Mirror Thermal Noise

R. Nawrodt[1], G. Hofmann[1], J. Komma[1], C. Schwarz[1], D. Heinert[1], D. Chen[2], K. Yamamoto[2], K. Craig[3], R. Douglas[3], J. Hough[3], K. Haughian[3], I. W. Martin[3], M. Masso Reid[3], P. G. Murray[3], S. Rowan[3], A. A. van Veggel[3], S. Reid[4], D. Vine[4]

on behalf of the ELiTES WP2 members

[1]Institut für Festkörperphysik, Friedrich-Schiller-Universität Jena, Helmholtzweg 5, D-07743 Jena, Germany

[2]Institute for Cosmic Ray Reserach, The University of Tokyo, 5-1-5 Kashiwa-no-Ha, Kashiwa, Chiba, 277-8585

[3]SUPA, School of Physics and Astronomy, Kelvin Building, University of Glasgow, Glasgow, G12 8QQ, UK

[4]University of the West of Scotland, School of Engineering and Computing, Paisley, Scotland, PA1 2BE, UK

ET-0003A-15

27/02/2015

ELiTES is a IRSES-Marie Curie initiative, GA 295153.

Contents

1 Introduction

One of the key features of the ELiTES program is research related to the cryogenic operation of optical components and its benefits for both current and future gravitational wave (GW) detectors. Within different exchange periods, various material properties in addition to basic thermal noise issues have been studied, with many new results being obtained for the GW community, e.g. mechanical loss of coatings [1–6] and bulk materials [7, 8], thermal properties of suspension elements [9] or steps towards the assembling of possible cryogenic suspensions [10] and bonding techniques [11].

The following report will summarize the activities regarding estimates and a comparison of mirror thermal noise of optics based on sapphire and silicon.

2 Thermal noise in optical components

In an interferometric gravitational wave detector, the positions of the end mirrors are monitored through use of a laser. The laser beams probe the mirror surface according to their intensity profile which is typically TEM_{00}[1] (see fig. 2.1). Thermal fluctuations of the surface of the mirror will therefore cause a phase noise in the reflected laser beam. Due to the profile of the laser beam, thermal fluctuations of the center parts of the surface will have a stronger influence onto the thermal noise sensed by the laser, because the intensity of the beam is highest there.

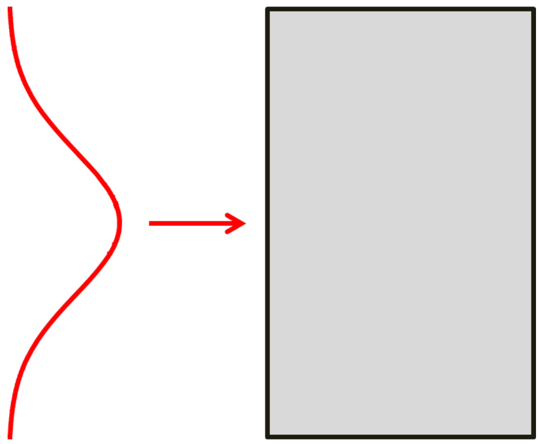

Figure 2.1: Gaussian read-out of the surface position of a test mass in an interferometric gravitational wave detector.

Phase fluctuations of the reflected light can be caused (or driven) by various different processes which are summarized here. The first noise arises from thermal fluctuations of the atoms of the mirror. This noise is called Brownian thermal noise and is given

[1] Other beam shapes have been proposed and are under investigation as well to further reduce thermal noise, see e.g. [12–15].

by [16–20]:

$$S_x^{\text{bulk}}(f,T) = \frac{2k_\text{B}T}{\pi^{3/2}f}\frac{1-\sigma^2}{wY}\phi_{\text{bulk}} \tag{2.1}$$

with the beam radius w, the Young's modulus and Poisson ratio of Y and σ and the mechanical loss of the bulk material ϕ_{bulk}. This result is valid for a semi-infinite size of mirror (or a small laser beam compared to the mirror size). A refined model for finite-sized mirrors is given by [19] and typically results in a correction factor in the order of a few percent.

The coating also contributes to Brownian thermal noise of the test mass as it is directly applied to the front surface of the test mass and thus is in direct interaction with the laser beam. The thermal noise contribution of the coating layer was calculated by [20]:

$$S_x^{\text{coat}}(f,T) = \frac{2k_\text{B}T}{\pi^2 f}\frac{t}{w^2 Y}\left(\frac{Y'}{Y}\phi_{||} + \frac{Y}{Y'}\phi_{\perp}\right) \tag{2.2}$$

where unprimed symbols indicate the bulk properties and primed ones the coating. Two different mechanical losses have been introduced for strains parallel and perpendicular to the coating surface. Again this estimation is valid if the test mass size is much larger than the beam diameter. Corrections for finite test masses are again in the percent region for a typical test mass of a GW detector and will be neglected here.

Another source of noise comes from fluctuations of the temperature in the different locations of the test mass that couple into the read-out via the material properties. As an example, in the case of the so-called thermo-elastic noise, temperature fluctuations lead to local expansion or contraction of the test mass by means of the coefficient of thermal expansion. Another related noise is thermo-refractive, where the coupling from temperature fluctuations into phase noise is linked through the temperature dependence of the refractive index. This effect is only relevant for parts where laser light of the read-out penetrates material, e.g. in the beam splitter, the input test mass or the first few coating layers of the end test mass. Since both thermo-elastic and thermo-refractive noise originate from the same process (the temperature fluctuation) they are correlated. A full treatment of both effects in coatings leads to the so called thermo-optic noise which was treated in [21, 22] and can lead to a lower total noise than the uncorrelated sum of both processes.

