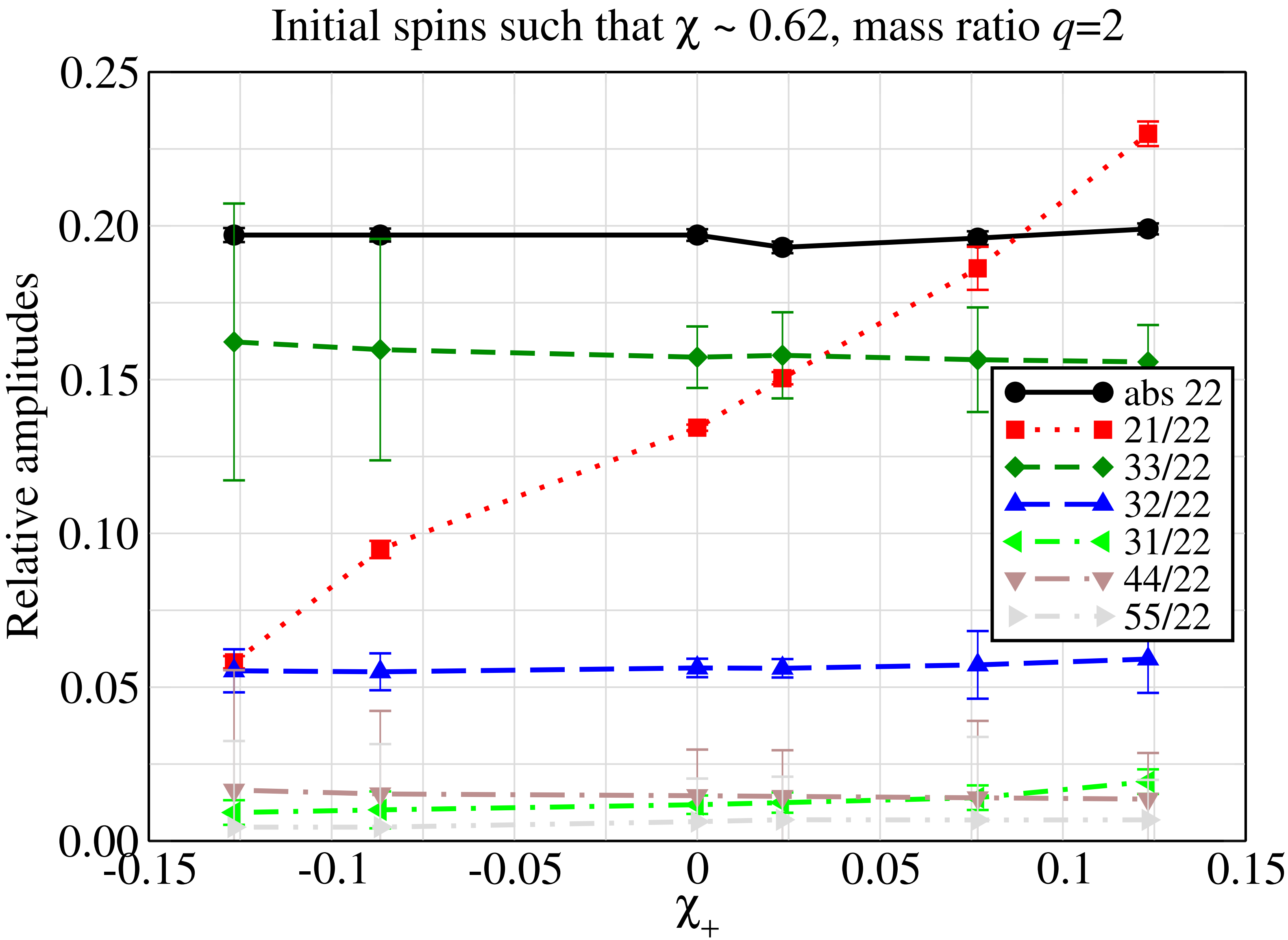
  - It is computationally expensive to carry out simulations in the full parameter space
- Analytically modelling the full space of waveforms is not easy
  - Carry out a study of binaries with black holes whose spins are aligned with the orbital angular momentum

