Characteristics of the rotor R4-07 for the O4 NCal system VIR-0861A-22

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Contents

1 Introduction

This note follows the same discussion made on the rotor R4-01 in VIR-0591C-22 and R4-05 in VIR-0859A-22. The drawings and figures of the rotor can be found in this first technical note.

The rotor has been engraved IPHC-R4-07 on one side and sandblasted on the other side.

2 Measurement method

To determine the geometry of the rotor we will use the same method as for R4-01 (see VIR-0591C-22). The thickness was measured using 24*2 = 48 points (see fig. 1), the outer diameter was measured in 4*2 = 8 points and the inner diameter using 4 points.

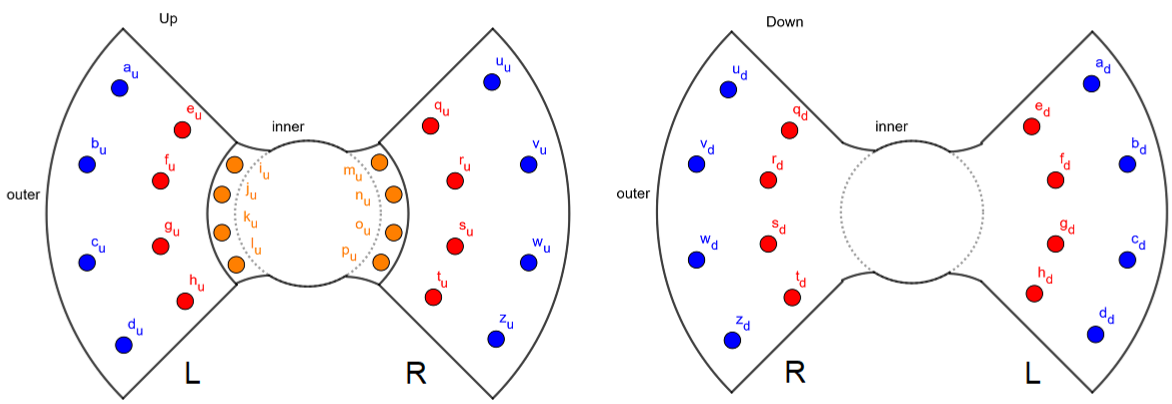

Figure 1: Outline of the faces of the rotor with the measurement points. Left figure is face up, right figure is face down. Sectors have been labelled L for left sector and R for right sector.

The tool used to measure the thickness and the outer diameter is a measuring column "Garant 44 5350_600 HC1" (see VIR-0160A-22) with a given precision of 1.8+L/600 μm (L the measured length in mm). A vernier caliper "TESA-CAL IP67" with a precision of 20 μm was used to measure the inner diameter.

The measuring column was operated on a metrology table with a value range from 0 to 2 μm. The rms of the 16 values is 0.9 μm.

We measured the opening angles of the sectors using a video measuring microscope "Garant MM2" (see VIR-0591C-22) with a given precision of 2.9+L/100 μm at 95% CL (L the measured length in mm).

2.1 Thermal effects and density

The rotor R4-07 has been machined from the same aluminum block as R4-05 described in VIR-0859A-22. As for R4-05 the results will be expressed at a reference temperature of 23°C.

The density of the rotor R4-07 is then 2810.8 $\pm$ 0.2 kg.m^{-3}. This density is measured in air, if the rotor is used under vacuum, the density should be increased by the air density (ρ_{air}=1.3 kg.m^{-3}).

3 Raw measurements of the rotor

This section presents the raw measurements made on the rotor at the ambient temperature of 24.5°C. Table 1 shows the thickness measurements according to the measurement points defined in figure 1. The rotor is laying on the table. The rotor surface as well as the table are not perfectly flat. Some space could be present in between that should be substracted when computing the rotor thickness as discussed later.

Measurement point	L sector		Measurement point	R sector	
	Up	Down		Up	Down
a	104.406	104.416	q	104.403	104.413
b	104.388	104.403	r	104.392	104.403
c	104.367	104.390	s	104.373	104.389
d	104.347	104.387	t	104.359	104.383
e	104.410	104.406	u	104.407	104.409
f	104.397	104.398	v	104.398	104.394
g	104.387	104.376	w	104.389	104.376
h	104.382	104.357	z	104.382	104.356
i	101.502		m	101.499	
j	101.501		n	101.497	
k	101.498		o	101.495	
l	101.499		p	101.496	

Table 1: Raw measurements of the height in mm for each point at 24.5°C on L and R sectors of R4-07.