Again, these effects can occur in both the bulk material as well as the coating. Thermo-elastic noise for a semi-infinite test mass is given by [18]:

$$S_{\text{TE}}^{\text{bulk}}(f,T) = \frac{4}{\pi^{5/2}f^2}\frac{\alpha^2\kappa(1+\sigma)k_\text{B}T^2}{(\rho C)^2 w^3} \tag{2.3}$$

with the thermal conductivity κ, the heat capacity C and mass density ρ of the bulk material. The correction for finite-sized samples is given by [19]. Additionally, the given result is only valid if the thermal fluctuation does not spread over a larger spot than the beam diameter during the time $1/f$. This especially occurs in highly conductive materials at low temperatures and low frequencies. A solution for this case is given by [23]:

$$S_{\mathrm{TE}}^{\mathrm{bulk}}(f,T) = \frac{4}{\sqrt{\pi}}\alpha^2(1+\sigma)^2\frac{k_{\mathrm{B}}T^2 w}{\rho C a^2} \times J\left(\Omega\right) \tag{2.4}$$

with the thermal diffusivity $a^2 = \kappa/(\rho C)$ and the parameter $\Omega = 2\pi f/\omega_c$ where $\omega_c = 2a^2/w^2$. The frequency dependent part of eq. (2.4) is given by:

$$J(\Omega) = \sqrt{\frac{2}{\pi^3}}\int_0^{+\infty} du \int_{-\infty}^{+\infty} dv \frac{u^3 e^{-u^2/2}}{(u^2+v^2)\left[(u^2+v^2)^2+\Omega^2\right]}. \tag{2.5}$$

This integral can be computed numerically. This task is straightforward but time consuming due to convergence issues. It can be shown [24] that eq. (2.5) can be rewritten in an analytical form:

$$J(\Omega) = \Re\left\{\frac{e^{i\Omega/2}(1-i\Omega)}{\Omega^2}\left(\mathrm{Erf}\left[\frac{\sqrt{\Omega}(1+i)}{2}\right]-1\right)\right\} + \frac{1}{\Omega^2} - \sqrt{\frac{1}{\pi\Omega^3}}. \tag{2.6}$$

Erf(...) is the error function and $\Re$(...) the real part. This analytical function can be easier calculated as the initial integral.

Thermo-optic noise in coating materials is much smaller than the Brownian noise of the coating material [21, 22, 25] and is thus not taken into account in this treatment. Also, as we are comparing end test masses here bulk thermo-refractive noise is omitted.

A summary of all noise sources with a focus on coatings can be found in [25].

3 Investigations of the mechanical loss

Following the treatment above, it becomes clear that the mechanical loss of bulk materials ϕ_{bulk} as well as the coatings ϕ_{coat} must be carefully evaluated as it gives direct rise to Brownian thermal noise which will limit the ultimate GW detector sensitivity [2,7]. While ideal elastic materials show an instantaneous reaction, anelastic materials exhibit a time dependence and thus a phase lag between stress and strain which is the mechanical loss angle ϕ. The following two sections will summarize the current status of the investigation of these losses which are of direct relevance to the ELiTES project.

3.1 Mechanical loss of bulk and coating materials

3.1.1 Investigations of the mechanical loss of silicon and sapphire

The mechanical loss is equivalent to the rate of energy dissipation within a material. Note that external losses, such as gas damping, can also contribute and therefore have to be minimized for the experiment. By recording the free amplitude ring down τ of the specimens resonant vibration at a certain resonant frequency f_0 one obtains the mechanical loss via:

$$\phi = \frac{1}{\pi f_0 \tau} . \tag{3.1}$$

Silicon and sapphire bulk samples of cylindrical shape have been measured in a temperature range of 5 K up to 300 K and were suspended by a single loop of tungsten wire in vacuum. Sapphire samples with different surface qualities of ground, inspection polished as well as optical polished were investigated. There it was found that a flat and smooth polished surface allows for the lowest losses while a rougher surface results in higher losses.

Two sapphire samples have been investigated within the framwork of ELiTES. One pure sample and one sample that was contaminated with chromium. These samples gave mechanical losses of 3×10^{-9} at 10 K and 1×10^{-8} at 20 K and about 1×10^{-7} at room