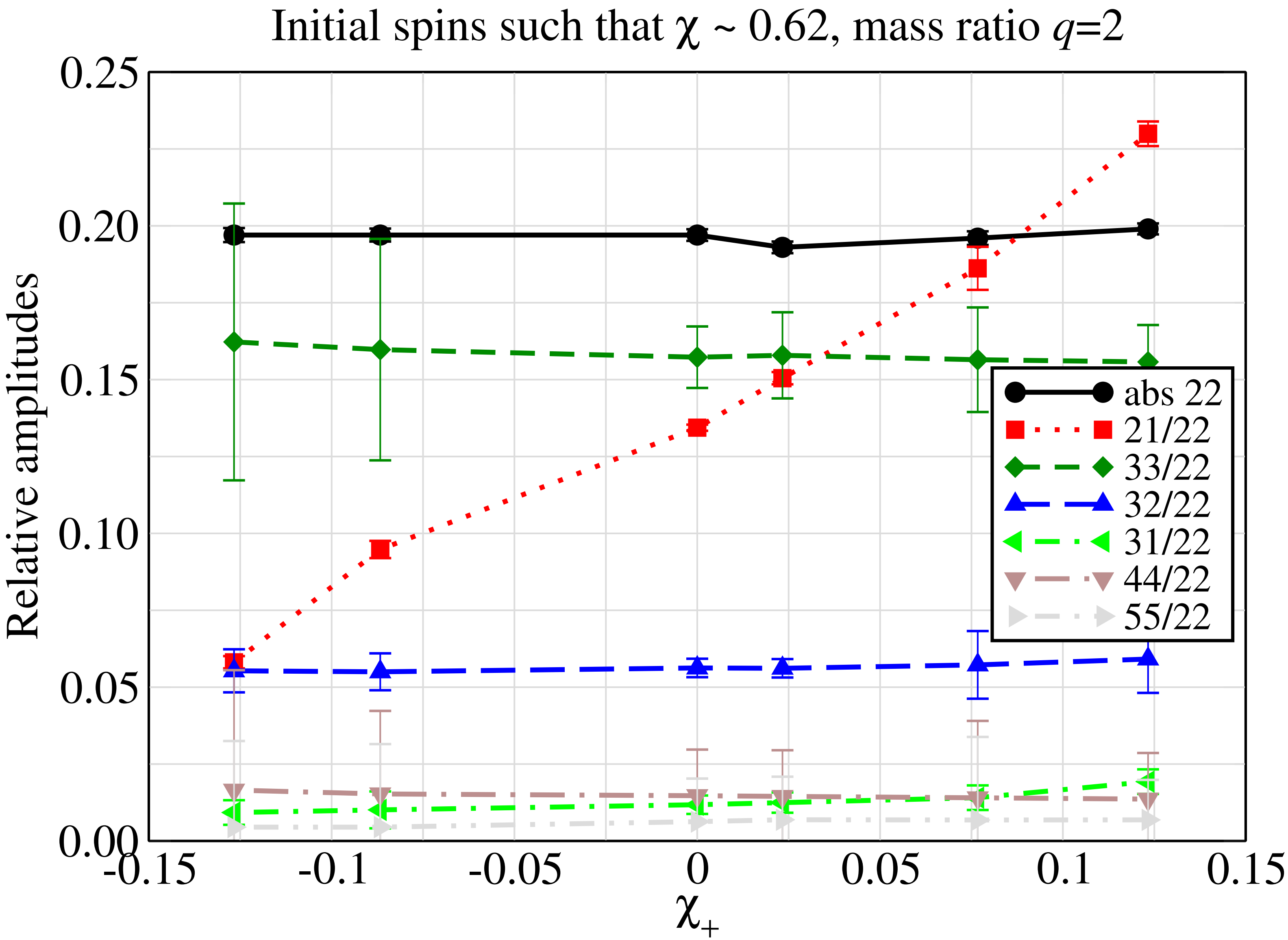
Parameter space of Simulations

q	χ_1	χ_2	χ_{eff}
2	0.16	-0.70	0.197
2	0.07	-0.40	0.102
2	-0.04	0.15	-0.045
2	-0.11	0.45	-0.130
2	-0.19	0.75	-0.220
3	0.00	0.00	0.000
3	-0.02	0.10	-0.025
3	-0.03	0.30	-0.056
3	-0.05	0.50	-0.094
3	-0.06	0.70	-0.125
4	-0.020	0.10	-0.024
4	-0.025	0.30	-0.047
4	-0.030	0.50	-0.071
4	-0.040	0.70	-0.098



