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# Advanced Virgo: A draft design for a non-degenerate power recycling cavity

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# Contents

# 1 Introduction

This document describes the design of the first draft layout for a non-degenerate power recycling cavity (PRC) for Advanced Virgo. The design closely follows the concept from Advanced LIGO [1]. The main difference is that the current Advanced LIGO design features a PRC of approximately 50 m length (single trip) whereas the Virgo vacuum system limits the length for Advanced Virgo to 25 m. To compensate for that, the design presented here features lenses in the input test masses (i.e. curved AR coated surfaces). We present some arguments for this choice, however, we have not performed a trade-off analysis regarding the use of lens-like substrates for the ITMs.

We have concentrated on the PRC in this document. The SRC design in Advanced LIGO differs slightly from the PRC design in order to improve the alignment sensing and control. The Advanced LIGO design choice for the SRC might be ideal for Advanced Virgo too but a detailed analysis will be done as soon as a more detailed design for the alignment sensing and control system of Advanced Virgo becomes available.

We have further ignored possible astigmatism of the beam inside the off-axis telescope. The telescope details as well as the PRC length needs to be optimised later on, depending on inputs from various other subsystems. However, the design concept presented here does not rely on exact values for the mirror curvatures or positions but can easily be adapted if necessary.

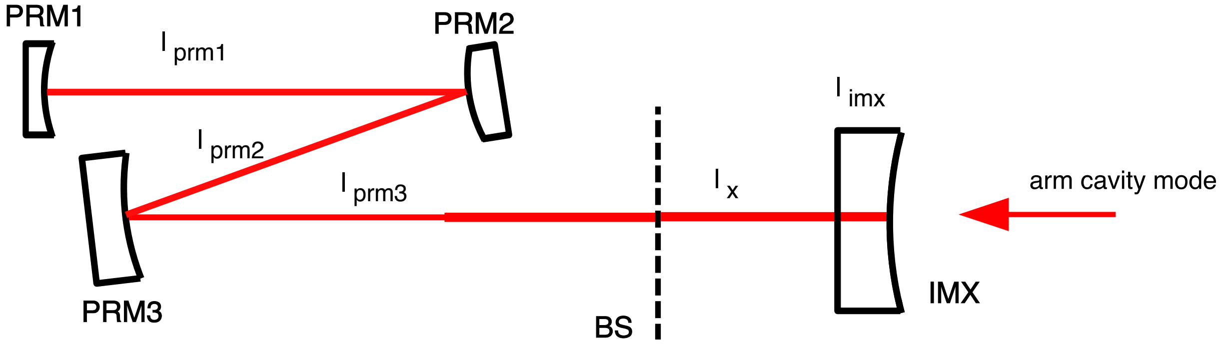

**Figure 1:** Simplified layout for a non-degenerate power recycling cavity, following the Advanced LIGO design. IMX represents the X-arm cavity input mirror, PRM1 is the power recycling mirror and PRM2 and PRM3 are turning mirrors that act as a kind of mode-matching telescope. PRM1 and PRM3 would be located inside the injection tower, PRM3 in the power recycling tank. The front face of IMX is shown to be flat in this picture, for the solution including a lens in the input test masses, this surface would be curved to create the lens effect.

## 1.1 Draft layout

Figure 1 shows a simplified layout of a non-degenerate PRC, following the Advanced LIGO design. We have performed the analysis with the X-arm only, so that we can ignore the beam splitter for the moment. In practice then IMX represents both input test masses and $l_x$ represents the common small Michelson length $(l_x + l_y)/2$.

The lengths have been chosen so that the mirrors fit into the current vacuum enclosure, i.e. PRM1 and PRM3 are to be mounted on the injection bench while PRM2 is housed in the power recycling tank.

The lengths in the PRC have to be fine-tuned later in connection with the modulation frequencies. For the moment we assume the following:

```
limx=.1;
```

```
lx=5.5; (this represents (lx+ly)/2 of the real ineterferometer)
lprm3=10.5;
lprm2=4.5;
lprm1=4.5;
```

This results in a cavity length of $L_{\mathrm{prc}} = 25.145\,\mathrm{m}$ and a free spectral range off FSR $\approx 6\,\mathrm{MHz}$.

