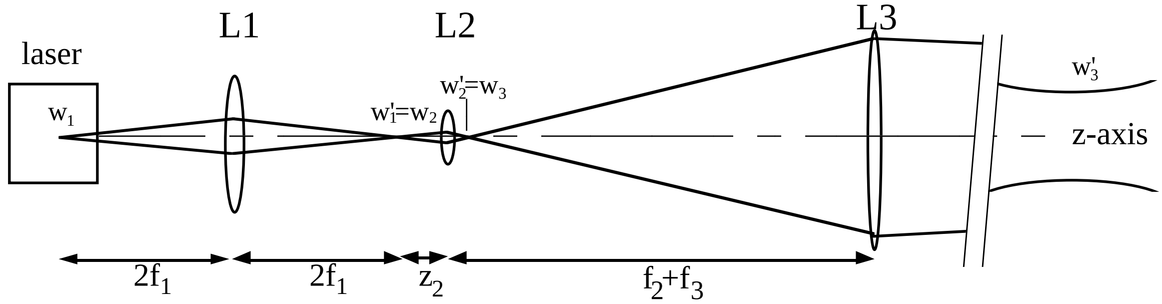
VIRGO	P J T 9 4 0 3 8
H. Heitmann 20.12.1994	**Subject :** **Design of a three-lens mode matching telescope**

Idea

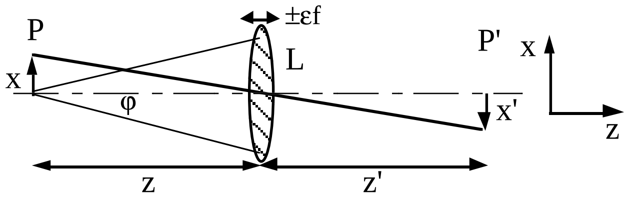
For the mode matching of a laser beam to a resonant cavity it is desirable to control size and position of the beam waist. This note suggests a simple three-lens design for a telescope which permits an independent control of both quantities by separating the functions of waist size change and displacement, which is not possible in two-lens systems. The principle can be illustrated as follows. The first lens (L_1) works approximately in the symmetrical 1:1 conjugate ratio, i.e. its output beam has the same waist size as the input beam. Under this condition, upon a slight shift of the lens position along the z axis, the output waist size w'_1 can be changed, to first order without changing the waist position. The following afocal telescope (L_2 and L_3) magnifies this beam waist by a factor α_t. If the distance z_2 is changed by Δz_2 (the telescope is shifted along the z axis), then the waist position will change by $\alpha_t^2\, \Delta z_2$ *without changing the waist size*.



The next two paragraphs treat this telescope in geometrical and Gaussian optics.

Geometrical optics

a) first lens (L_1)



The (transverse) magnification of a simple lens L is given by

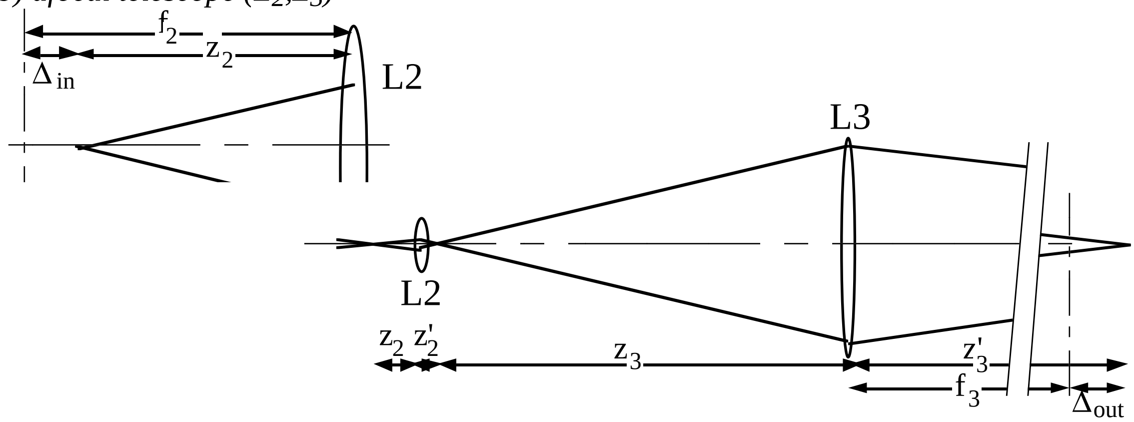
$$\alpha = \frac{x'}{x} = \frac{1}{z/f-1} = \frac{1}{1+\varepsilon} \approx 1-\varepsilon \quad \text{for } z/f = 2+\varepsilon \quad ; \qquad (1)$$