Initial spins such that χ ~ 0.62, mass ratio q=2
Relative amplitudes
0.25
0.20
0.15
0.10
0.05
0.00
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
χ+
abs 22
21/22
33/22
32/22
31/22
44/22
55/22

22 and 33 mode amplitudes are constant, 21 mode varies with *total* spin $\chi_+ = (m_1\,\chi_1 + m_2\,\chi_2)/M_{in}$



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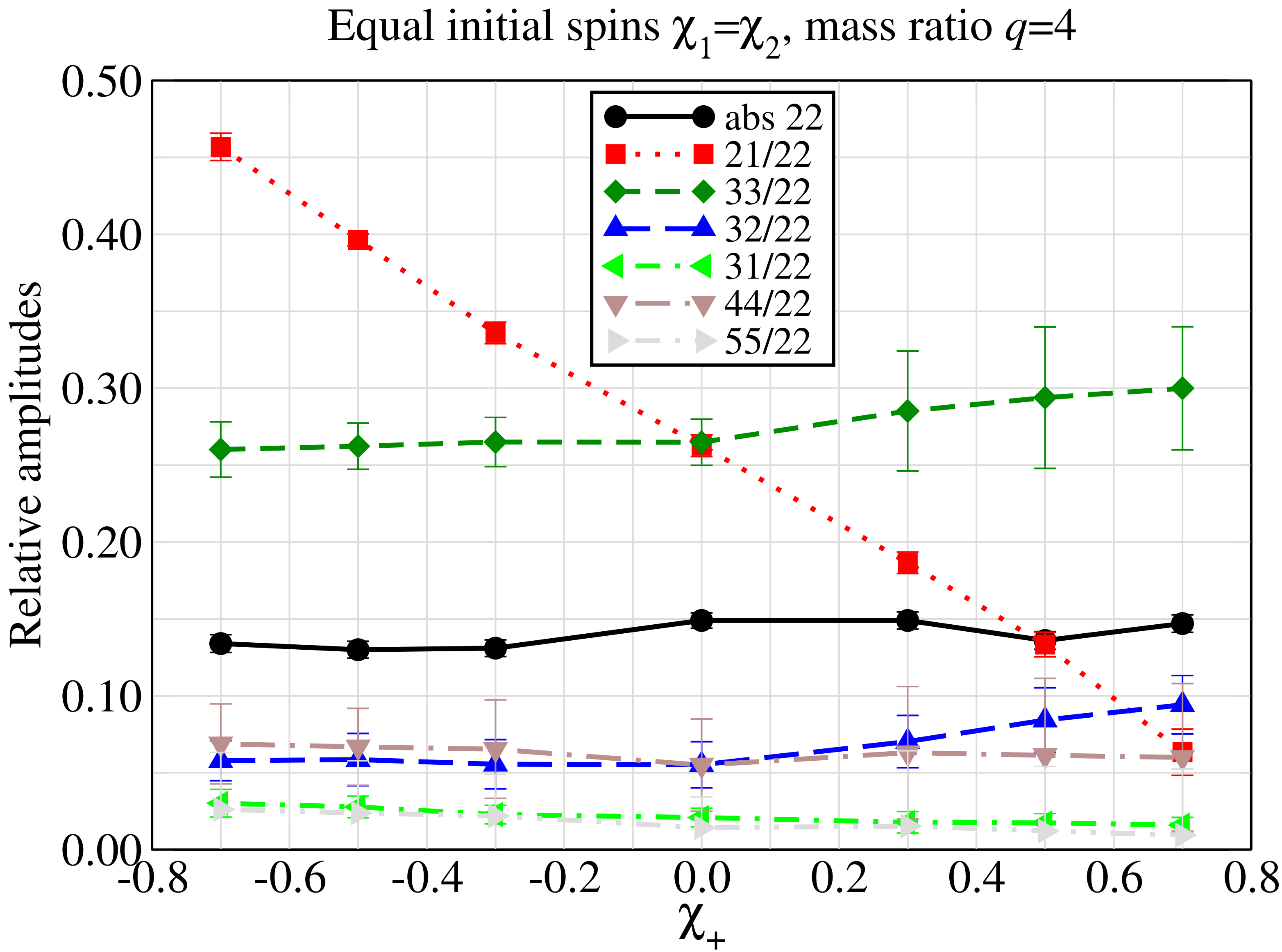