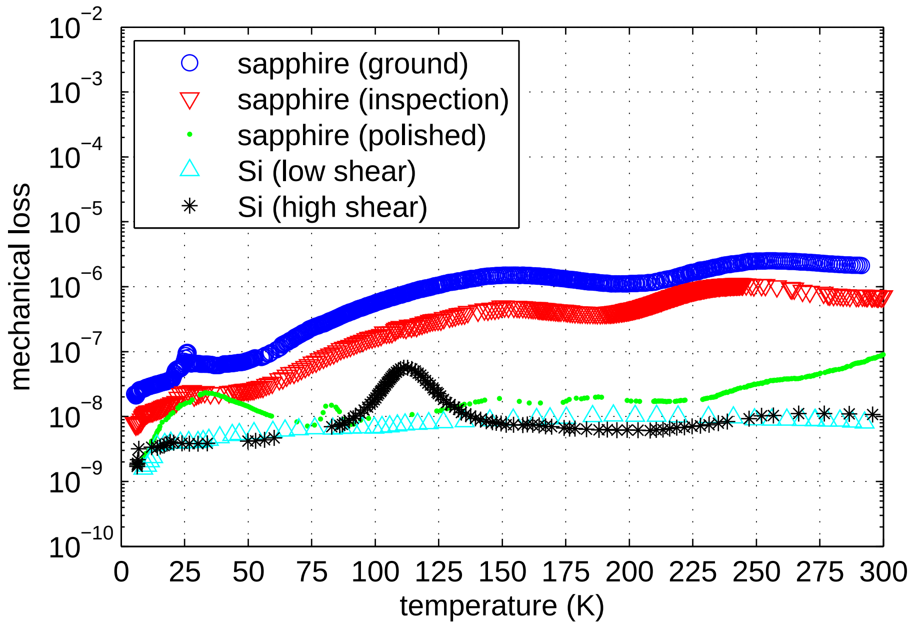

Figure 3.1: The mechanical loss of silicon and sapphire. Below 20 K both materials have a loss equal or lower than 10^{-8}. Due to shear energy in the modes used silicon exhibits a loss peak around 120 K. The surface quality of the samples has a large influence on the measured losses.

temperature. The effect of impurities was investigated using the chromium containing sample. At cryogenic temperatures around 30 K a peak in the loss of pure and doped sapphire was observed and assigned to loss from phonon-phonon interaction. The related theory is given by Akhieser and corresponds for data below 80 K (for details see [7]).

Several silicon samples with a different doping concentration of phosphorous (n-type) and boron (p-type) as well as intrinsic material have been under investigation. There, at 10 K the loss was found to reach even lower levels of about 2×10^{-9} and 4×10^{-9} at 20 K for almost all samples. Also, at 300 K it is low and reaching 8×10^{-9}. Further a thermally activated loss peak around 120 K has been correlated to the amount of shear energy in a resonant vibration. While at this temperature the loss is increased by nearly one order of magnitude at low temperatures the same results can be obtained.

Fig. 3.1 summarizes these results comparing different sapphire and silicon samples. The quality of the surface strongly influences the measured mechanical loss. In the case of silicon a loss peak at 120 K was observed that is strongly dependent on the amount of shear energy of the mode that is used for measurement. An additional correlation with intrinsic oxygen defects is currently done.

3.1.2 Investigations of the mechanical loss of tantala and silica

Early research into coating materials for interferometric gravitational wave detectors was focussed on designing silica/tantala multilayer stacks, optimised to have very low optical absorption at 1064 nm. These coatings were ion-beam sputtered (IBS), due to their good adhesion, density, low levels of impurities and lower optical absorption and then typically annealed at 450-550 °C to further reduce optical absorption [25]. Mechanical dissipation of such coatings is a significant source of thermal noise in interferometric gravitational wave detectors. Investigations revealed that the multilayer coating loss was predominantly due to losses in the high index tantala coating layers. The losses of the tantala and silica components were found to be $(4.4 \pm 0.2) \times 10^{-4}$ and $(0.5 \pm 0.3) \times 10^{-4}$ respectively [26]. The combined dissipation of a silica/tantala multilayer was then found to be $2-4 \times 10^{-4}$ [20,27]. With mechanical losses of bulk silica measured as low as 1×10^{-9} [28], these higher mechanical losses, in addition to the location of the coatings with respect to the probing laser beam, make the thermal noise associated with mirror coatings significantly higher than the substrate thermal noise.

Several studies have been undertaken to reduce the coating mechanical loss without adversely affecting the optical properties. Doping the tantala layers with titania showed a significant reduction in the coating loss at room temperature. Studies of coatings deposited with increasing percentages of titania doping, by LMA in Lyon, were made with the largest reduction in coating loss ($\sim$ 40%) observed with a titania concentration of 22.5% [29,30]. Coatings sputtered by CSIRO with 15% titania showed a similar level in the reduction of loss [30]. Variations in the levels of loss of coatings produced by different vendors have been observed, suggesting sputtering parameters, often proprietary, have a significant effect on the coating loss. A similar reduction in the loss has been observed though the majority of the 10-300 K temperature range [31].

Studies of the temperature dependence of the dissipation of mirror coating materials, [31–33] will allow the coating thermal noise to be estimated at low temperatures. In addition, any observed temperature dependences give insights into the underlying loss mechanisms [34]. Loss temperature dissipation peaks have been observed in tantala and detailed studies of the effect of heat-treatment on the temperature dependence of the loss of tantala have been carried out [2]. Measurements have shown that titania doping may reduce the activation energy of the dissipation peak at 20 K [31]. Studies of alternative dopants have been carried out, however titania, to date, is the only dopant found to reduce both the mechanical loss and optical absorption [35].

A dissipation peak ($\sim$ 20 K) has been observed on a single layer of IBS silica [36]; a similar magnitude to that measured on tantala. Therefore for a multilayer silica/tantala coating it is expected that both of the coating materials will make a significant contribution to the total coating loss at cryogenic temperatures. The loss peak observed in a 1 μm thick IBS silica coating heat-treated at 600 °C is at a significantly lower temperature than the well-known dissipation peak in bulk silica, which may imply a different microscopic dissipation mechanism is responsible for the peak [36].

Mechanical loss measurements of an Advanced LIGO silica/titania-doped tantala coating stack applied to a silicon cantilever has been studied between 10 and 300 K [37] show a low-temperature loss peak. However, the temperature of the loss peak is somewhat higher than would be predicted by the single-layer measurements detailed above, possibly due to slightly differing heat treatments and doping concentrations between the various coatings.