The distance between two conjugate points (P,P') on the axis is

$$z+z' = \frac{z^2}{z-f} = f\left(4 + \frac{\varepsilon^2}{1+\varepsilon}\right) \quad . \qquad (2)$$

As it can be seen, by changing the lens position along the z axis the magnification of L can be varied in the neighborhood of unity without, to first order, changing the axial position of P' with respect to P.

b) afocal telescope (L_2,L_3)



In an afocal telescope, the image side focus of the entrance lens coincides with the object side focus of the output lens. The (transverse) telescope magnification is given by the product of the magnifications of the individual lenses:

$$\alpha_t = \frac{x_3'}{x_2} = \alpha_2\,\alpha_3\,,\quad \text{where}\quad \alpha_2 = \frac{f_2}{z_2 - f_2} \quad \text{and} \quad \alpha_3 = \frac{f_3}{z_3 - f_3} \quad . \qquad (3)$$

With $z_3 = f_2 + f_3 - z_2'$ it follows $\alpha_t = f_3/f_2$. A similar calculation can be carried out for the beam opening angle φ, leading to an angular magnification of $\varphi_3'/\varphi_2 = 1/\alpha_t = f_2/f_3$.

The output focus position is given by

$$z_3' = f_3 + \left(\frac{f_3^2}{f_2^2}\right)(f_2 - z_2) \quad \text{or} \quad \Delta_{out} = \alpha_t^2\,\Delta_{in} \quad , \qquad (4)$$

where $\Delta_{in} = f_2 - z_2$ and $\Delta_{out} = z_3' - f_3$. This means, that when there is a point source in the object focal plane of L_2, then its image will be in the image side focal plane of L_3; if the point source is displaced in axial direction by Δ_{in}, then its image on the other side of the telescope will shift by $\alpha_t^2\,\Delta_{in}$ in the same direction. We can thus define a longitudinal magnification, whose value is the square of the transversal one.

A transition to Gaussian optics can now be made by relating the opening angle of the beam to the waist size via $\varphi = \lambda/\pi w_o$, and equating the location of the beam focus to the beam waist position. Thus the output waist can be shifted in axial direction by shifting the input waist. The transformation of the waist diameter is governed by the angular magnification, which is independent of the position. Thus the output waist can be shifted without affecting the waist size.

Gaussian optics

The formulæ for Gaussian beams are quite similar to the ones for geometrical optics. One has just to replace the axial distance z by the complex confocal parameter $q = z + i\,z_R$, where z is the distance from the waist, and $z_R = \pi w_o^2/\lambda$ is the Rayleigh length. The beam transformation by a lens becomes then

$$\frac{1}{f} = \frac{1}{q} - \frac{1}{q'} \quad , \qquad (5)$$

where the q's are measured at the lens, using the correct signs [1]. From this, another form relating the waist positions can be deduced:

$$\frac{1}{c + c_R^2/(c-1)} + \frac{1}{c'} = 1 \quad , \qquad (6)$$

where c = z/f etc. are the distances normalized by the lens focal length [2,3].