Table 2 displays the diameter measurements. The measurements were made on 4*2 diameters (two parts of each diameter, the up and down sides of the rotor).

Measurement point	Up	Down
1	207.974	207.957
2	207.987	207.984
3	207.985	207.985
4	207.950	207.958

Table 2: Raw measurements of the diameter in mm for each point at 24.5°C on R4-07.

Theoretical values were taken for the inner radius r_{min} = 29 mm and the up face radius for the counterweight $r_{\text{counterweight}}$ = 40 mm (see drawing in VIR-0591C-22).

4 Extracting the geometrical parameters

4.1 Thickness

We need to correct the possible gap between the rotor and the measuring table. Assuming that the table is flatter than the rotor surface we can extract the gap from the measurement of the top surface considering the plane tangents to the highest points (asking them to be on both sectors). For this rotor these points are a, e, q for the up and e, q, u for the down face (see fig. 1). Using the measurements in table 1 we can compute a plane equation for each side of the rotor in cartesian coordinates:

$$\text{Up plane equation} : z = 6.65 \times 10^{-5}x - 1.27 \times 10^{-4}y + 104.42 \quad (1)$$

$$\text{Down plane equation : } z = 6.65 \times 10^{-5}x - 3.20 \times 10^{-4}y + 104.42 \quad (2)$$

Using eqs. (1) and (2) the gap can be determined, see table 3. The maximum rms of the gap for each sector is 25 μm.

Measurement point	L sector		Measurement point	R sector	
	Up	Down		Up	Down
a	0	-4	q	0	0
b	21	17	r	16	17
c	46	47	s	32	38
d	71	72	t	37	50
e	0	0	u	10	0
f	15	15	v	22	26
g	28	31	w	49	55
h	36	45	z	71	84

Table 3: Gap computed in μm on up and down sides of both sectors of R4-07.

We can then compute the rotor thickness for each point by removing these gaps. If one of the raw values is lower than the corrected thickness we take this lowest value. The value of each point is shown in table 4 at 23°C.

Measurement point	L sector	Measurement point	R sector
a	104.399	q	104.409
b	104.384	r	104.399
c	104.363	s	104.385
d	104.343	t	104.379
e	104.403	u	104.402
f	104.393	v	104.390
g	104.383	w	104.385
h	104.378	z	104.379
i	101.498	m	101.495
j	101.497	n	101.493
k	101.494	o	101.491
l	101.495	p	101.492

Table 4: Measurements of the thickness in mm for each point at 23°C on L and R sectors of R4-07.

4.2 Radius

Using comparators while the rotor is rotating on its axis we can determine the deformation on both sectors and compute different radii values. Table 5 shows the raw measurements using comparators on L and R sectors. The measurements were made on the up, center and down sides of L and R sectors using three comparators for a total of 5*3*2=30 points (the first and last points are near the edge of the sectors).

Measurement point	L sector			R sector		
	Up	Center	Down	Up	Center	Down
A	10	0	-10	-10	-10	0
B	30	35	45	30	30	35
C	40	45	50	40	45	40
D	30	35	35	35	30	30
E	0	0	0	0	-5	-5

Table 5: Raw measurements in μm of the comparators for the L and R sectors of R4-07.

The zeroing of the comparators was made arbitrarily close to the edge of the sector. The offsets shown in table 5 are measured relative to this reference.

To compute the radius per measurement point we use the following process: First we compute the mean deformation for one comparator. Then we remove this mean deformation to each measurement of this comparator. The corrected shift value is added to the mean radius of 103.983 mm computed using table 2 at 23°C. This process is repeated for each comparator. The final radius for each point are shown in table 6.