Loss measurements on silica/tantala multilayer coating deposited at NAOJ on a sapphire disc show no evidence of a sharp peak in the coating loss for an as-deposited coating, while a coating heat-treated at 500 °C did show a loss peak at 28 K; matching both the position and magnitude of the loss peak of the Advanced LIGO coating [1].

4 Thermal noise of silicon and sapphire based mirrors

4.1 Parameter overview

This section will give an overview of the material parameters that will be used in the thermal noise estimates for sapphire and silicon based mirrors.

4.1.1 Design parameters of the optics for KAGRA and ET-LF

The geometrical as well as the optical parameters that will be used in the thermal noise estimates are summarised in tab. 4.1. In case of the silicon-based mirror the orientation of the silicon bulk material was chosen to be Si(100). This orientation has the best possible matching of the Young's modulus to the coating and will thus lead to a minimum contribution of thermal noise. However, this choice needs to be carefully reviewed once the quality of the silicon and the level of noise of the coatings is known. Some remarks on the choice of the crystal orientation of a silicon sample is made in section 4.3.

Table 4.1: Geometrical and optical parameters of KAGRA and ET used for thermal noise estimates.

	KAGRA [38]	ET-LF [39]
material	sapphire	silicon
orientation	c-axis	[100]
wavelength (nm)	1064	1550
diameter (mm)	220	450
thickness (mm)	150	450
beam radius (mm)	40	90

4.1.2 Material properties

The relevant material properties in order to investigate the mirror thermal noise of a silicon- and sapphire-based mirror have been studied within the ELiTES project during the last years. Several reports and publications on bulk properties [7,8] as well as coating materials [1, 2] have been produced. Table 4.2 summarizes the relevant parameters for the materials required for the estimates in this report.

Table 4.2: Material properties of bulk and coating materials used for thermal noise estimates for a KAGRA and an ET mirror. All parameters are given for a operational temperature 20 K. The parameters given for the coating are averaged values based on material properties given in tab. 4.3 except the mechanical loss.

	sapphire	silicon	coating	unit
density	3980 [8]	2330 [8]	3550	kg/m^3
heat capacity	0.8 [8]	3.4 [8]	20	J/kg K
thermal cond.	15700 [8]	4940 [8]	0.2	W/m K
thermal expans.	6×10^{-9} [8]	-3×10^{-9} [8]	-1.8×10^{-6}	1/K
Youngs modulus	400	130 [40]	96	GPa
Poisson ratio	0.2	0.23	0.19	
mech. loss	10^{-8}	2×10^{-9}	7×10^{-4} [1, 37]	

In order to estimate the averaged coating parameters a certain coating design needs to be assumed. So far, there has not been a final decision on both - KAGRA and ET - so a coating similar to the aLIGO coating which is currently well characterised [37] will be assumed. Measurements of a multilayer coating in Japan revealed similar losses at 20 K [1] although being manufactured by a different vendor.

Although both detectors are operating at different wavelengths, the evaluation of the coatings and their properties can be done in one step as changing the wavelength gives a proportional change of values that cancels during averaging. All values for 1550 nm have been obtained by adapting the layer thickness to the new wavelength and refractive indices. Small corrections due to dispersion have been neglected here. The thermal parameters are obtained at room temperature from [41]. Assuming a similar temperature dependence of the glassy coating materials as silica the data was then estimated for low temperatures.

The averaging procedure for the multilayer coating follows [41] defining first a volume

Table 4.3: Coating parameters used to obtain the averaged values given in tab. 4.2. All parameters are given for 20 K except the refractive indices and the mass density which are assumed to be temperature independent.

		tantala	silica	unit
density		6000 [2]	2200	kg/m^3
total thickness	1064 nm	2.1 [37]	3.8 [37]	μm
	1550 nm	3.1	5.6	μm
refractive index	1064 nm	2.0962 [42]	1.4496 [43]	
	1550 nm	2.0856 [42]	1.4440 [43]	
Youngs modulus		140 [2]	72 [2]	GPa
Poisson ratio		0.23 [2]	0.17 [2]	
heat capacity	300 K	306 [41]	738 [44]	J/kg K
	20 K	10.4	25.2 [44]	J/kg K
thermal cond.	300 K	33 [41]	1.38 [41]	W/m K
	20 K	3.1	0.13 [45]	W/m K
thermal expans.	300 K	3.6×10^{-6} [41]	5.1×10^{-7} [41]	1/K
	20 K	-4.1×10^{-6}	-5.8×10^{-7} [46]	1/K

averaging operator:

$$\langle X \rangle_{\text{avg}} = \frac{t_a}{t_a + t_b} X_a + \frac{t_b}{t_a + t_b} X_b. \tag{4.1}$$

t_a and t_b are the total thicknesses of the coating layer a and b and X is the property to be averaged over the volume. Following [41] the thermal properties are given by:

$$C_{\text{coating}} = \langle C \rangle_{\text{avg}} \tag{4.2}$$

$$1/\kappa_{\text{coating}} = \langle 1/\kappa \rangle_{\text{avg}} \tag{4.3}$$

$$\tag{4.4}$$

using the defined averaging operator. The obtained values are presented in tab. 4.2.