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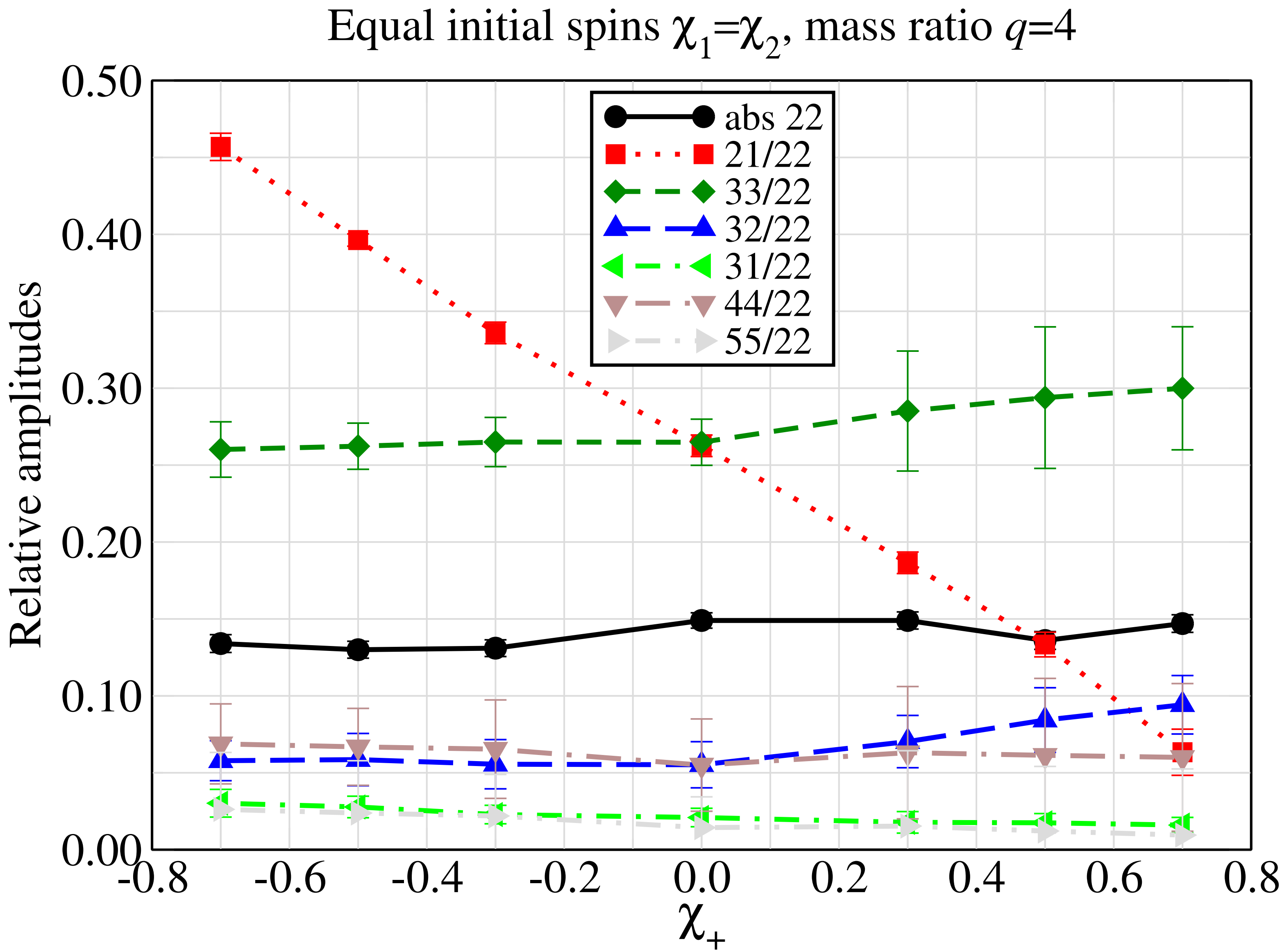
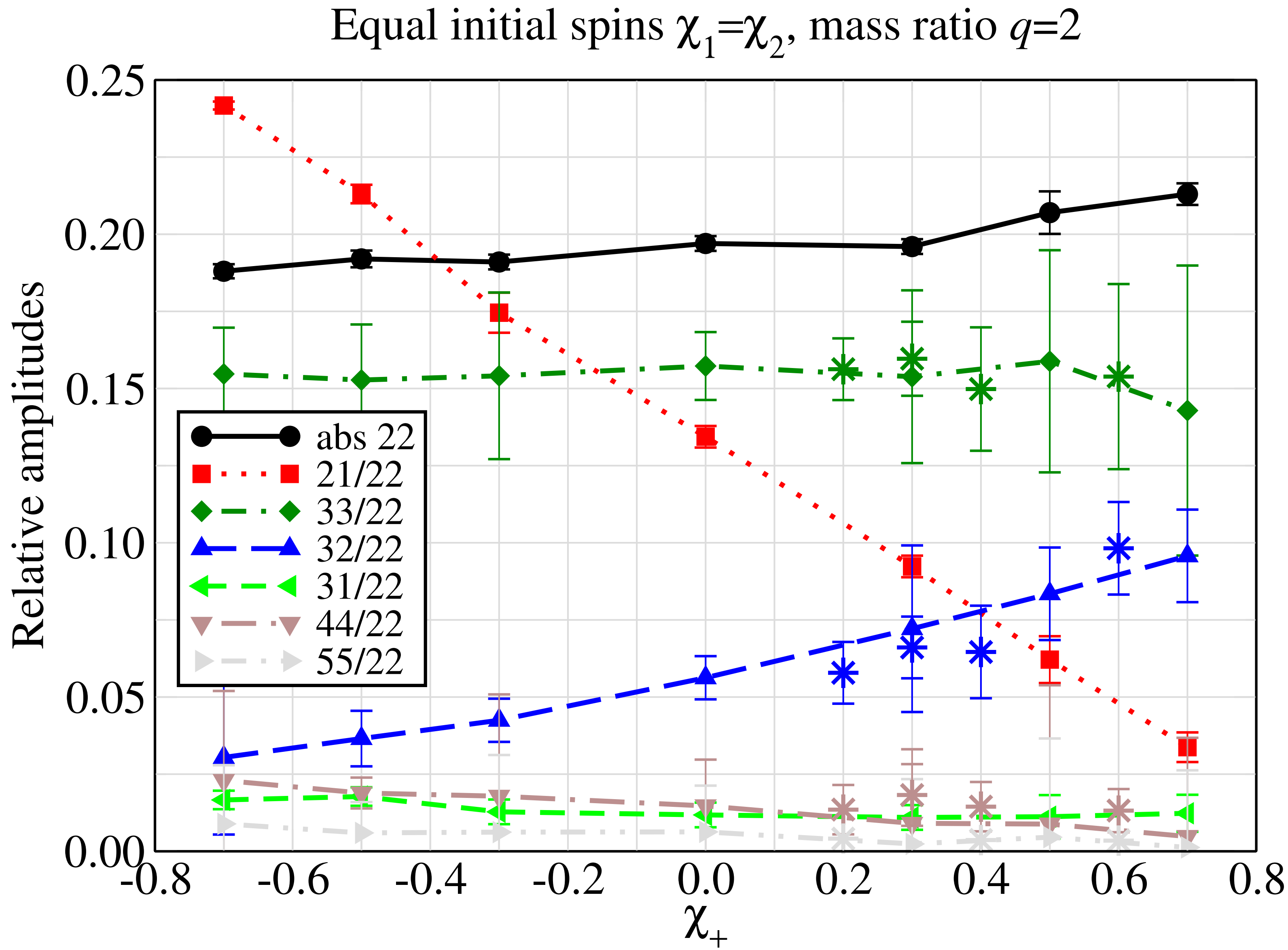
- Yes; 22 and 33 remain roughly constant and 21 varies

