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Advanced Virgo output mode cleaner: specifications

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1 Introduction

The specifications for the Advanced Virgo output mode cleaner are detailed in this note. The parameters to be defined are: the finesse, the length, the waist, the geometry and the type of cavity (monolitic as for Virgo, or not). The precision on the length control is also discussed.
First, the finesse should be set so that the losses inside the cavity are not too high. Since DC readout will be used the output mode cleaner has to filter not only the carrier higher order modes but also the sidebands from the modulation frequencies and their higher order modes in order to keep a low shot noise. The light back-scattered by the cavity should be low enough in order to avoid to add any phase noise to the sensitivity. The use of a monolitic cavity will be decided on the basis of the needed length but also on the induced additional losses and thermal effects.
The parameters given in the Advanced Virgo conceptual design [1] are used to define these specifications. It is also assumed that a ring cavity will be used in order to be compliant with HOLMs but the case of a triangular cavity is also discussed.

2 Power contributions to the dark fringe

The total power transmitted to the dark fringe, for DC readout with detuned SR, is given by:

$$P_{df} = \frac{P_0 G_{rec}}{2}\left((1-C) + \frac{1}{2}\left(\frac{2F}{\pi}\frac{4\pi}{\lambda}L_{off}\right)^2\right)\frac{T_{SR}}{4} + P_{SB1} + P_{SB2} + P_{SB3} \qquad (1)$$

where P_0 is the ITF input power, G_{rec} is the recycling gain, T_{SR} the transmission of the signal recycling mirror, L_{off} the differential arm offset and P_{SBi} the power of the sidebands at the modulation frequency f_i. These last terms are given by $P_{SBi} = 2T_i P_0 (m_i/2)^2$ where m_i is the modulation index and T_i the transmission factor to the anti-symmetric port. The Advanced Virgo locking scheme is not yet defined and might depend on the use of degenerate or non-degenerate recycling cavities. In the case of degenerate cavities the prefered scheme is to keep 2 modulation frequencies à la Virgo ($f_1 \simeq 6$ MHz resonant only in the power recycling cavity and $f_3 \simeq 8$ MHz not resonant in any cavity) and a third one, $f_2 \simeq 80$ MHz resonant in both the power and signal recycling cavities [2]. A small Schnupp asymmetry (few cm) ensures that f_1 and f_2 are not transmitted to the dark port ($T_1 \simeq 10^{-5}$, $T_3 < T_1$). The third modulation frequency, f_2 is also chosen such that it is transmitted to the anti-symmetric port: $T_2 \simeq 0.5$ [2].

The following values (from the Advanced Virgo conceptual design [1] and ISC studies [2]) are used throughout this note: $P_0 = 125$ W, $G_{rec} = 20$, $F = 880$, $T_{SR} = 0.043$, $f_2 = 80$ MHz, $T_2 = 0.5$, $m_i = 0.1$, $T_3 < T_1 < 10^{-4}$. The value of the differential offset L_{off} has to be well above the locking accuracy and such that it's power contribution to the dark fringe lies well above the contribution from the contrast detect (see Eq. 1). This last requirement will probably be the most stringent: After the output mode cleaner the contrast defect is given by the asymmetry of losses inside the Fabry-Perot cavities:

$$1 - C = \frac{(\Delta r)^2}{2} = \frac{1}{2}\left(\frac{F\Delta P}{\pi}\right)^2 \tag{2}$$

where Δr is the difference of reflectivity (in amplitude) and ΔP is the losses asymmetry. Requiring that the power due to contrast defect does not increase the shot noise by more than 1% gives the following relation:

$$L_{off} > \frac{\Delta P(ppm)}{8\sqrt{2}\pi} 10^{-11}\mathrm{m} \tag{3}$$

and differential losses of 30 ppm give $L_{off} > 10^{-11}$ m. This value is used in the following.

With the value of the parameters given above the contributions to the dark fringe power are: P_{1-C}=0.5 mW, P_{Loff} = 28 mW, $P_{SB1} < 60$ μW, P_{SB2} = 310 mW, $P_{SB3} < 60$ μW.