4.2 Thermal noise of a silicon- and sapphire-based mirrors

Based on the thermal noise theory, as well as the material parameters and geometry of KAGRA and ET given in the previous sections, it is possible to estimate the mirror thermal noise for a silicon- and sapphire-based mirror. In fig. 4.1 and fig. 4.2 a direct comparison of the different noise sources of such a mirror is given.

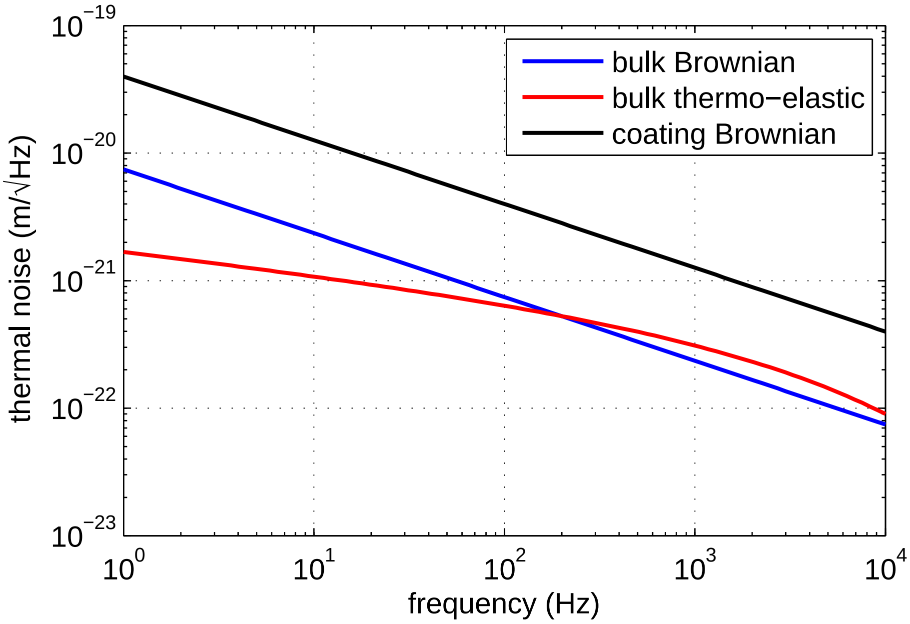

Figure 4.1: Comparison of the most important noise sources in a sapphire-based mirror for KAGRA.

In both cases the Brownian coating thermal noise dominates the total mirror thermal noise throughout the frequency range from 1 Hz up to 10 kHz. The bulk Brownian noise is comparable in both cases. Bulk thermo-elastic noise is slightly higher than bulk Brownian noise for sapphire above 200 Hz but also never exceeds the coatings Brownian noise. Due to the larger beam diameter the overall thermal noise of the ET-LF mirror is smaller than the KAGRA mirror.

4.3 Crystal orientation of a silicon-based mirror

The choice of the optimum orientation of the bulk silicon material depends on different aspects. First of all, the crystal orientation strongly influences the mechanical loss. As shown in fig. 3.1 some intrinsic defects in silicon are very sensitive to the crystal alignment and the excitation of modes having different shear energy contributions. The origin of this behaviour is an intrinsic defect symmetry that leads to dissipation processes and thus an increased mechanical loss at certain temperatures. In the presented case a loss process around 125 K is observed and results in a large measured mechanical loss of a mode is

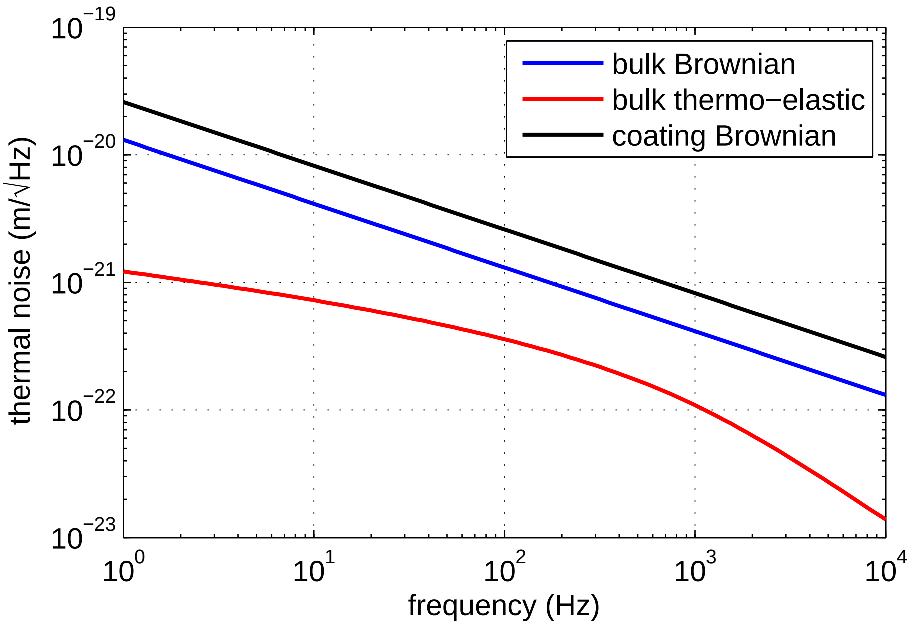

Figure 4.2: Comparison of the most important noise sources in a silicon-based mirror for the Einstein Telescope.

used that has a large amount of shear energy. This process can be linked to interstitial oxygen in silicon that is present from the growing process (in this case Czochralski, see for details e.g. [7]). Thus, in silicon samples having a large amount of interstitially bound oxygen an orientation is to be chosen that avoids an activation of this defect. A more detailed treatment reveals that Si(100) is a suitable choice in such a case.

