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Thermal correction of mirror deformations by scanning with a heating laser - noise considerations

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VIRGO * A joint CNRS-INFN Project
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1 Introduction

Partial absorption of the high laser power in the Virgo arm cavities by the mirror substrate or the coating leads to surface deformations and thermal lensing, which compromises mode matching and power buildup in the interferometer [1]. A technique for measurement and correction of these deformations has been proposed and tested in the GEO collaboration [2, 3]. Here we are interested in the correction method, which consists in scanning a strongly absorbed laser beam (e.g. CO_2) over the mirror surface; the laser power (or scanning speed) is modulated in order to heat more where originally less intra-cavity power was absorbed.

The scanning, although much faster than the thermal relaxation time of the mirror, will cause time-dependent mirror deformations. Here we investigate their influence on the VIRGO sensitivity. Here we consider only a rectangular scanning pattern.

2 Model

An order-of-magnitude estimation can be obtained by considering a constant (non-correcting) TV-like scanning: A quadratic field of width and height L on the mirror is subdivided in N horizontal stripes; the heating beam scans over each stripe from left to right and then jumps to the next one below, or to the beginning of the first one if none is left. The beam has a quadratic beam shape and fills the width of a stripe without spilling into the upper or lower one. The irradiation time is much shorter than the time scale for heat conduction and thermal equilibration of the mirror. The heating beam is absorbed in the surface layer of the mirror, and the irradiated point is immediately deformed in proportion to the absorbed energy, so that the deformation linearly increases during exposure to the beam. We consider the steady-state case: During one complete scan cycle, there is no average change of the mirror shape. This means that the deformation acquired during irradiation relaxes completely until the next hit of the heating beam, and the surface of each zone just oscillates around its equilibrium position with a sawtooth-like characteristic. The amplitude U of this oscillation is given by

$$U \;=\; k\,t_{ir}\,I_s \;=\; \frac{k\,I_s}{f_s\,N^2} \quad ,$$

where f_s is the scan frequency, t_{ir} the irradiation time per point, I_s the absorbed power density and k the thermal deformation coefficient of the surface. The latter can be obtained by evaluating the data in [4] to be about $6 \times 10^{-7} \frac{\mathrm{m}}{\mathrm{sec\,W/m^2}}$.

During the scan, the movement of the mirror surface as a whole (neglecting wavefront distortions), seen by a reflected plane wave, is given by

$$u^p(t) \;=\; \frac{1}{L^2}\int\!\!\int u(x,y,t)\,dx\,dy \;=\; 0 \quad .$$

Since two points on the surface behave the same way apart a time delay, the surface average is equivalent to a time average over one point, which is, by definition of u, zero. Hence for a plane wave the mirror surface does not move. If, however, the mirror is part of a Fabry-Perot cavity with a resonant TEM_{00} mode, the latter sees a mirror movement of

$$\begin{aligned} u^{00}(t) &= \;<\psi_{00}\,|\,u(x,y,t)\,|\,\psi_{00}> \\ &= \frac{2}{\pi w_0^2 L^2}\int\!\!\int exp(-2\frac{x^2+y^2}{w^2})\;u(x,y,t)\,dx\,dy \\ &\neq\; 0 \quad ; \end{aligned} \tag{1}$$

the time averaging argument from above does not hold here, because of the inhomogeneously (Gaussian) weighted surface averaging. So the effective cavity length oscillates during the scanning.

N	16	Number of rows in scan
n	8...64	Number of samples over one scan beam width
L	200 mm	Height / width of scanned field
f_s	5 Hz	Scan frequency
	$6 \times 10^{-7} \frac{\mathrm{m}}{\mathrm{sec\,W/m^2}}$	Thermal deformation coefficient
P_s	1 W	Heating laser power
P	20 kW	Intra-cavity YAG power
w	55 mm	Gaussian beam radius of YAG on mirror
A	3 ppm	YAG absorption in coating

Table 1: Parameters used for numerical calculations

3 Numerical analysis

The power of the heating laser was chosen such that its power density, averaged over the scanned area, was about twice the maximum absorbed power density of the YAG beam, in order to be able to sufficiently heat the areas far from the cavity axis. The mirror oscillation $u^{00}(t)$ was calculated using the discrete form of Eq. 1:

$$u^{00}(t) = \frac{2}{{}^{2}_{0} n N w^{2}} \sum_{i=1}^{N} \sum_{j=1}^{n \cdot N} exp(-2\frac{x_{ij}^2 + y_{ij}^2}{w^2})\ u_{ij}(t)$$

with the parameters given in Tab. 1. Over the width of the scan beam, n sample points are taken; this number is adjusted according to the upper spectral frequency of interest.

