Modelling uncertainty of the O4 NCal rotors using FROMAGE VIR-0854A-24

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1 Introduction

Initially, to compute the uncertainty of the FROMAGE modelling method on the signal, we compared the results of the simple and the advanced models simulations. The modelling uncertainty was taken as the difference between both models. This was reported in the technical notes describing each rotor, see for instance section 7.5.5 of VIR-0591C-22 for the first aluminum rotor R4-01 and section 6.5.3 of VIR-0203A-24 for the first PVC rotor R4-10. However, this approach overestimates the uncertainties, as the uncertainty in thickness and radius is already accounted for in the rotor geometry uncertainty. Moreover, this method does not assess modelling errors in FROMAGE.

A more detailed method was therefore developed and is described in this technical note. The results of this method were reported in the publication describing the NCal system at the start of O4b [1].

Additionally, reassessed measurement results for the thickness and external radius of some rotors, along with values for the second set of PVC rotors from VIR-0441A-24, are presented in tables 1 to 3.

2 Modelling method

If we had a perfect knowledge of the rotor geometry with a very large number of measuring points, the NCal signal could be computed accurately using FROMAGE. But we use fairly large elements in FROMAGE, with only one measurement for the thickness and radius. To compute the NCal signal uncertainty, we now make the assumption that the thickness and radius fluctuations within an element are not larger than the fluctuation measured between rotor elements. Then we made 1000 FROMAGE simulations where the thickness and radius are randomly changed following the normal distributions:

- Thickness $X(b) \sim \mathcal{N}(\mu = b_{nom}, \sigma = \delta b)$, with $b_{nom} = 104.4$ mm the nominal thickness of the rotor and δb taken for each rotor from table 1.
- Outer radius $X(r_{max}) \sim \mathcal{N}(\mu = r_{max,nom}, \sigma = \delta r_{max})$, with $r_{max,nom} = 104$ mm the nominal rotor radius and δr_{max} taken for each rotor from table 2.

Rotor	Thickness b at 23°C			
	mean value b left sector [mm]	mean value b right sector [mm]	uncertainty δb [μm]	NCal 2f signal uncertainty $\delta b/b$ [%]
R4-01	104.217	104.210	6	**0.006**
R4-02	104.279	104.305	11	**0.011**
R4-03	104.300	104.324	82	**0.082**
R4-04	104.349	104.345	10	**0.010**
R4-05	104.441	104.404	55	**0.052**
R4-06	104.337	104.342	33	**0.032**
R4-07	104.356	104.335	14	**0.013**
R4-08	104.237	104.236	15	**0.014**
R4-10	104.416	104.415	9	**0.008**
R4-11	104.407	104.411	12	**0.011**
R4-12	104.400	104.399	11	**0.010**
R4-13	104.418	104.414	10	**0.010**
R4-14	104.444	104.441	11	**0.010**
R4-15	104.422	104.418	9	**0.008**
R4-16	104.419	104.419	8	**0.008**
R4-17	104.416	104.410	9	**0.009**

Table 1: Thickness of the sectors from the advanced rotors geometry. Updated values are highlighted in yellow.

Rotor	Radius r_{max} at 23°C			
	mean value r_{max} left sector [mm]	mean value r_{max} right sector [mm]	uncertainty δr_{max} [μm]	NCal 2f signal uncertainty $4\delta r_{max}/r_{max}$ [%]
R4-01	104.006	103.987	12	**0.046**
R4-02	104.017	104.016	10	**0.039**
R4-03	104.023	104.021	11	**0.040**
R4-04	104.010	104.016	14	**0.052**
R4-05	103.971	103.968	14	**0.055**
R4-06	103.985	103.978	14	**0.059**
R4-07	103.990	104.987	16	**0.055**
R4-08	103.901	103.882	12	**0.046**
R4-10	103.840	103.838	5	**0.018**
R4-11	103.690	103.686	10	**0.039**
R4-12	103.922	103.903	12	**0.046**
R4-13	103.896	103.903	8	**0.031**
R4-14	104.047	104.071	15	**0.057**
R4-15	104.089	104.081	10	**0.040**
R4-16	104.063	104.042	18	**0.071**
R4-17	104.024	104.045	17	**0.066**

Table 2: Radius of the sectors from the advanced rotors geometry. Updated values are highlighted in yellow.

We use FROMAGE to include the machining imperfections which are expected to be correlated within each lathe pass. Therefore the elements within the colored sub-sectors represented in fig. 1 have the same fluctuation to the nominal value. The outer radius is computed independently for both sectors. The RMS of the 1000 simulated mirror displacement due to each rotor geometry is converted to the relative uncertainty shown in table 3. This value will be taken as the modelling uncertainty for each rotor, accounting for the radius and thickness uncertainties. They include the uncertainties of the measuring tools which are anyway small compared to the surface defects. This table has been made on June 2024. Of course any future machining of the rotors to reduce unbalances and surface imperfections would change these values.