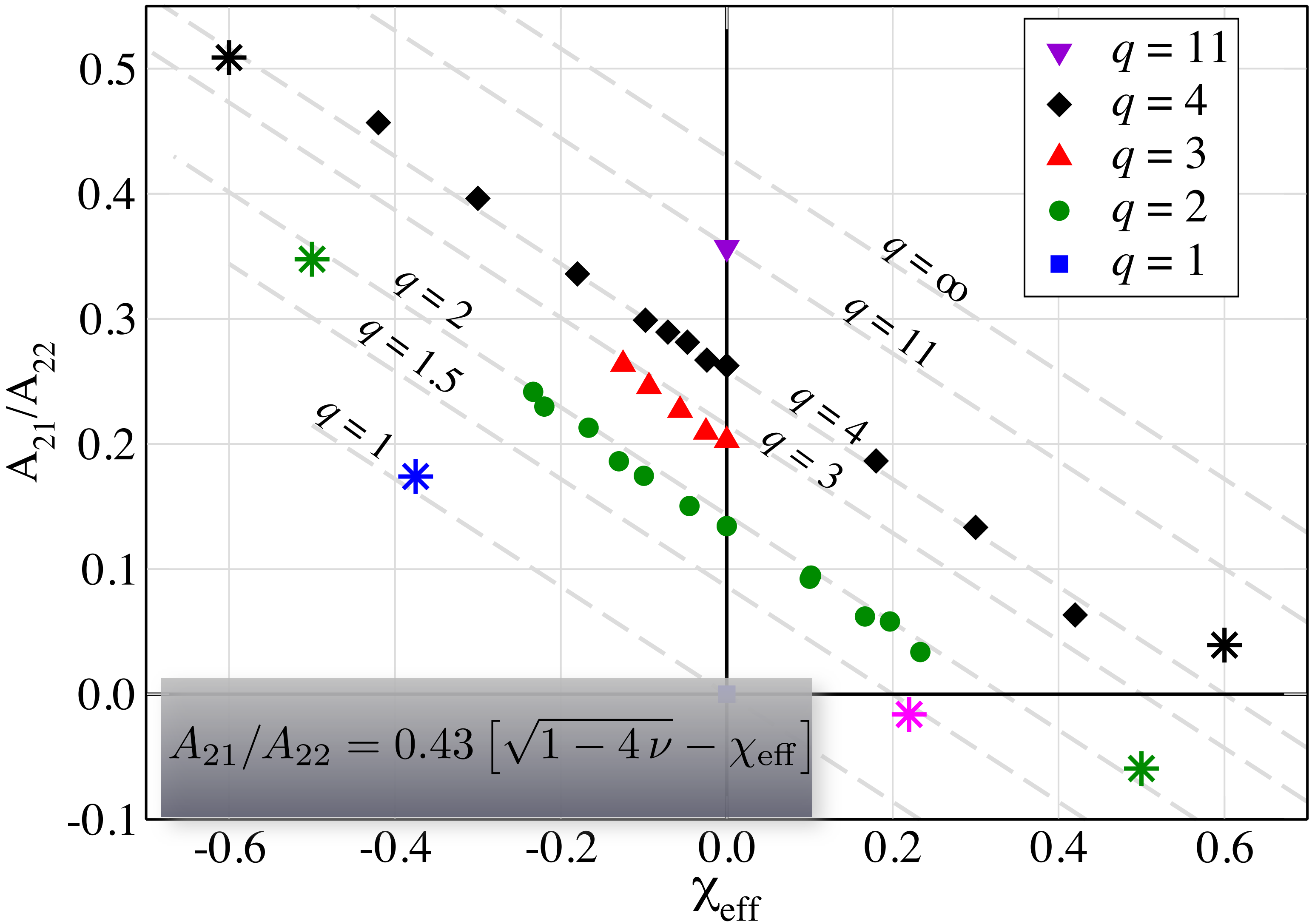


Is this generic? Seen in many other cases



A new parameterization

$$\chi_{\rm eff} = \frac{1}{2}(\sqrt{1-4\nu}\,\chi_1 + \chi_-), \quad \chi_- = \frac{m_1\chi_1 - m_2\chi_2}{M_{in}}, \quad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}, \quad q = \frac{m_1}{m_2}$$



$$A_{21}/A_{22} = 0.43\left[\sqrt{1-4\nu} - \chi_{\rm eff}\right]$$

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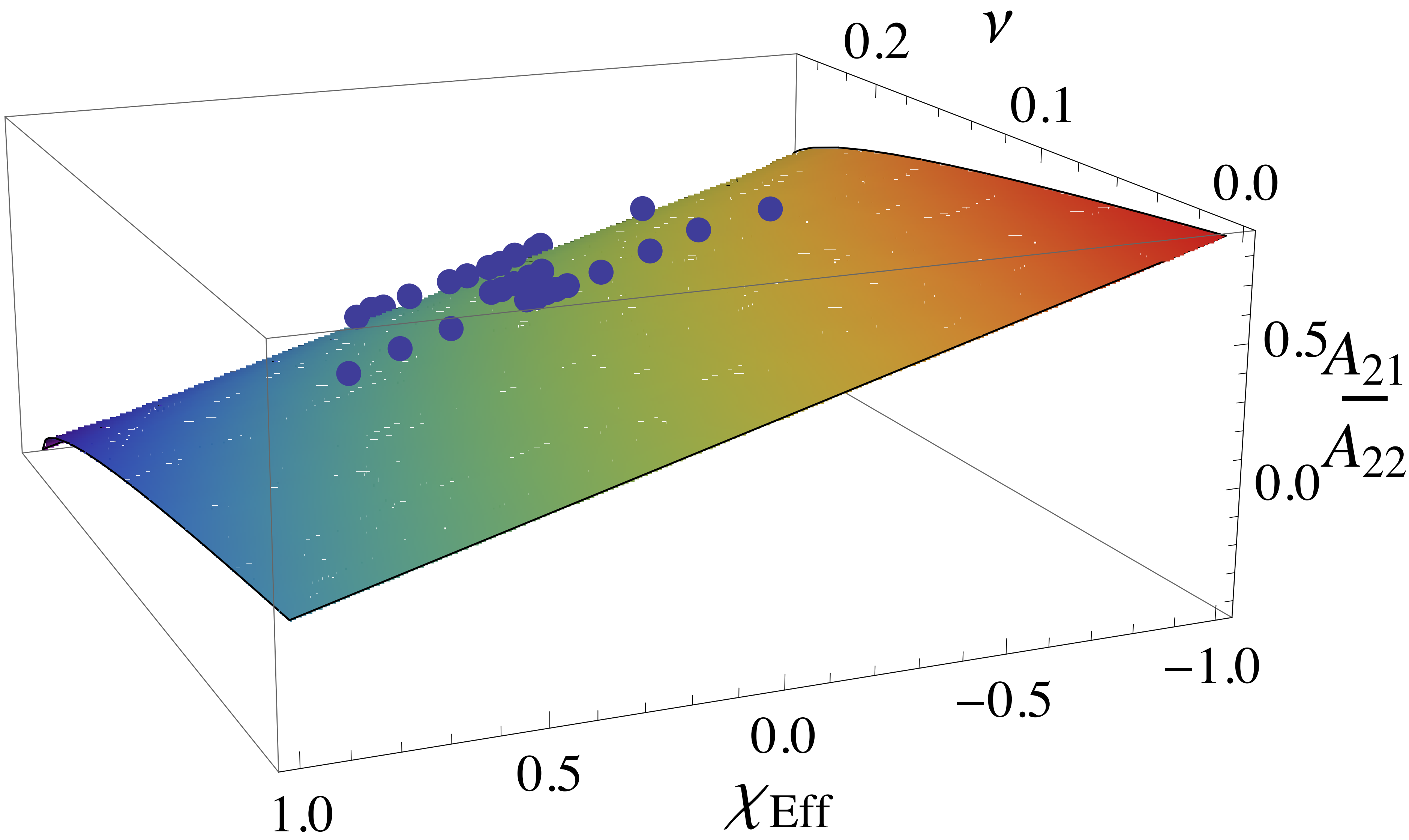
  - Vary by 20% when spins change from -0.8 to +0.8
  - Spins have a negligible effect on 22 and 33 amplitudes