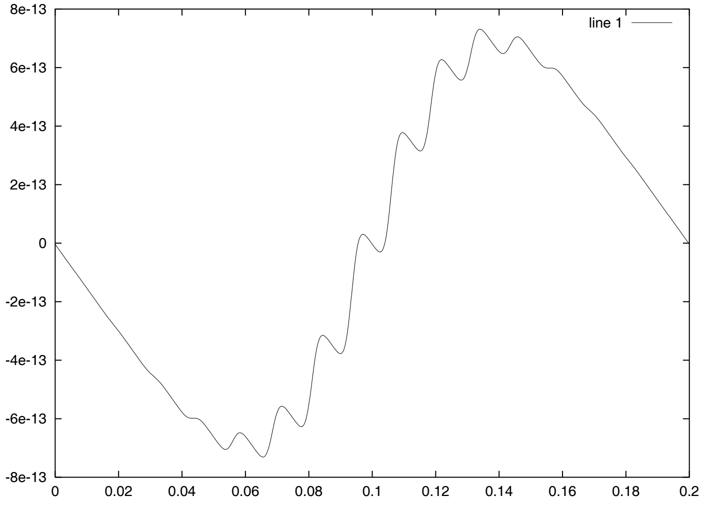

Figure 1: Effective cavity length change seen by the resonant TEM_{00} mode during one scan cycle.

The result is given in Fig. 1. With the bare eye, one recognizes an oscillation at the scan frequency f_s, contributing almost all of the peak-to-peak amplitude of 1.5 pm, and one at the line frequency $N \cdot f_s$. Fig. 2 shows the spectrum. Apart from the dominant peaks at f_s and $N \cdot f_s$ there is a multitude

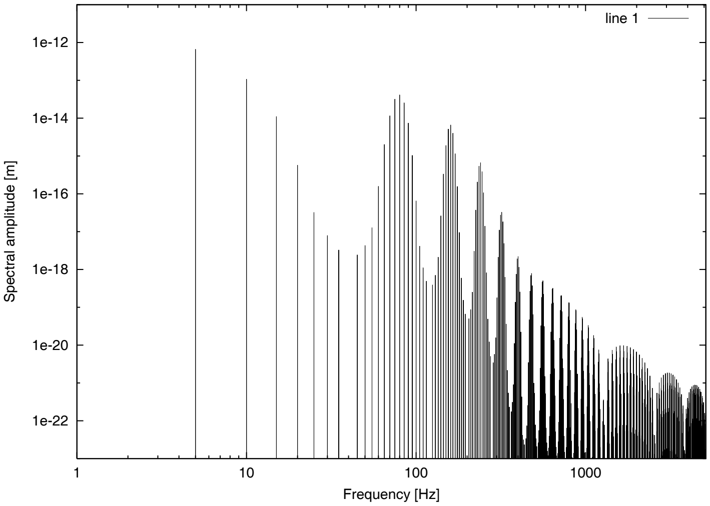

Figure 2: Amplitudes of the various spectral components of the effective mirror movement of Fig. 1.

of other frequencies at multiples of f_s. While the low-frequency contribution may be corrected by the interferometer locking servo, the higher frequency components, although rapidly decreasing in amplitude, are more worrying. Fig. 3 shows the effect of reducing the scan frequency by a factor of 10. Obviously the slow oscillations become 10 times stronger, because each zone has more time to heat up and cool down, but at high frequencies the amplitudes become much weaker. In Fig. 4, the scan resolution N was reduced while maintaining the total sweep area. The low frequency contribution is the same, but one observes reduced peak heights a high frequencies.

A better way to reduce perturbations in the Virgo sensitivity range may be to increase the scan frequency by not using mechanical actuators, but e.g. acousto-optic modulators (AOM) for beam deflection. Fig. 5 shows a simulation for a whole-surface scan frequency f_s of 5 kHz. Apart from being at the upper edge of the detection range, the fundamental peak has also a decreased amplitude due to the thermal inertia of the mirror.