A second influence arises from the orientation dependence of the Young's modulus. Si(100) has the smallest, Si(111) the largest Young's modulus amongst all other orientations [40]. Brownian thermal noise of the bulk sample is dependent on the Young's modulus regarding eq. (2.1) and thus Si(111) would result in the minimum bulk Brownain noise.

The third (and in this report used) influence dominates the other effects if coatings Brownian noise is the leading noise term. In this case, the bulk's Young's modulus should match the coating's Young's modulus as good as possible (see eq. (2.2)) to minimize coating as well as bulk Brownian noise.

5 Summary

Two example mirrors have been compared in this report. One is sapphire-based and its parameters or similar to the ones used for the KAGRA detectors. The other mirror is identical to the proposed ET-LF mirror. The operational temperature was chosen to be 20 K in both cases. The thermal noise behaviour of both mirrors is comparable and is limited by the Brownian thermal noise of the coatings with originates from the mechanical losses of the constituting materials for the thin layers, in this case silica and tantala. Any improvement of these coatings (e.g. by means of better designs [47], novel coatings techniques [3,4], alternative coatings based on microstructured surfaces [48,49]or crystalline materials [50, 51]) will directly translate through to a significant reduction of the overall thermal noise and thus increase the sensitivity of the detector. If the coating thermal noise is lowered by any of these or other novel techniques the choice of the crystal orientation of the bulk material needs to be reviewed regarding its Young's modulus, dissipation mechanisms and coating matching.

Within the ELiTES program, novel findings regarding the mechanical loss behaviour of the coating materials [1] as well as the bulk materials [7] have been investigated and quantified. The findings presented within this report provide fundamental understanding on future research activities within KAGRA (Japan) and ET (Europe) in order to improve future GW detector sensitivities.

Bibliography

[1] E. Hirose, K. Craig, H. Ishitsuka, I. W. Martin, N. Mio, S. Moriwaki, P. G. Murray, M. Ohashi, S. Rowan, Y. Sakakibara, T. Suzuki, K. Waseda, K. Watanabe, and K. Yamamoto, "Mechanical loss of a multilayer tantala/silica coating on a sapphire disk at cryogenic temperatures: Toward the KAGRA gravitational wave detector," *Phys. Rev. D*, vol. 90, p. 102004, Nov 2014.

[2] P. G. Murray and I. W. Martin, "Properties of Optical Coatings." ET-0003A-14 available at https://tds.ego-gw.it/ql/?i=ET, 2014.

[3] I. M. Pinto, "Nanolayered Material - Coating Prototypes." ET-0005B-14 available at https://tds.ego-gw.it/ql/?i=ET, 2014.

[4] I. M. Pinto, "Coating Optimization." ET-0006B-14 available at https://tds.ego-gw.it/ql/?i=ET, 2014.

[5] I. M. Pinto, "Material Losses from TNI Coating Noise Measurements." ET-0010A-14 available at https://tds.ego-gw.it/ql/?i=ET, 2014.

[6] S. Reid, "Coating Deposition Techniques." ET-0004A-14 available at https://tds.ego-gw.it/ql/?i=ET, 2014.

[7] R. Nawrodt, D. Heinert, G. Hofmann, P. G. Murray, and C. Schwarz, "Mechanical loss in bulk materials." ET-0002A-13 available at https://tds.ego-gw.it/ql/?i=ET, 2013.

[8] D. Heinert, C. Schwarz, G. Hofmann, J. Komma, and R. Nawrodt, "Database of Bulk Material Properties." ET-0001A-14 available at https://tds.ego-gw.it/ql/?i=ET, 2014.

[9] A. Khalaidovski, G. Hofmann, D. Chen, J. Komma, C. Schwarz, C. Tokoku, N. Kimura, T. Suzuki, A. O. Scheie, E. Majorana, R. Nawrodt, and K. Yamamoto, "Evaluation of heat extraction through sapphire fibers for the GW observatory KAGRA," *Class. Quantum Grav.*, vol. 31, no. 10, p. 105004, 2014.

[10] E. Hirose, Y. Sakakibara, Y. Igarashi, and T. Ishii, "Sapphire screws and strength test on them at liquid nitrogen temperature," *Rev. Sci. Instrum.*, vol. 85, no. 10, pp. –, 2014.

[11] R. Douglas, A. A. van Veggel, L. Cunningham, K. Haughian, J. Hough, and S. Rowan, "Cryogenic and room temperature strength of sapphire jointed by hydroxide-catalysis bonding," *Class. Quantum Grav.*, vol. 31, no. 4, p. 045001, 2014.

[12] J.-Y. Vinet, "Mirror thermal noise in flat-beam cavities for advanced gravitational wave interferometers," *Classical and Quantum Gravity*, vol. 22, no. 7, p. 1395, 2005.

[13] B. Mours, E. Tournefier, and J.-Y. Vinet, "Thermal noise reduction in interferometric gravitational wave antennas: using high order tem modes," *Classical and Quantum Gravity*, vol. 23, no. 20, p. 5777, 2006.

[14] S. Chelkowski, S. Hild, and A. Freise, "Prospects of higher-order laguerre-gauss modes in future gravitational wave detectors," *Phys. Rev. D*, vol. 79, p. 122002, Jun 2009.