### 1.2 Arm cavity mode

The power recycling cavity must of course be mode-matched to the arm cavities. However, the arm cavity design is still work in progress and thus the cavity mode not defined. In this document we make use of the latest proposal: The arm cavity mode features a beam waist of $w_0 = 9.7\,\mathrm{mm}$ at $z = 1400\,\mathrm{m}$, a beam size on the input mirror of $w_1 = 50\,\mathrm{mm}$ and a Gaussian beam parameter on the input mirror of $q = 1400 + 275.838\mathrm{i}$. Please note that the main features of the design do not depend on the arm cavity mode. Only the exact values for the mirror curvatures need to be re-computed if a different arm cavity mode is selected.

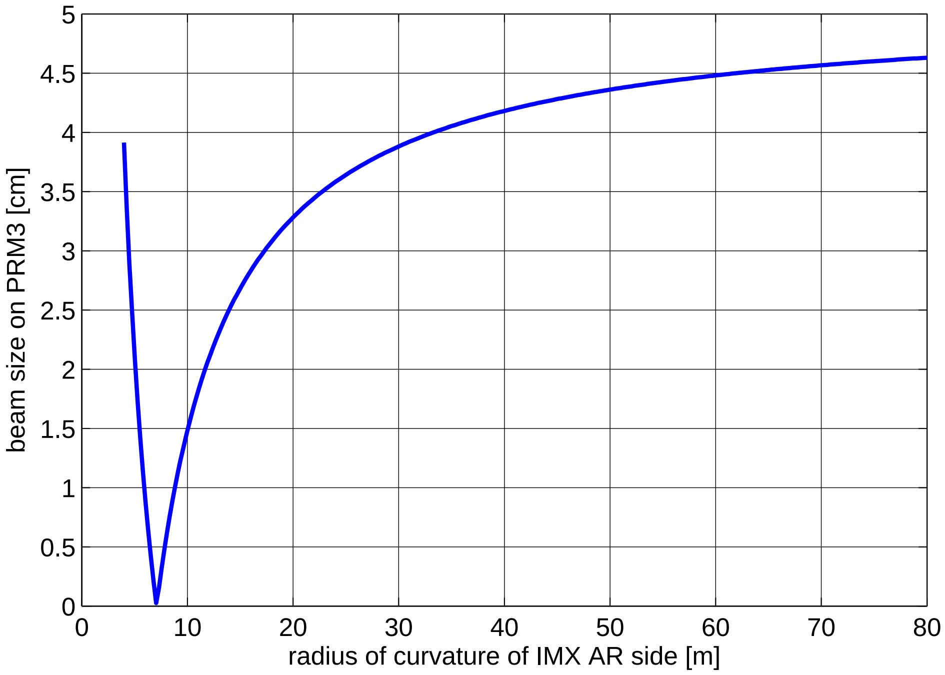

**Figure 2:** The beam size on PRM3 as a function of the radius of curvature of the AR coated side of the input mirrors.

## 2 Design procedure

The design concept is described in [1]. In the following we try to recreate the following features of the Advacned LIGO design:

1. the PRC should have a single trip Gouy phase of 160 deg
2. a beam waist is located between PRM1 and PRM2
3. the beam size at PRM1 is reasonably large