4 Conclusions

Intuitively one might expect that it is better to use high scan frequencies in order to minimize the amplitude of the residual mirror pulsation. However, it seems to be better to use low scan frequencies for reducing the spectral components in the detection range of VIRGO. Low scan resolutions are favorable because they further reduce the frequencies involved in the scan. Perhaps other scan methods have better high-frequency characteristics; for example, circular scanning might be more appropriate for the circular symmetry of the YAG beam.

It remains to be determined whether the line spectrum generated by scanning is compatible with the Virgo requirements. A way to avoid this concern is to use non-mechanical beam deflectors, which allow pushing the fundamental frequency beyond the Virgo detection range.

The time-variable deformation of the mirror has the additional effect of creating higher-order TEM_{mn} modes on reflection, thus causing losses, matching effects etc. Whether these are worth while looking at, is yet unknown. Anyway, they have been neglected here.

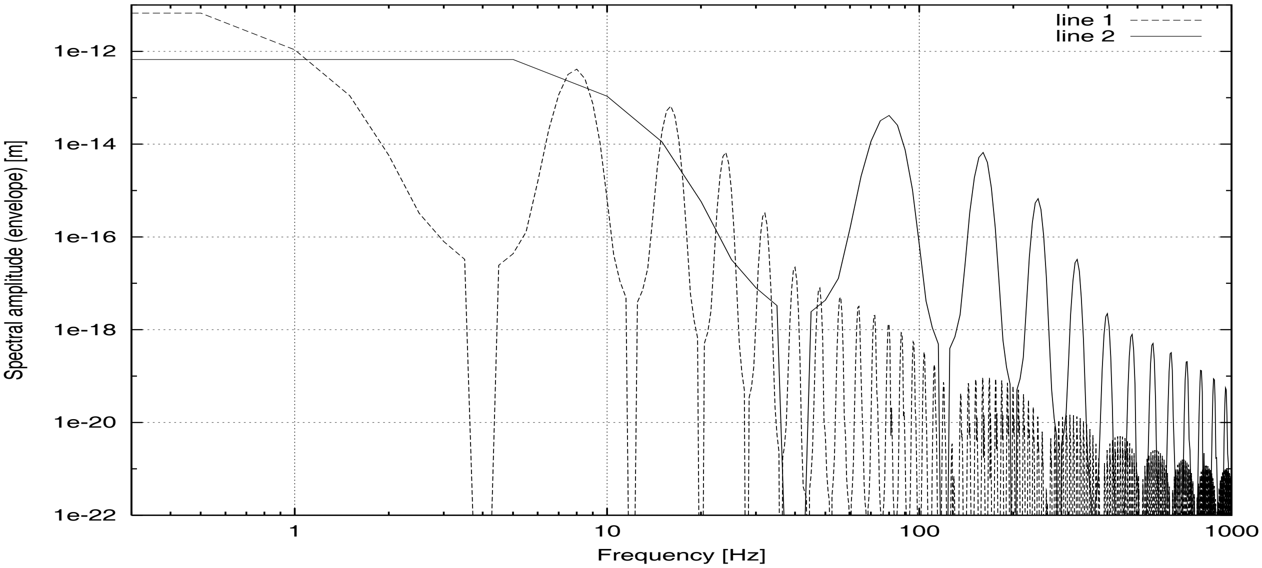

Figure 3: Comparison of spectra for f_s=0.5 Hz (line 1) and 5 Hz (line 2). The envelope of the spectral lines is shown for facilitating comparison.