a) first lens (L_1)
Putting $z = -z'$ and $z_R = z_R'$ in (5) leads to the symmetric case

$$c_{I/II} = 1 \pm \sqrt{1-c_R^2} \approx \begin{cases} 2-c_R^2/2 \\ c_R^2/2 \end{cases} , \qquad (7)$$

which tends to the geometrical optics result for $c_R \to 0$ ($z_R \ll f$). Introducing a small detuning such that $c = c_I + \varepsilon$, leads to

$$c + c' \approx 4 + \varepsilon^2 - c_R^2(1-2\varepsilon) \quad . \qquad (8)$$

In the last equation there appears a term proportional to ε, if $c_R \neq 0$; this means that for a Gaussian beam the object-image waist distance changes a little bit on lens displacement in the symmetric case. So one needs to depart somewhat from the symmetric case to find a situation where the distance between input and output waist does not change. The lens magnification for Gaussian optics is given by [3]

$$\alpha = \frac{1}{\sqrt{(1-c)^2+c_R^2}} ,$$

and from (6) one gets the distance between object and image waist :

$$c + c' = \frac{c_R^2\,(c+1) + c^2\,(c-1)}{c_R^2 + (c-1)^2} \quad . \qquad (9)$$

If c+c' is not to change with c, the derivative of (9) must be zero; one finds 4 solutions satisfying this condition:

$$c_{1/2/3/4} = 1 \pm \sqrt{\frac{1}{2}\,(1-2c_R^2 \pm \sqrt{1-8c_R^2})} \qquad (10)$$

$$\approx \text{ (for small } c_R) \quad 1 \pm (1-\frac{3}{2}\,c_R^2) \quad \text{and} \quad 1 \pm c_R \quad . \qquad (11)$$

Including a small deviation ε, the magnifications for these 4 cases are:

Solution	1	2	3	4
Normalized object distance z/f = c =	$2-\frac{3}{2}c_R^2 + \varepsilon$	$\frac{3}{2}c_R^2 + \varepsilon$	$1 + c_R + \varepsilon$	$1 - c_R + \varepsilon$
Magnification α ≈	$1 + c_R^2 - \varepsilon$	$1 + c_R^2 + \varepsilon$	$\frac{1}{\sqrt{2c_R}}\left(1-\frac{\varepsilon}{2c_R}\right)$	$\frac{1}{\sqrt{2c_R}}\left(1+\frac{\varepsilon}{2c_R}\right)$

The first solution ($c \approx 2-3c_R^2/2$, $\alpha \approx 1+ c_R^2$) is similar to the classical symmetric case ($c=2$, $\alpha=1$) as treated in the last paragraph.

Thus one sees that also in the case of Gaussian beams the magnification of a lens can be varied without, to first order, changing the position of the output waist with respect to the input waist. If the Rayleigh length is small compared to the focal length of the lens, then the geometrical optics result can be applied.

b) afocal telescope (L_2,L_3)
From (5) and simple geometrical considerations, one obtains the beam transformations by L_2 and L_3:

$$q_2' = -\frac{f_2}{Q_2}(f_2+Q_2) \qquad (Q_2 = q_2-f_2 = \Delta_{in} + i\,z_{R2})$$
$$q_3 = q_2'+f_2+f_3 \qquad (12)$$

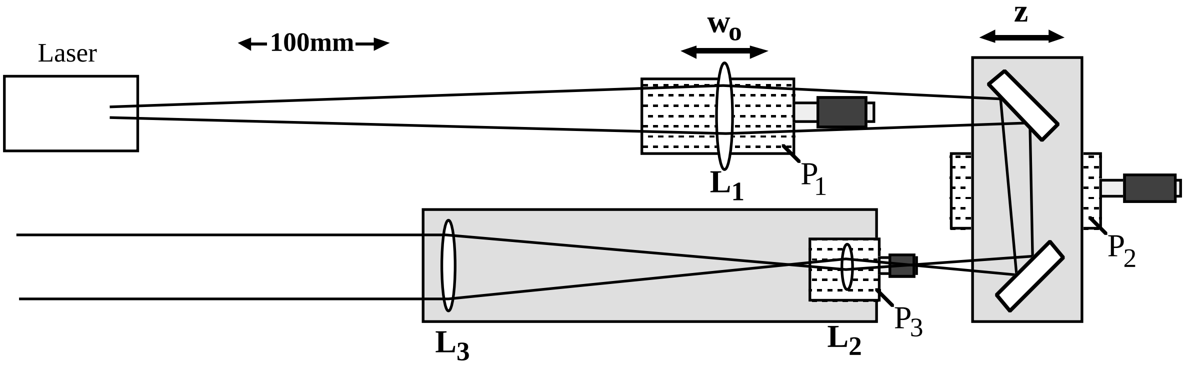
$$Q_3' = \frac{f_3^2}{f_3 - q_3} \qquad (Q_3' = q_3' + f_3 = \Delta_{out} + i\, z_{R3}) \quad .$$