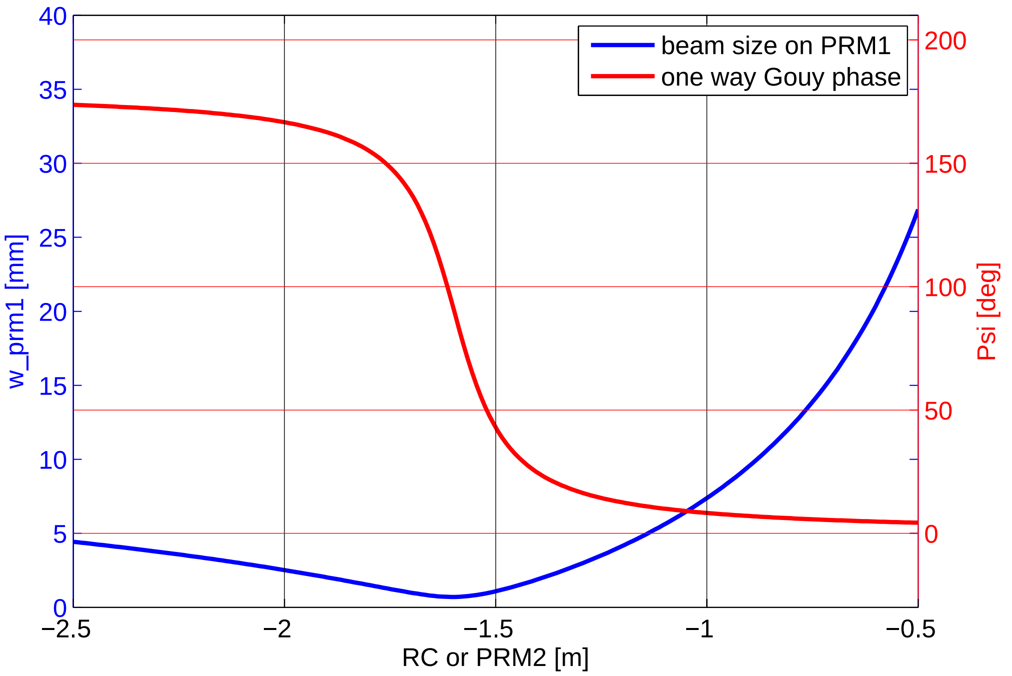

**Figure 3:** Beam size on PRM1 and single-trip Gouy phase inside the PRC as a function of $R_{\mathrm{C,PRM2}}$. Note that $R_{\mathrm{C,PRM1}}$ is automatically tuned during this computation.

The first point from this list can only be obtained if somewhere inside the PRC the Rayleigh range is similar to or smaller than the typical distance between the cavity optics ($\approx 5\,\mathrm{m}$). The Rayleigh range is defined as:

$$z_{\mathrm{R}} = \frac{\pi w_0^2}{\lambda} \tag{1}$$

Thus we need a beam waist of about 1 millimeter size (or smaller) which demands at least one strong focusing element in the PRC.

The second point on the list above then leads to the layout as shown in Figure 1: PRM3 is used as a the main focusing element and PRM2 is used as a defocussing element which can be used to fine tune the parameters of the waist in between PRM2 and PRM1 (i.e. to fulfil requirement number three from the list above). If the curvatures of IMX, PRM3 and PRM2 are set, the curvature of PRM1 follows automatically from the fact that the cavity must have a stable eigenmode.

## 2.1 Lenses in the input mirror substrates

As we will show below, the relatively short length of the PRC makes it difficult to obtain a working design if only PRM3 is used as a focusing element. (even when a folded path is used as shown in Figure 1).

The purpose of a lens inside the input mirror substrates would be to reduce the beam size on PRM3 and thus loosen the requirements for the curvatures of PRM3 and PRM2. Figure 2 shows the beam size on PRM3 as a function of the radius of curvature of the AR coated face of IMX and IMY. We have chosen rather arbitrarily a curvature of $R_{\mathrm{C,IMAR}} = 10.5$ for the solution presented below. This yields minimum size of PRM3 of approximately 10 cm.