$$\hat{A}_{21} \equiv A_{21}/A_{22} = 0.43\left[\sqrt{1-4\,\nu} - \chi_{\rm eff}\right]$$



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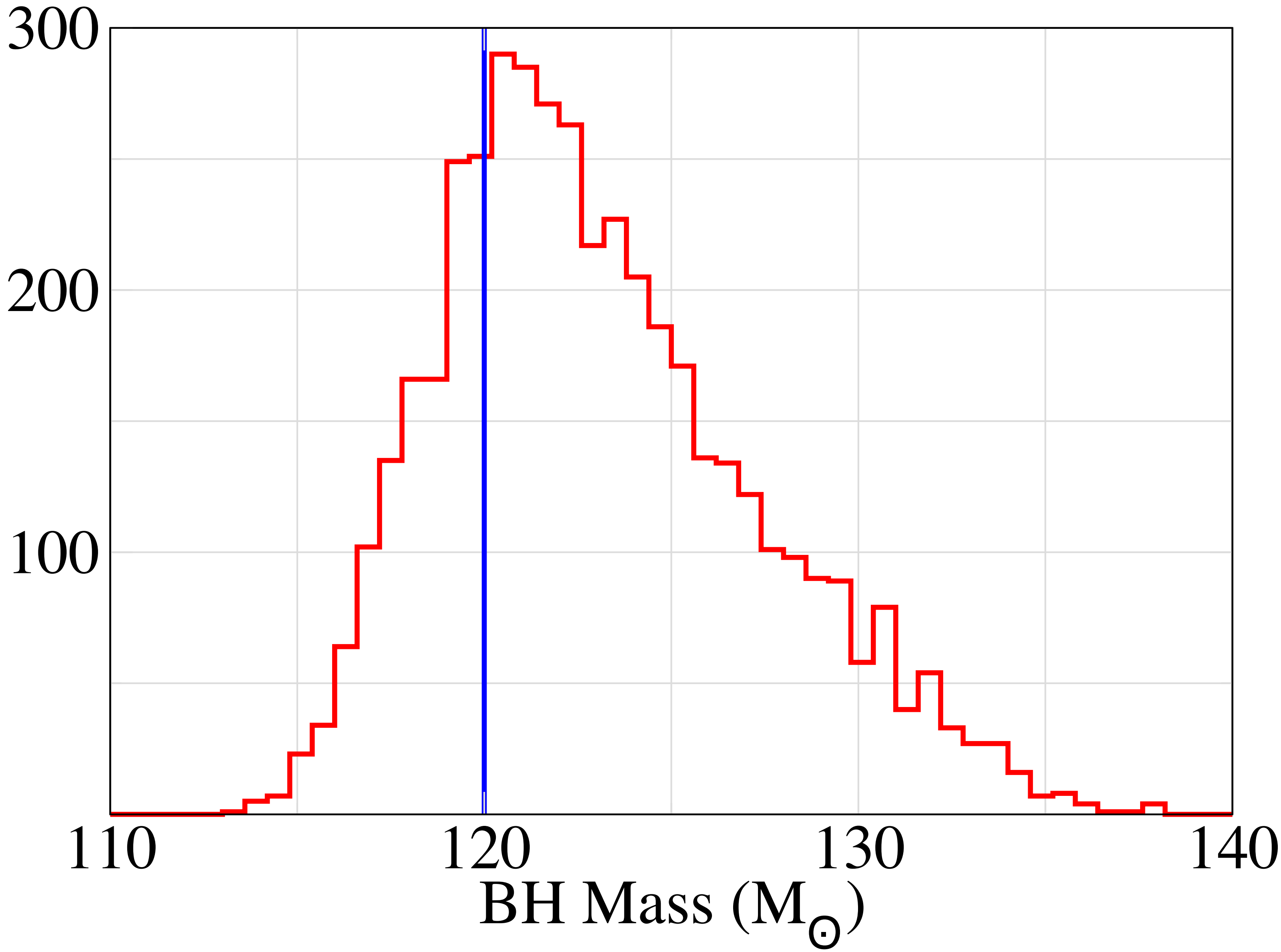
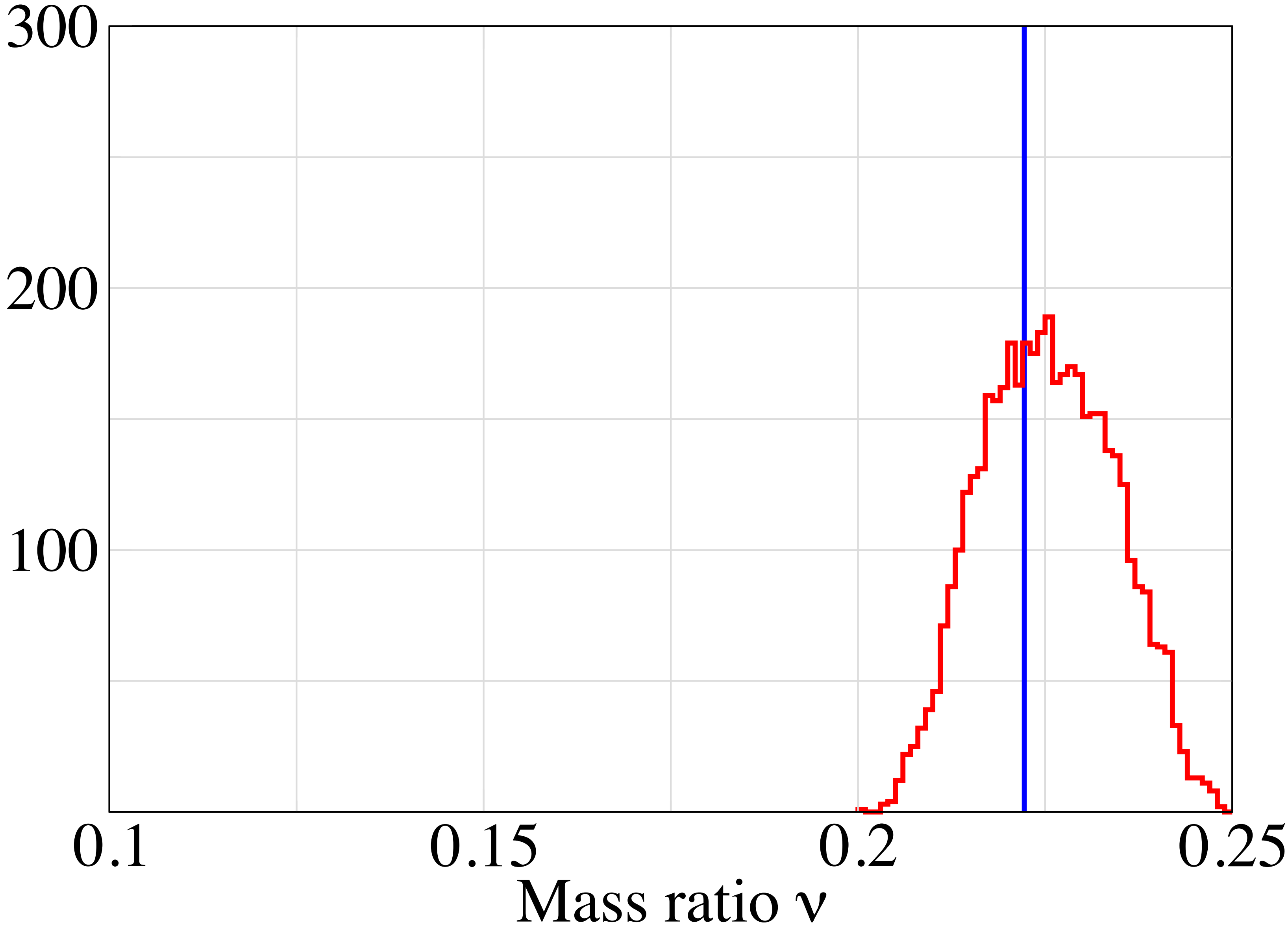
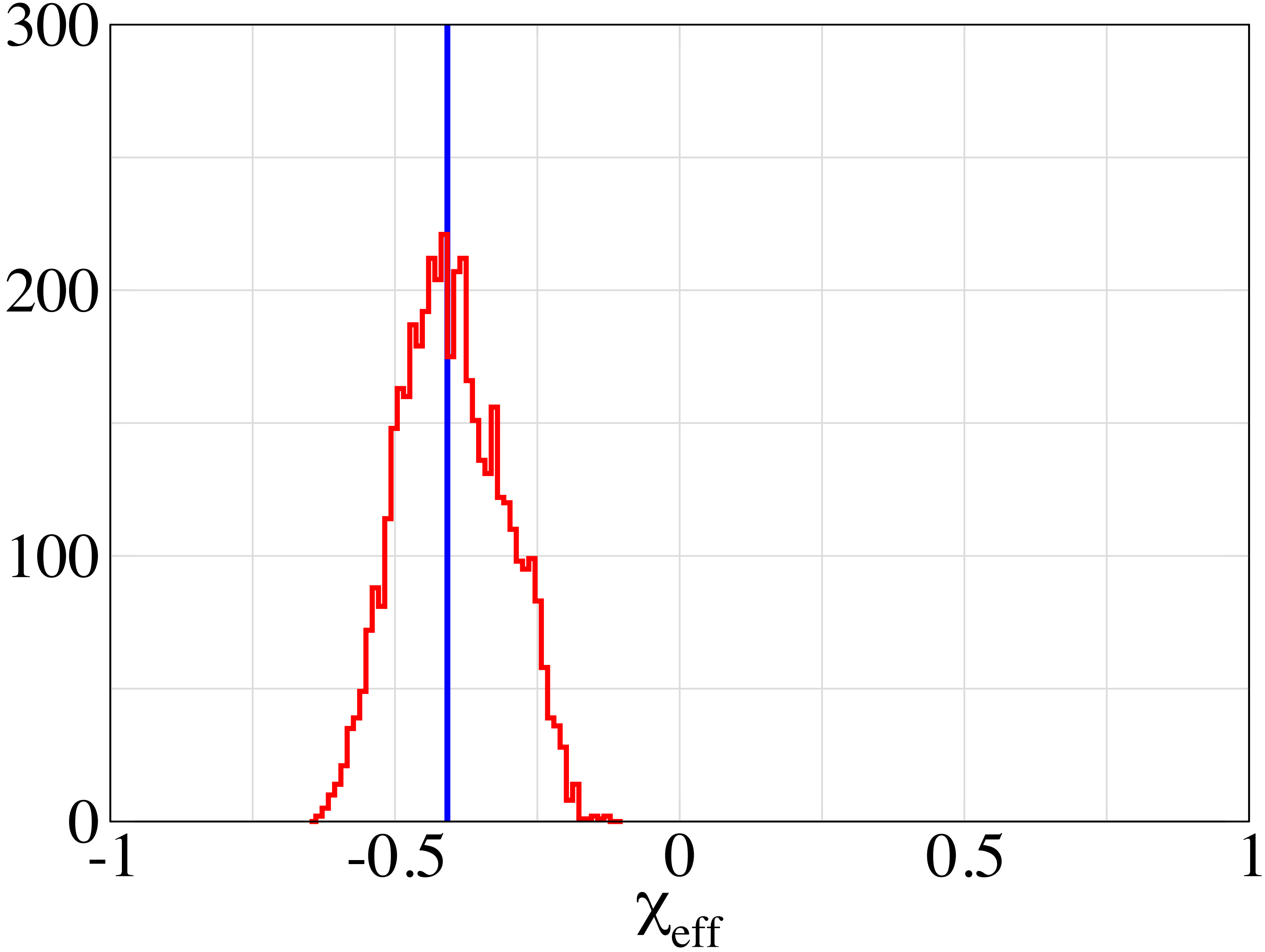
Posterior PDF of Black Hole Mass