Measurement point	L sector			R sector		
	Up	Center	Down	Up	Center	Down
A	103.976	103.966	103.954	103.956	103.956	103.964
B	103.996	104.001	104.009	103.996	103.996	103.999
C	104.006	104.011	104.014	104.006	104.011	104.004
D	103.996	104.001	103.999	104.001	103.996	103.994
E	103.966	103.966	103.964	103.966	103.961	103.959

Table 6: Radius measurements in mm at 24.5°C for the L and R sectors of R4-07.

5 Characterization of the rotor using a simple model

5.1 Theoretical model of the rotor

As explained in VIR-0591C-22 the theoretical strain of the mirror at 2f using FROMAGE v1r2 with the rotor design parameters:

$$\text{strain(2f)} = \frac{2.1217 \times 10^{-18}}{(2f_{rot})^2}$$

This strain value will be compared to models based on the measurements of the rotor.

5.2 Thickness

A simple model can be used to determine a mean value for the thickness and its uncertainty.

As shown on fig. 1, a total of 16 points were used to compute the thickness of each sector. In this case we will not consider the inner points so that we obtain uniform sectors.

For the simple model we take the thickness as the mean value of table 4: 104.384 mm at 23°C. Since we have a limited number of measurement points, to be conservative we take the thickness uncertainty as the rms of table 4 (18.6 μm) to which we add linearly the metrology table uncertainty (0.9 μm) and the tool uncertainty (2.0 μm). Therefore, for this simple model, the thickness is 104.384 $\pm$ 0.021 mm.

5.3 Radius

For the simple model we take the radius as the mean value of table 6: 103.983 mm at 23°C. Using a linear sum of the rms of table 6 (20.4 μm) and the tool uncertainty (2.2 μm) we take an uncertainty of 23 μm on the mean radius.

We have to point out that we do not take into account the fact that the sectors might not be centered on the same axis. Therefore the uncertainty might be underestimated. We will then consider each sector individually later.

5.4 Expected NCal signal and uncertainties

The geometry used to describe the rotor as a simple model is represented in figure 2.

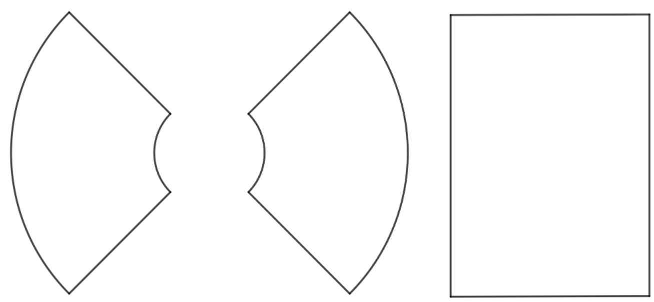

Figure 2: Simple model geometry used to describe the rotor. Left is a front view, right is a side view.

Using the analytical equation of the strain at 2f (see eq.8 in Newtonian calibrator tests during the Virgo O3 data taking) we compute with our parameters (d = 1.7 m and an angle ϕ = 34.7°):
strain(2f) $= \frac{2.1200 \times 10^{-18}}{(2f_{rot})^2}$.

Using FROMAGE on this geometry we compute the following 2f strain on the mirror at a distance of 1.7m and an angle of 34.7°:
strain(2f) $= \frac{2.1201 \times 10^{-18}}{(2f_{rot})^2}$

Comparing the theoretical model strain with the simple model at 2f using FROMAGE we obtain a relative deviation of 0.075%.

Comparing the analytical strain at 2f with FROMAGE we obtain a relative deviation of 0.005%.

The uncertainties considered for this model are displayed in table 7.

R4-07 rotor parameter simple model (23°C)			NCal 2f signal uncertainty	
name	value	uncertainty	formula	value (%)
Density ρ (kg.m^{-3})	2810.8	0.2	$\delta\rho/\rho$	0.007
Thickness b (mm)	104.384	2.1×10^{-2}	$\delta b/b$	0.021
r_{max} (mm)	103.983	2.3×10^{-2}	$4\delta r_{max}/r_{max}$	0.087
G (m^3.kg^{-1}.s^{-2})	$6.674\,30 \times 10^{-11}$	1.5×10^{-15}	$\delta G/G$	0.002
Temperature T (°C)	23	3	$\left\lvert \frac{\partial h}{\partial T} \right\rvert \frac{\Delta T}{h}$	0.014
Quadratic sum				0.090

Table 7: Uncertainties on the amplitude of the calibration signal at 2f from the R4-07 rotor simple model geometry..