[15] S. Gras, D. G. Blair, and L. Ju, "Opto-acoustic interactions in gravitational wave detectors: Comparing flat-top beams with gaussian beams," *Phys. Rev. D*, vol. 81, p. 042001, Feb 2010.

[16] F. Bondu, P. Hello, and J.-Y. Vinet, "Thermal noise in mirrors of interferometric gravitational wave antennas," *Physics Letters A*, vol. 246, no. 3, pp. 227–236, 1998.

[17] Y. Levin, "Internal thermal noise in the ligo test masses: A direct approach," *Phys. Rev. D*, vol. 57, pp. 659–663, Jan 1998.

[18] V. Braginsky, M. Gorodetsky, and S. Vyatchanin, "Thermodynamical fluctuations and photo-thermal shot noise in gravitational wave antennae," *Physics Letters A*, vol. 264, no. 1, pp. 1 – 10, 1999.

[19] Y. T. Liu and K. S. Thorne, "Thermoelastic noise and homogeneous thermal noise in finite sized gravitational-wave test masses," *Phys. Rev. D*, vol. 62, p. 122002, Nov 2000.

[20] G. M. Harry, A. M. Gretarsson, P. R. Saulson, S. E. Kittelberger, S. D. Penn, W. J. Startin, S. Rowan, M. M. Fejer, D. R. M. Crooks, G. Cagnoli, J. Hough, and N. Nakagawa, "Thermal noise in interferometric gravitational wave detectors due to dielectric optical coatings," *Classical and Quantum Gravity*, vol. 19, no. 5, p. 897, 2002.

[21] M. Evans, S. Ballmer, M. Fejer, P. Fritschel, G. Harry, and G. Ogin, “Thermo-optic noise in coated mirrors for high-precision optical measurements,” *Phys. Rev. D*, vol. 78, p. 102003, Nov 2008.

[22] M. L. Gorodetsky, “Thermal noises and noise compensation in high-reflection multilayer coating,” *Physics Letters A*, vol. 372, no. 46, pp. 6813 – 6822, 2008.

[23] M. Cerdonio, L. Conti, A. Heidmann, and M. Pinard, “Thermoelastic effects at low temperatures and quantum limits in displacement measurements,” *Phys. Rev. D*, vol. 63, p. 082003, Mar 2001.

[24] K. Somiya, K. Kokeyama, and R. Nawrodt, “Remarks on thermoelastic effects at low temperatures and quantum limits in displacement measurements,” *Phys. Rev. D*, vol. 82, p. 127101, Dec 2010.

[25] G. Harry, T. Bodiya, and R. DeSalvo, *Optical Coatings and Thermal Noise in Precision Measurement.* Cambridge University Press, 2012.

[26] S. D. Penn, P. H. Sneddon, H. Armandula, J. C. Betzwieser, G. Cagnoli, J. Camp, D. R. M. Crooks, M. M. Fejer, A. M. Gretarsson, G. M. Harry, J. Hough, S. E. Kittelberger, M. J. Mortonson, R. Route, S. Rowan, and C. C. Vassiliou, “Mechanical loss in tantala/silica dielectric mirror coatings,” *Classical and Quantum Gravity*, vol. 20, no. 13, p. 2917, 2003.

[27] D. R. M. Crooks, P. Sneddon, G. Cagnoli, J. Hough, S. Rowan, M. M. Fejer, E. Gustafson, R. Route, N. Nakagawa, D. Gretarsson, “Excess mechanical loss associated with dielectric mirror coatings on test masses in interferometric gravitational wave detectors,” *Classical and Quantum Gravity*, vol. 19, no. 5, p. 883, 2002.

[28] A. Ageev, B. C. Palmer, A. D. Felice, S. D. Penn, and P. R. Saulson, “Very high quality factor measured in annealed fused silica,” *Classical and Quantum Gravity*, vol. 21, no. 16, p. 3887, 2004.

[29] G. M. Harry, M. R. Abernathy, A. E. Becerra-Toledo, H. Armandula, E. Black, K. Dooley, M. Eichenfield, C. Nwabugwu, A. Villar, D. R. M. Crooks, G. Cagnoli, J. Hough, C. R. How, I. MacLaren, P. Murray, S. Reid, S. Rowan, P. H. Sneddon, M. M. Fejer, R. Route, S. D. Penn, P. Ganau, J.-M. Mackowski, C. Michel, L. Pinard, and A. Remillieux, “Titania-doped tantala/silica coatings for gravitational-wave detection,” *Classical and Quantum Gravity*, vol. 24, no. 2, p. 405, 2007.

[30] P. G. Murray, *Measurement of the mechanical loss of test mass materials for advanced gravitational wave detectors.* PhD thesis, University of Glasgow, 2008.

[31] I. Martin, E. Chalkley, R. Nawrodt, H. Armandula, R. Bassiri, C. Comtet, M. Fejer, A. Gretarsson, G. Harry, D. Heinert, *et al.*, “Comparison of the temperature dependence of the mechanical dissipation in thin films of ta2o5 and ta2o5 doped with tio2,” *Classical and Quantum Gravity*, vol. 26, no. 15, p. 155012, 2009.

[32] I. Martin, H. Armandula, C. Comtet, M. Fejer, A. Gretarsson, G. Harry, J. Hough, J. M. Mackowski, I. MacLaren, C. Michel, *et al.*, “Measurements of a low-temperature mechanical dissipation peak in a single layer of ta2o5 doped with tio2,” *Classical and Quantum gravity*, vol. 25, no. 5, p. 055005, 2008.