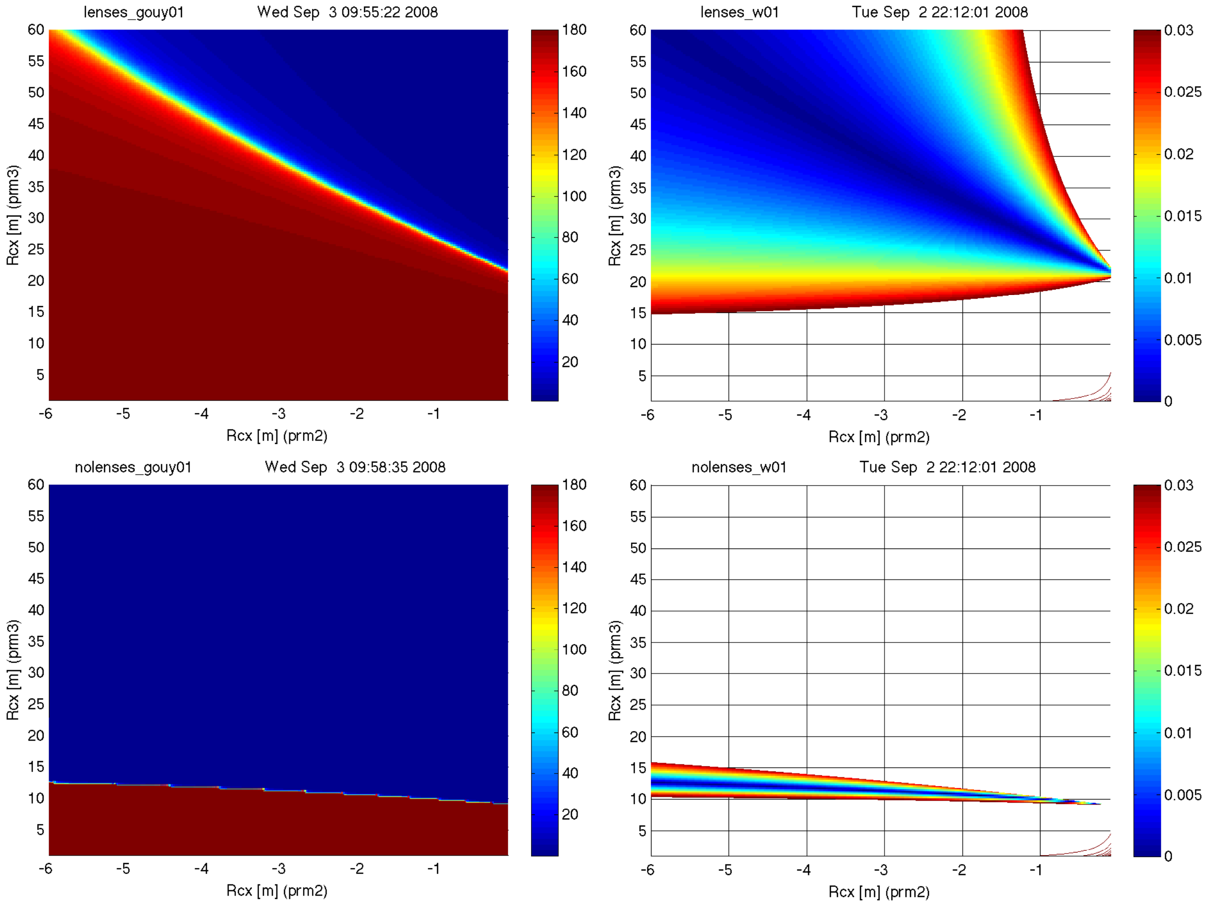

**Figure 4:** The two upper plots depict two parameters of a non-degenerate PRC when the input test masses feature a lens of $R_{\mathrm{C,IMXAR}} = 10.5\,\mathrm{m}$. The left plot shows the single trip Gouy phase as the function of the radius of curvature of PRM2 and PRM3. The right plot shows the beam size on PRM1 in the same parameter space (the color scale is in meters). The two lower plots show the same data for a design in which the input test masses have a flat secondary surface.

## 2.2 Curvature of PRM3 and PRM2

The main aim of the non-degenerate cavity design is to have a specific non-zero Gouy phase. The Adligo design of the PRC features a single-trip Gouy phase of 160 deg. Figure 3 shows the beam size on PRM1 and the single-trip Gouy phase as function of $R_{\mathrm{C,PRM2}}$. $R_{\mathrm{C,PRM1}}$ has been automatically adjusted during this calculation, the other parameters correspond to the design presented below. First of all, this plot recreates almost exactly a similar plot shown in [1]. It further illustrates nicely that only in a very small area of the parameter space the Gouy phase is not 0 nor 180 deg, i.e. is the cavity not-degenerate. However, if the telescope is designed to be within this range it is easy to adjust the telescope for any required Gouy phase. It can also be seen that the region of non-degeneracy corresponds to very small beam sizes.

The aim is to optimise the curvatures of PRM2 and PRM3 such that we achieve a Gouy phase of 160 deg while retaining a large enough beam. The plots in Figure 4 show a scan of the useful parameter space for these two curvatures. Again, we observe that the area with useful numerical values of the Gouy phase always correspond to the smallest beam size. It is also obvious that without the lens the useful area becomes much more narrow. In order to illustrate this further, we have extracted the subset of data corresponding to a Gouy phase of 160 deg. The corresponding beam size as a function of $R_{\mathrm{C,PRM2}}$ is shown in Figure 5. This plot does shows that:

- the beam size on PRM1 is very small in almost all cases,
- a strong lens in the input test mass increases the beam size,
- and a very strong, negative curvature on PRM2 increase the beam size.

For the moment we have no information about the negative impact of strong curvatures at the various optical components on the interferometer performance. As an educated guess we have chosen as radii of curvature to be close to the following values: $R_{\mathrm{C,PRM2}} = -2\,\mathrm{m}$, $R_{\mathrm{C,PRM3}} = 30\,\mathrm{m}$ and $R_{\mathrm{C,IMXAR}} = 10.5\,\mathrm{m}$.