6 Characterization of the rotor using an advanced model

6.1 Thickness

A more advanced model can be used considering the deformations on the surfaces of the sectors for better accuracy. Each measurement point of table 4 can be considered as a sub-sector with its own thickness.

The uncertainty on this value is more complex to evaluate. As a conservative approach we use the maximum rms of the deviation to a plane for each sector (25 μm see section 4.1) to which we add linearly the uncertainty on the flatness of the measurement table (0.9 μm) as well as the measurement tool (2.0 μm). The total uncertainty on the thickness is 29 μm.

6.2 Radius

On fig. 1 we divided the external sectors in 4 sub-sectors for each sector (blue points). We convert the point of table 6 to the grid of fig. 1 by averaging the two closest values and converting them to 23°C. The results are shown in table 8. We notice that the L sector is on average 3 μm larger than the R sector.

Radius	L sector			R sector		
	Up	Center	Down	Up	Center	Down
1	103.982	103.980	103.978	103.972	103.972	103.978
2	103.997	104.002	104.008	103.997	104.000	103.998
3	103.997	104.002	104.003	104.000	104.000	103.996
4	103.977	103.980	103.978	103.980	103.975	103.973

Table 8: Radius measurements (in mm at 23°C) for the L and R sectors of R4-07.

The maximum rms of the radii for each sector is 12.2 μm. The tool uncertainty is 2.2 μm. Like for the thickness we use a linear sum and find the uncertainty on both radii to be 14 μm.

6.3 Counterweight

A counterweight has been designed to reduce the unbalance of the rotor using the same method as R4-06 in section 6.3 of VIR-XXXXX-22. This counterweight is made of aluminum 2017 ($\rho_{\mathrm{Al}_{2017}} = 2790$ kg.m^{-3}) and the geometry is shown in fig. 3.

The dimensions of the counterweight are: an inner radius of 10 mm, an outer radius of 40 mm and a material cut with a chord of 65.22 mm to balance the rotor (see hatched are on fig. 3).

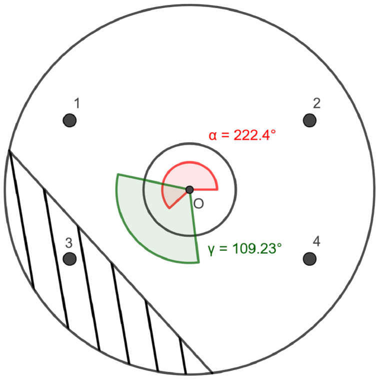

Figure 3: Outline of the counterweight for R4-07. The hatched area represents the material removed.

6.4 Opening angles and asymmetry

The opening angles of the full and empty sectors have been measured using a video microscope with the same method as for R4-01. The uncertainty on α = 0.2 mrad is the same as for R4-01 giving an uncertainty of 4×10^{-6} % on the 2f signal which is neglectable.

The measurements are shown in table 9. The center value corresponds to the mean of up and down measurements.

Opening angle	Up	Center	Down
L	1.57084	1.57078	1.57073
R	1.57058	1.57066	1.57074
L-R	1.57096	1.57104	1.57112
R-L	1.57080	1.57069	1.57059

Table 9: Opening angle measurements in rad for the L, R full sectors and L-R, R-L empty sectors of R4-07.

These measurements allow us to compute the signal with different opening angles and an asymmetry between the sectors. These measured opening angles will be included in the advanced model described in the next section.

6.5 Expected NCal signals and uncertainties

6.5.1 Advanced geometry including chamfers and counterweight

The geometry used to describe the rotor as an advanced model is represented in figure 4. The external parts of the sectors are divided in 3 sub-sectors each to correspond to the different radii determined. In addition we include the counterweight, the screws, the screw holes, the opening angles and asymmetry of the sectors.