[33] K. Yamamoto, S. Miyoki, T. Uchiyama, H. Ishitsuka, M. Ohashi, K. Kuroda, T. Tomaru, N. Sato, T. Suzuki, T. Haruyama, A. Yamamoto, T. Shintomi, K. Numata, K. Waseda, K. Ito, and K. Watanabe, “Measurement of the mechanical loss of a cooled reflective coating for gravitational wave detection,” *Phys. Rev. D*, vol. 74, p. 022002, Jul 2006.

[34] A. Nowick, *Anelastic Relaxation In Crystalline Solids.* Materials science and technology, Elsevier Science, 2012.

[35] R. Flaminio, J. Franc, C. Michel, N. Morgado, L. Pinard, and B. Sassolas, “A study of coating mechanical and optical losses in view of reducing mirror thermal noise in gravitational wave detectors,” *Classical and Quantum Gravity*, vol. 27, no. 8, p. 084030, 2010.

[36] I. Martin, R. Nawrodt, K. Craig, C. Schwarz, R. Bassiri, G. Harry, J. Hough, S. Penn, S. Reid, R. Robie, *et al.*, “Low temperature mechanical dissipation of an ion-beam sputtered silica film,” *Classical and Quantum Gravity*, vol. 31, no. 3, p. 035019, 2014.

[37] M. Granata, K. Craig, G. Cagnoli, C. Carcy, W. Cunningham, J. Degallaix, R. Flaminio, D. Forest, M. Hart, J.-S. Hennig, J. Hough, I. MacLaren, I. W. Martin, C. Michel, N. Morgado, S. Otmani, L. Pinard, and S. Rowan, “Cryogenic measurements of mechanical loss of high-reflectivity coating and estimation of thermal noise,” *Opt. Lett.*, vol. 38, pp. 5268–5271, Dec 2013.

[38] K. Somiya, “Detector configuration of KAGRA–the Japanese cryogenic gravitational-wave detector,” *Class. Quantum Grav.*, vol. 29, no. 12, p. 124007, 2012.

[39] "Einstein telescope design study document." ET-0106C-10 available at http://www.et-gw.eu/etdsdocument, 2011.

[40] J. Wartman and R. Evans, "Young's modulus, shear modulus and poisson's ratio in silicon and germanium," *J. Appl. Phys.*, vol. 36, pp. 153–156, 1965.

[41] M. M. Fejer, S. Rowan, G. Cagnoli, D. R. M. Crooks, A. Gretarsson, G. M. Harry, J. Hough, S. D. Penn, P. H. Sneddon, and S. P. Vyatchanin, "Thermoelastic dissipation in inhomogeneous media: loss measurements and displacement noise in coated test masses for interferometric gravitational wave detectors," *Phys. Rev. D*, vol. 70, p. 082003, Oct 2004.

[42] L. Gao, F. Lemarchand, and M. Lequime, "Exploitation of multiple incidences spectrometric measurements for thin film reverse engineering," *Opt. Express*, vol. 20, pp. 15734–15751, Jul 2012.

[43] I. H. Malitson, "Interspecimen comparison of the refractive index of fused silica," *J. Opt. Soc. Am.*, vol. 55, pp. 1205–1208, Oct 1965.

[44] R. C. Lord and J. C. Morrow, "Calculation of the heat capacity of α quartz and vitreous silica from spectroscopic data," *The Journal of Chemical Physics*, vol. 26, no. 2, pp. 230–232, 1957.

[45] D. H. Damon, "Thermal Conductivity of Vitreous Silica at Low Temperatures," *Phys. Rev. B*, vol. 8, pp. 5860–5865, Dec 1973.

[46] G. White, "Thermal expansion of reference materials: copper, silica and silicon," *Journal of Physics D: Applied Physics*, vol. 6, no. 17, p. 2070, 1973.

[47] J. Agresti, G. Castaldi, R. DeSalvo, V. Galdi, V. Pierro, and I. M. Pinto, "Optimized multilayer dielectric mirror coatings for gravitational wave interferometers," 2006.

[48] F. Brückner, D. Friedrich, T. Clausnitzer, M. Britzger, O. Burmeister, K. Danzmann, E.-B. Kley, A. Tünnermann, and R. Schnabel, "Realization of a monolithic high-reflectivity cavity mirror from a single silicon crystal," *Physical review letters*, vol. 104, no. 16, p. 163903, 2010.

[49] D. Heinert, S. Kroker, D. Friedrich, S. Hild, E.-B. Kley, S. Leavey, I. W. Martin, R. Nawrodt, A. Tünnermann, S. P. Vyatchanin, *et al.*, "Calculation of thermal noise in grating reflectors," *Physical Review D*, vol. 88, no. 4, p. 042001, 2013.

[50] G. D. Cole, W. Zhang, M. J. Martin, J. Ye, and M. Aspelmeyer, “Tenfold reduction of brownian noise in high-reflectivity optical coatings,” *Nature Photonics*, vol. 7, no. 8, pp. 644–650, 2013.

[51] A. Cumming, K. Craig, I. Martin, R. Bassiri, L. Cunningham, M. Fejer, J. Harris, K. Haughian, D. Heinert, B. Lantz, *et al.*, “Measurement of the mechanical loss of prototype gap/algap crystalline coatings for future gravitational wave detectors,” *Classical and Quantum Gravity*, vol. 32, no. 3, p. 035002, 2015.