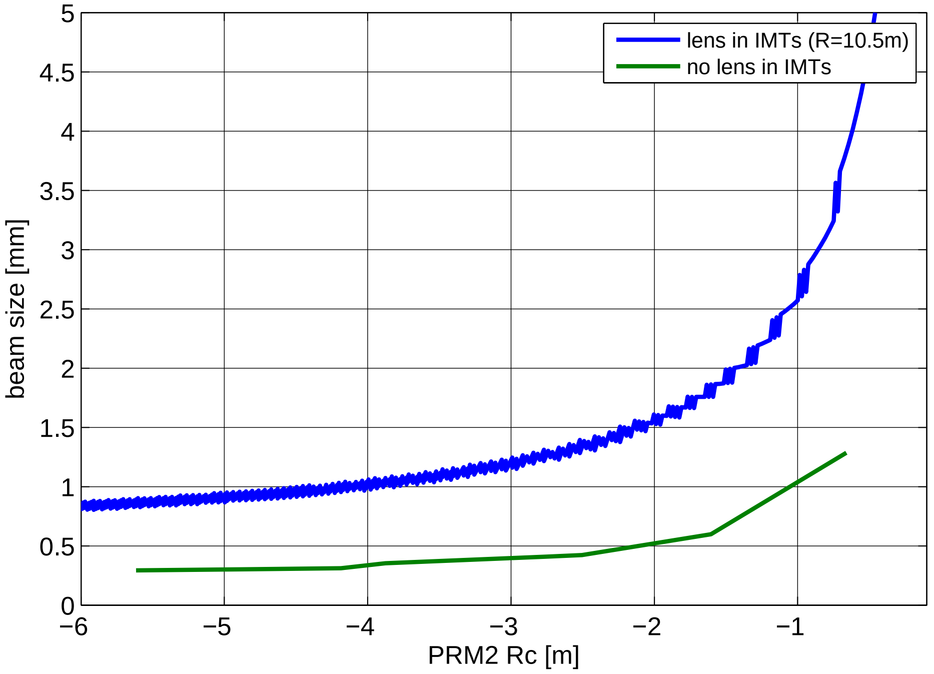

**Figure 5:** The beam size on PRM1 as a function of the radius of curvature of PRM2. Note that PRM3 is automatically adjusted to provide a single-trip Gouy phase of 160 deg.

# 3 Results

We have used the curvatures as derived above as a starting point for a automatic parameter optimisation: In this we have manually set $R_{\mathrm{C,IMXAR}} = 10.5\,\mathrm{m}$ and $R_{\mathrm{C,PRM3}} = 30\,\mathrm{m}$. The other parameters have then been computed automatically. This yields the results listed in the following table (beam sizes are given as the radius in [mm], distance to waist and radius of curvature are given in [m]):

```
RCimxAR=10.5;
RCprm3=30;
RCprm2=-1.8492;
RCprm1=2.1178;

arm cavity mode in vacuum q =1400 + i 275.838
arm cavity mode in substrate q =1427.97 + i 194.083
   |RCimxAR i1|RCimxAR o2|        BS|  RCprm3 1|  RCprm3 2|  RCprm2 1|  RCprm2 2|  RCprm1 1|  RCprm1 2|
w: |        50|        50|      38.5|      16.5|      16.5|      2.17|      2.17|      1.77|      1.77|
w0:|      6.74|     0.162|     0.162|     0.162|     0.106|     0.106|     0.395|     0.395|     0.395|
z: |  1.43e+03|     -23.9|     -18.4|     -7.91|     -5.18|    -0.679|     -2.49|      2.01|     -2.01|
Rc:|  1.45e+03|     -23.9|     -18.4|     -7.91|     -5.18|     -0.68|     -2.57|      2.12|     -2.12|
roundtrip Gouy phase Psi=5.56514 rad
single trip Gouy phase Psi=159.43 deg
```

## 3.1 Conclusion

This document presents a first draft design for an non-degenerate power recycling cavity for Advanced Virgo. The design is closely related to the that of the Advanced LIGO power recycling cavity. The lengths used in this design put the optics into the existing vacuum enclosure but require a major redesign of the injection and detection bench because each of these would now need to accommodate two mirrors from the respective recycling cavity.

The draft design presented here includes lenses in the input test masses and thus manages to gives values for the beam sizes and curvatures very similar to the Advanced LIGO case. Further development of this design needs input regarding the following issues:

- constraints for positions and size of mirrors on the injection and detection bench (INJ, DET)
- constraints on the PRC lengths from choice of modulation frequencies, mode cleaner length etc (ISC)
- impact of strong curvatures on the interferometers performance (probably TCS and ACS ??)
- impact of small beam sizes on the alignment system (ASC)
- impact of the round trip Gouy phase on the systems of ICS, ASC

# References

[1] M.A. Arain and G. Mueller, *Design of the advanced ligo recycling cavities*, Optics Express **Vol. 16, Issue 14** (2008), 10018–10032. 2, 3, 5