Here the beam parameters q_2' and q_3 are measured at their respective lenses, and Q_2 and Q_3' at the object / image focal planes of L_2 / L_3. Then one gets

$$Q_3' = \alpha_t^2\, Q_2 \quad \text{or} \quad \Delta_{out} + i\pi\frac{w_3^2}{\lambda} = (\alpha_t^2\, \Delta_{in}) + i\pi\frac{(\alpha_t w_2)^2}{\lambda} \quad , \qquad (13)$$

which gives $w_3 = \alpha_t w_2$ and $\Delta_{out} = \alpha_t^2\, \Delta_{in}$. Thus, as for the geometrical optics case, the telescope magnifies the waist *size* by α_t and an input waist *position* change by α_t^2.

Example: input telescope for the Orsay mode cleaner prototype



The Orsay mode cleaner prototype is a 30 m long ring cavity, whose optical components are suspended as pendulums under vacuum. The beam from a compact (palm top) Nd:YAG ring laser must be made resonant in it, and consequenctly needs to be matched. As an application of the above considerations, this section gives the constuctive details of an appropriate telescope.

The optical characteristics are:

		laser	MC cavity
waist size	w_o	0.1 mm	3.7 mm
Rayleigh length	z_R	30 mm	40.4 m
beam half angle	φ	3.4 mrad	90 µrad
length	l		30 m
mirror curvature	R		85 m

For the first lens, a focal length of 200 mm was chosen in order to give enough space (400 mm to each side) for additional optical components. So c_R is small (0.15), and the geometrical approximation for the first lens is quite good. L_1 reproduces the input beam waist at a distance of 786mm from the laser, with the possibility of a magnification or reduction. For the following afocal telescope (L_2,L_3) a magnification α_t=37 is required in order to increase the beam waist from 0.1 to 3.7 mm. Space constraints forbid a very long focal length for L_3; the values chosen for f_2 and f_3 are 6.4mm and 250mm, respectively; the distance between L_2 and L_3 is thus 256.4mm. This length should be quite stable, since otherwise the telescope will no longer be afocal as assumed in the calculations, and because a change of 1mm would lead to an output waist shift of 12m. The magnification resulting from the lens data is 39; the resulting slight waist size mismatch must be corrected by shifting L_1. The beam path between L_1 and the afocal telescope is folded using two flat mirrors, which saves space and, by axial shifts, gives the possibility of changing the distance L_1–L_2 without changing the other distances. Thus the waist size can be changed by shifting L_1 with P_1, and the waist position by moving the block carrying the two flat mirrors with P_2. P_3 serves for fine tuning the distance L_2–L_3.

From the above formulæ, the sensitivity to shifts Δz_P of the translational stages P_1 and P_2 can be calculated:

waist size	**waist position**
$\frac{\Delta w_3'}{w_3'} = (1+c_R^2)\,\frac{\Delta z_{P1}}{f}$	$\Delta z_3' = \alpha_t\, 2\Delta z_{P2}$

From this results the maximum tuning range, supposing that P_1 and P_2 can be shifted by ±25mm:

$\Delta z_{P1} = \pm$ 25mm max. => $\frac{\Delta w_3'}{w_3'} = \pm 12.5\%$ max.(f=200mm) => w_3' = 3.25 ... 4.15mm	$\Delta z_{P2} = \pm$ 25mm max. => $\Delta z_3' = \pm 1.7\, z_R$ max. = ± 68.5m

The tuning range of w_3' can be further increased by reducing the focal length of L_1 (e.g. f=100mm).

[1] H. Kogelnik, T. Li, "Laser beams and resonators"; Appl. Opt. **5**, 1550 (1966)
[2] S.A. Self, "Focusing of spherical optical beams", Appl. Opt. **22**, 658 (1983)
[3] Melles Griot, Optics Guide