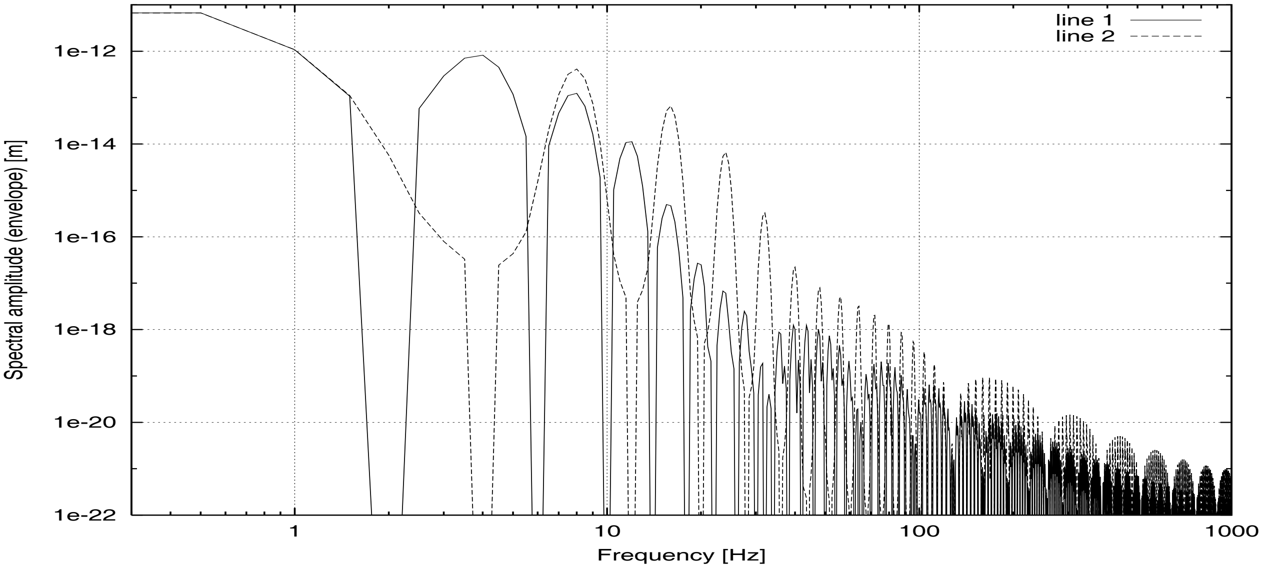

Figure 4: Spectra for N=8 (line 1) and N=16 (line 2) at f_s=0.5 Hz.

References

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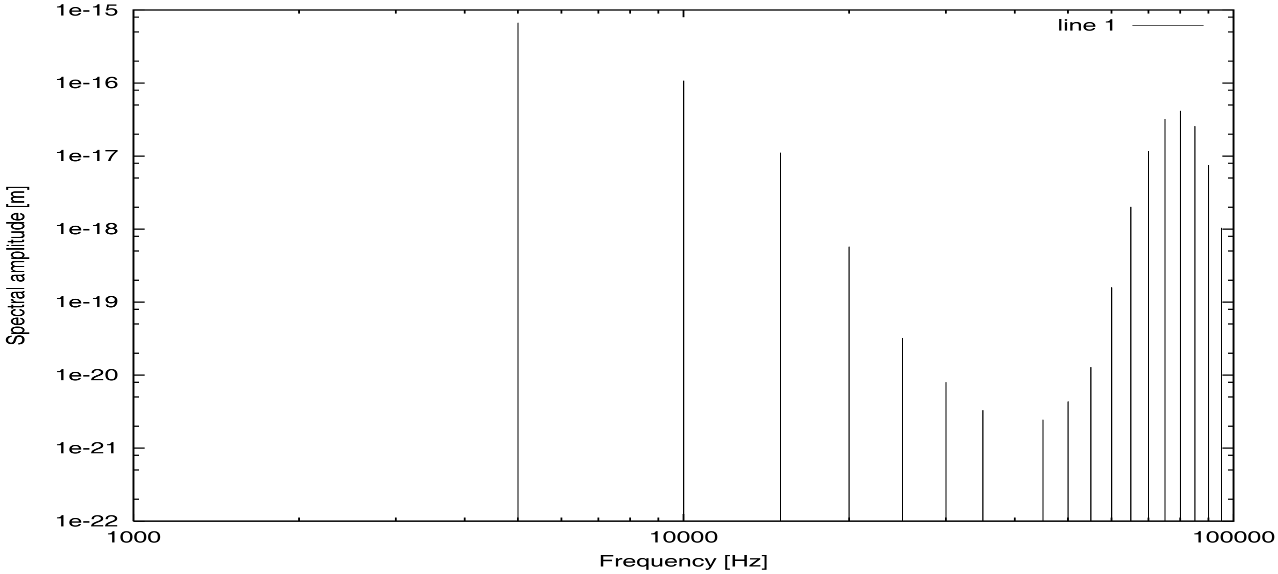

Figure 5: Spectum of mirror surface movement for N=16 at f_s=5 kHz.

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