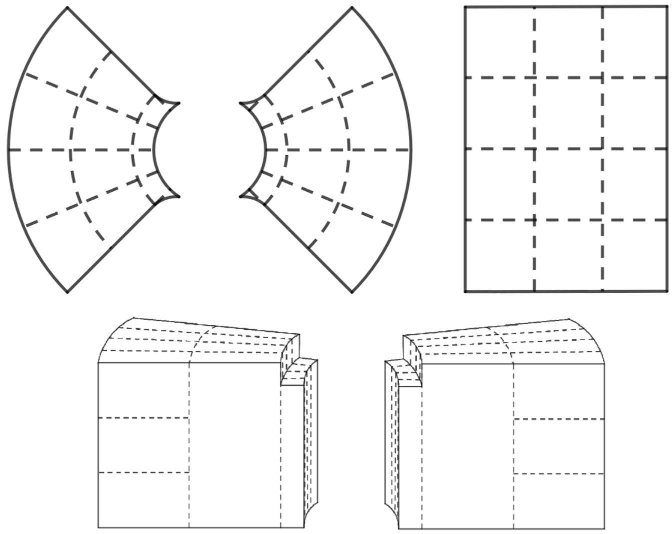

Figure 4: Advanced model geometry used to describe the rotor. Top left is a front view, top right is a side view (external sub-sectors) and bottom is a tilted view of the sectors. Only the 4 external part sectors are divided in 3 sub-sectors each. The chamfers are visible on the inner radius.

Using FROMAGE on this geometry gives the following strains:

- strain(1f) = $\dfrac{3.1396 \times 10^{-20}}{(1f_{rot})^2}$
- strain(2f) = $\dfrac{2.1193 \times 10^{-18}}{(2f_{rot})^2}$
- strain(3f) = $\dfrac{4.1207 \times 10^{-23}}{(3f_{rot})^2}$

The relative deviation to the simple model at 2f is 0.038%.

6.5.2 Remaining geometry uncertainty

Since R4-02 and following have been machined with a better precision than R4-01 as seen from the lower 1f value. The remaining geometry uncertainties are taking as for R4-01.

6.5.3 Uncertainties

To set an uncertainty on the strain(2f) from the description of the geometry we take the difference between the simple model (strain(2f) = $2.1201 \times 10^{-18}/(2f)^2$) and the advanced model (strain(2f) = $2.1193 \times 10^{-18}/(2f)^2$). This deviation, 0.038%, is reported in table 10 as modelling uncertainty.

The uncertainties considered for this full model are displayed in table 10.

R4-07 rotor parameter advanced model (23°C)			NCal 2f signal uncertainty	
name	mean value	uncertainty	formula	value (%)
Density ρ (kg.m^{-3})	2810.8	0.2	$\delta\rho/\rho$	0.007
Thickness b left sector (12 sub-sectors) (mm) Thickness b right sector (12 sub-sectors) (mm)	104.381 104.386	2.9×10^{-2}	$\delta b/b$	0.028
r_{max} left sector (12 ext sub-sectors) (mm) r_{max} right sector (12 ext sub-sectors) (mm)	103.990 103.987	1.4×10^{-2}	$4\delta r_{max}/r_{max}$	0.055
G (m^3.kg^{-1}.s^{-2})	$6.674\,30 \times 10^{-11}$	1.5×10^{-15}	$\delta G/G$	0.002
Temperature T (°C)	23	3	$\left\lvert\frac{\partial h}{\partial T}\right\rvert \frac{\Delta T}{h}$	0.014
Modelling Uncertainty				0.038
FROMAGE grid uncertainty				0.005
Opening angle and sector asymmetry uncertainty				$<5 \times 10^{-4}$
Remaining geometry uncertainty				$<5 \times 10^{-4}$
Total uncertainty from the rotor (quadratic sum)				0.074

Table 10: Uncertainties on the amplitude of the calibration signal at 2f from the R4-07 rotor advanced model geometry at 23°C.