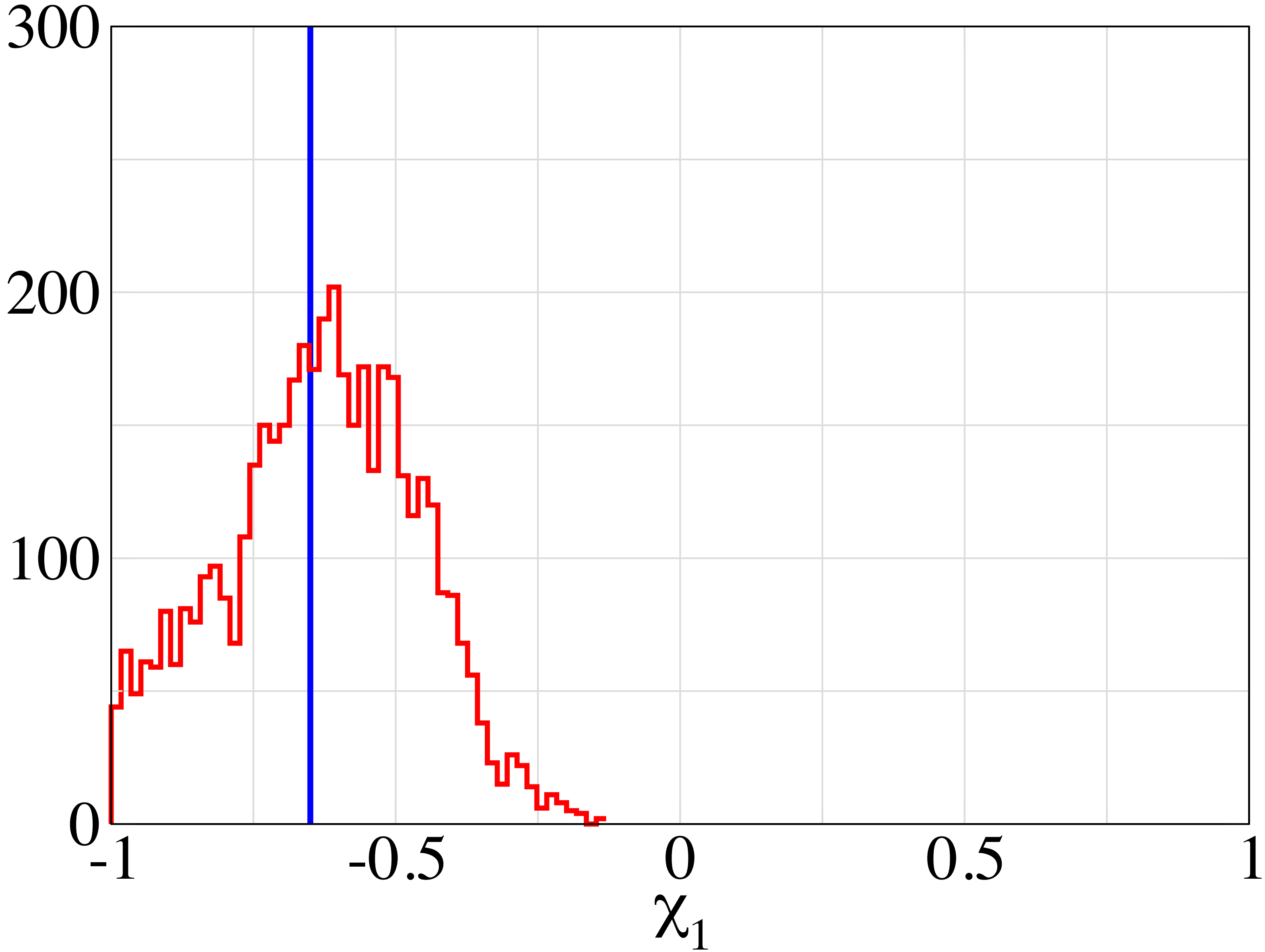
Posterior PDF of Mass Ratio



Posterior PDF of Effective Spin



Posterior PDF of Component Spin



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  - Einstein Telescope or LISA would be required for precise tests of these predictions