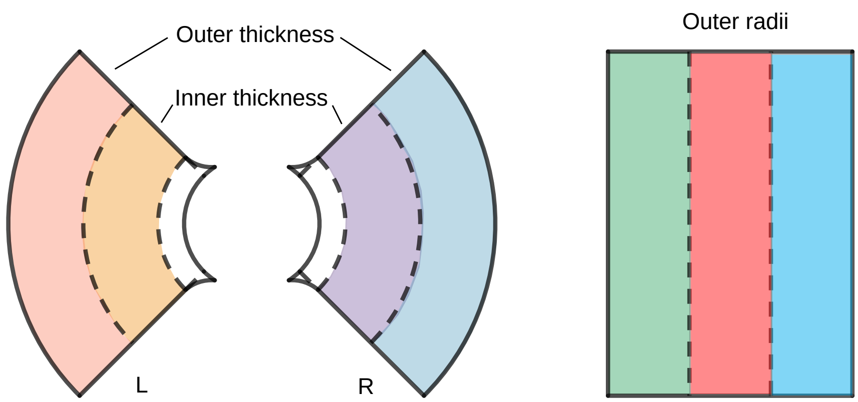

Figure 1: Left is a top view of the rotor model used in FROMAGE, right is a side view of a sector. Each colored sub-sector is expected to vary from the nominal value of thickness and radius following a normal distribution.

Rotor	NCal 2f signal modelling uncertainty [%]
R4-01	0.018
R4-02	0.018
R4-03	0.044
R4-04	0.023
R4-05	0.034
R4-06	0.026
R4-07	0.026
R4-08	0.019
R4-10	0.010
R4-11	0.017
R4-12	0.020
R4-13	0.013
R4-14	0.023
R4-15	0.016
R4-16	0.029
R4-17	0.026

Table 3: Relative uncertainty on the gravitational signal at twice the rotor frequency from the modelling method of each rotor.

3 Combined uncertainty

Based on the previous section, table 4 summarizes the uncertainties on the signal emitted by a PVC rotor (R4-12) for a mirror at 1.7 m. Since most of these uncertainties are uncorrelated, we add them quadratically to compute the overall uncertainty. The only correlated uncertainties are due to the measuring column which enter both in the density and modelling uncertainty. However, since their effects are anti-correlated, adding them quadratically is a conservative choice. The 2f signal and its uncertainty for each rotor is presented in table 5. The signal was computed using FROMAGE at a nominal distance from the mirror $d = 1.7$ m, an angle to the beam axis $\phi = 34.7°$ and a twist $\psi = 12°$. They depend on the measured geometry and therefore on the machining defects which are rotor dependent.

R4-12 rotor parameter advanced model (23°C)	2f signal uncertainty [%]
Density ρ	0.014
Temperature T	0.024
Opening angle and sector asymmetry	$<4 \times 10^{-5}$
Rotor flat surfaces offsets	$<5 \times 10^{-4}$
Modelling Uncertainty	0.020
FROMAGE grid uncertainty	0.005
Gravitational constant G	0.002
Total uncertainty from the rotor (quadratic sum)	0.035

Table 4: Uncertainties on the amplitude of the calibration signal at 2f from the R4-12 rotor advanced model geometry at 23°C.

Rotor	2f strain signal	2f signal uncertainty[%]
R4-01	2.2175×10^{-18}	0.021
R4-02	2.2205×10^{-18}	0.021
R4-03	2.2213×10^{-18}	0.045
R4-04	2.2214×10^{-18}	0.026
R4-05	2.2213×10^{-18}	0.036
R4-06	2.2214×10^{-18}	0.028
R4-07	2.2212×10^{-18}	0.028
R4-08	2.2108×10^{-18}	0.019
R4-10	1.1335×10^{-18}	0.030
R4-11	1.1262×10^{-18}	0.033
R4-12	1.1364×10^{-18}	0.035
R4-13	1.1361×10^{-18}	0.031
R4-14	1.1434×10^{-18}	0.036
R4-15	1.1431×10^{-18}	0.032
R4-16	1.1434×10^{-18}	0.040
R4-17	1.1422×10^{-18}	0.038

Table 5: Uncertainties on the amplitude of the calibration signal at 2f for each rotor.

References

1. Aubin, F. *et al. The Virgo Newtonian calibration system for the O4 observing run* 2024. eprint: arXiv:2406.10028.