A Appendix

```
### This is a cfg file for a more realistic geometry of the mirror and the Virgo NCal R4-07 (2022)

### ALL THE OBJECTS ARE DEFINED IN THE MIRROR'S FRAME (O,x,y,z),
### with O the center of the mirror, x axis along the ITF's beam toward the beam-splitter,
### y axis orthogonal to x in the plane of the ITF,
### z axis orthogonal to the plane of the ITF upward

### MIRROR DEFINITION

GRID_SIZE 12 30 8

CYLINDER 2202. 0 0.175 0.2 360 0 0 0

GRID_SIZE 1 1 1

# Defining the flats on the edge of the mirror
CUT_CYL 2202. 0.175 0.2 0.05 0 0
CUT_CYL 2202. 0.175 0.2 0.05 0 180

# Defining the ears and anchors of the mirror
CUBOID 2202. 0.090 0.010 0.015  0     0.1782 -0.0125
CUBOID 2202. 0.090 0.010 0.015  0    -0.1782 -0.0125
CUBOID 2202. 0.039 0.008 0.008 -0.02 -0.1772 -0.024
CUBOID 2202. 0.039 0.008 0.008 -0.02  0.1772 -0.024
CUBOID 2202. 0.039 0.008 0.008  0.02 -0.1772 -0.024
CUBOID 2202. 0.039 0.008 0.008  0.02  0.1772 -0.024

### ROTOR DEFINITION: CYLINDER DENSITY INNER_RADIUS OUTER_RADIUS THICKNESS OPEN_ANGLE r z theta

ROTOR_CYLINDRICAL 1.7 34.7 0 0

### COUNTERWEIGHT 2790.
GRID_SIZE 16 65 40
CYLINDER 2790. 0.010 0.040 0.003 360 0 0.0478580987761587 0
GRID_SIZE 1 1 1
CUT_CYL 2790. 0.040 0.003 0.06522 0.0478580987761587 222.4

### SCREW HOLES
GRID_SIZE 1 1 1
CYLINDER -2810.8 0 0.0015 0.012 360 0.03 0.04474920341 150
CYLINDER -2810.8 0 0.0015 0.012 360 0.03 0.04474770347 30
CYLINDER -2810.8 0 0.0015 0.012 360 0.03 0.04474770347 210
CYLINDER -2810.8 0 0.0015 0.012 360 0.03 0.04474620352 330

CYLINDER -2790. 0 0.0015 0.002 360 0.03 0.0478580987761587 150
CYLINDER -2790. 0 0.0015 0.002 360 0.03 0.0478580987761587 30
CYLINDER -2790. 0 0.0015 0.002 360 0.03 0.0478580987761587 210
CYLINDER -2790. 0 0.0015 0.002 360 0.03 0.0478580987761587 330

### SCREWS COUNTERWEIGHT
GRID_SIZE 1 1 1
CYLINDER 7600. 0 0.0015 0.00965 360 0.03 0.04303309877616 150
CYLINDER 7600. 0 0.0015 0.00965 360 0.03 0.04303309877616 30
CYLINDER 7600. 0 0.0015 0.00965 360 0.03 0.04303309877616 210
```

```
CYLINDER 7600. 0 0.0015 0.00965 360 0.03 0.04303309877616 330

# TRES RAPIDE
#GRID_SIZE 4 4 4
# RAPIDE
GRID_SIZE 8 17 14
# LENT
#GRID_SIZE 8 65 40

### L sector

## Inner part
OUTER_FILLET 2810.8 0.029 0.101498 -0.002917 0.01 -11.2499 146.2503
CYLINDER 2810.8 0.029 0.04 0.101498 22.4998 0 -0.002917 146.2503
CYLINDER 2810.8 0.029 0.04 0.101497 22.4998 0 -0.002919 168.7501
CYLINDER 2810.8 0.029 0.04 0.101494 22.4998 0 -0.002917 191.2499
CYLINDER 2810.8 0.029 0.04 0.101495 22.4998 0 -0.002917 213.7497
OUTER_FILLET 2810.8 0.029 0.101495 -0.002917 0.01 11.2499 213.7497

## Middle part
CYLINDER 2810.8 0.04 0.071997 0.104403 22.4998 0 0 146.2503
CYLINDER 2810.8 0.04 0.071997 0.104393 22.4998 0 0 168.7501
CYLINDER 2810.8 0.04 0.071997 0.104383 22.4998 0 0 191.2499
CYLINDER 2810.8 0.04 0.071997 0.104378 22.4998 0 0 213.7497

## Outer part
CYLINDER 2810.8 0.071997 0.103982 0.03479955469965 22.5006 0 0.03479955469965 146.2491
CYLINDER 2810.8 0.071997 0.103980 0.03479955469965 22.4998 0 0 146.2503
CYLINDER 2810.8 0.071997 0.103978 0.03479955469965 22.4990 0 -0.03479955469965 146.2515

CYLINDER 2810.8 0.071997 0.103997 0.0347947682216 22.5006 0 0.0347947682216 168.7497
CYLINDER 2810.8 0.071997 0.104002 0.0347947682216 22.4998 0 0 168.7501
CYLINDER 2810.8 0.071997 0.104008 0.0347947682216 22.4990 0 -0.0347947682216 168.7505

CYLINDER 2810.8 0.071997 0.103997 0.034787768469 22.5006 0 0.034787768469 191.2503
CYLINDER 2810.8 0.071997 0.104002 0.034787768469 22.4998 0 0 191.2499
CYLINDER 2810.8 0.071997 0.104003 0.034787768469 22.4990 0 -0.034787768469 191.2495

CYLINDER 2810.8 0.071997 0.103977 0.0347811020387333 22.5006 0 0.0347811020387333 213.7509
CYLINDER 2810.8 0.071997 0.103980 0.0347811020387333 22.4998 0 0 213.7497
CYLINDER 2810.8 0.071997 0.103978 0.0347811020387333 22.4990 0 -0.0347811020387333 213.7485

### R sector

## Inner part
OUTER_FILLET 2810.8 0.029 0.101495 -0.002878 0.01 11.2490 33.7375
CYLINDER 2810.8 0.029 0.04 0.101495 22.4980 0 -0.002878 33.7375
CYLINDER 2810.8 0.029 0.04 0.101493 22.4980 0 -0.002878 11.2394
CYLINDER 2810.8 0.029 0.04 0.101491 22.4980 0 -0.002878 348.7414
CYLINDER 2810.8 0.029 0.04 0.101492 22.4980 0 -0.002875 326.2434
OUTER_FILLET 2810.8 0.029 0.101492 -0.002875 0.01 -11.2490 326.2434

## Middle part
CYLINDER 2810.8 0.04 0.071997 0.104409 22.4980 0 0 33.7375
CYLINDER 2810.8 0.04 0.071997 0.104399 22.4980 0 0 11.2394
CYLINDER 2810.8 0.04 0.071997 0.104385 22.4980 0 0 348.7414
```

```
CYLINDER 2810.8 0.04 0.071997 0.104379 22.4980 0 0 326.2434

## Outer part
CYLINDER 2810.8 0.071997 0.103973 0.0348007680092 22.4969 0 0.0348007680092 33.7357
CYLINDER 2810.8 0.071997 0.103970 0.0348007680092 22.4980 0 0 33.7375
CYLINDER 2810.8 0.071997 0.103971 0.0348007680092 22.4992 0 -0.0348007680092 33.7392

CYLINDER 2810.8 0.071997 0.103993 0.0347967681508 22.4969 0 0.0347967681508 11.2389
CYLINDER 2810.8 0.071997 0.103990 0.0347967681508 22.4980 0 0 11.2394
CYLINDER 2810.8 0.071997 0.103993 0.0347967681508 22.4992 0 -0.0347967681508 11.2400

CYLINDER 2810.8 0.071997 0.103988 0.0347907683632 22.4969 0 0.0347907683632 348.7420
CYLINDER 2810.8 0.071997 0.103982 0.0347907683632 22.4980 0 0 348.7414
CYLINDER 2810.8 0.071997 0.103991 0.0347907683632 22.4992 0 -0.0347907683632 348.7409

CYLINDER 2810.8 0.071997 0.103958 0.0347841019325333 22.4969 0 0.0347841019325333 326.2451
CYLINDER 2810.8 0.071997 0.103957 0.0347841019325333 22.4980 0 0 326.2434
CYLINDER 2810.8 0.071997 0.103968 0.0347841019325333 22.4992 0 -0.0347841019325333 326.2417

### GENERAL PARAMETERS

STEP 22.5 16

ARM_LENGTH 3000

SIGNAL 3
```