3 Finesse

The finesse of the output mode cleaner is driven by

- the losses inside the cavity: these should be kept below 1%
- the filtering needed for the sidebands: their contribution should not increase the shot noise by more than 1%
- the control: the finesse should remain low enough to ensure that the lock of the cavity is feasible

3.1 Losses

The total losses inside the cavity are given by:

$$L = 4P\frac{F_{OMC}}{\pi} \tag{4}$$

where P is the loss per face and the factor 4 is for the 4 faces of the cavity (to be replaced by 3 for a triangular cavity). A reasonable value for the losses is P = 30 ppm: a recently coated triangular cavity showed diffraction losses of 50 to 100 ppm per face, which can probably be improved with a better polishing, and good superpolished mirrors usually show diffraction losses of the order of 10 ppm. Keeping losses below 1% implies then $F_{OMC} < 260$. The lock should be feasible for this value of the finesse. F_{OMC} = 200 is used in the following.

3.2 Filtering of the sidebands

The power from the first and third modulation frequencies are already small enough and do not need any additional filtering, while the high contribution from the second modulation implies a filtering larger than 550 at 80 MHz. The output mode cleaner transmission at f_2 is given by:

$$T(f_2) = \frac{1}{1 + (\frac{2F_{OMC}}{\pi})^2 \sin^2(\frac{2\pi f_2}{c} l_{opt})} \tag{5}$$

where l_{opt} is the optical cavity length (defined by half of a round trip). The optical length is related to the geometrical length, l, of the cavity by: $l_{opt} = n \times i \times l$ where $i = 1$ for a triangular cavity and $i = 2$ for a ring cavity and n is the refraction index of the cavity. A filtering larger than 550 at 80 MHz implies $F_{OMC} \times l_{opt} > 21$: with $F_{OMC} = 200$ the requirement on the optical length is therefore $l_{opt} > 11$ cm. This is small enough so that the cavity can fit on a single suspended optical bench.

4 Beam waist

The waist of the cavity should be large enough in order to keep the back-scattered light by the cavity within the requirements [4]. This is discussed in Section 6. Constraints on the waist are given by the filtering needed for HO modes, which is discussed in this Section.
The waist of the cavity is given by:

$$w_0^2 = \frac{\lambda}{n\pi}\sqrt{l(\rho - l)} \simeq \frac{\lambda}{n\pi}\sqrt{l\rho} \tag{6}$$

where ρ is the radius of curvature of the curved face(s), l is the half (quarter) round trip cavity length for a triangular (ring) cavity (defined as the geometrical length) and n is the index of the cavity substrate. The approximation is valid for $l << \rho$ which is the case here. The product ρl depends on the filtering needed for the higher order modes which is discussed here.

4.1 Filtering of the higher order modes of the carrier

No simulation of the higher order mode content of the carrier has been performed for Advanced Virgo but it seems reasonable to request a filtering similar to the Virgo case, i.e. a factor 100 for the lowest order modes. The transmission for the TEM$_{nm}$ mode is:

$$T_{nm} = \frac{1}{1 + (\frac{2F_{OMC}}{\pi})^2 \sin^2(i(n+m)\mathrm{acos}(\sqrt{g} + \frac{\phi_n}{2}))} \tag{7}$$

where $\phi_n = \pi$ for a triangular cavity and odd n, $\phi_n = 0$ otherwise, and $g = 1 - \frac{l}{\rho}$. For $l << \rho$, $T_{01} < 1\%$ implies:

$$\rho l < 1\% \times \left(\frac{2F_{OMC} l_{opt}}{n\pi}\right)^2 \tag{8}$$

where the product $F_{OMC} l_{opt}$ is constrained by the filtering needed for f_2 (see Section 3). Using the lowest allowed value ($F_{OMC} l_{opt} = 21$) leads to $w_0 < 470$ μm.

4.2 Filtering of the higher order modes of the sideband

The most stringent requirement on ρ (and therefore on the waist) could come from the higher order modes of the sidebands: the power at the modulation frequency f_2 is expected to be 10 times higher than the power of the carrier (see Section 2), so higher order modes at f_2 could also give a significant contribution. The separation frequency between two modes is given by:

$$f_{sep} = \frac{c}{2\pi n} \frac{1}{\sqrt{\rho l}} \tag{9}$$

One should make sure that the lowest order modes of f_2 do not fall too close to the carrier TEM_{00} (for example, if $f_{sep} = f_2$ the first order mode of SB2 is transmitted with the TEM_{00} of the carrier). There are two possibilities:

1- The safest is that the first order mode falls on the other side of the carrier TEM_{00}, i.e. $f_{sep} > 2 \times f_2$ (in fact slightly less than $2 \times f_2$ should be fine but it does not change significantly the result). The largest waist that can be used for $f_2 = 80$ MHz is around $w_0 < 220$ μm. This limit goes up if a smaller f_2 is used. This corresponds to a radius of curvature of the order of 2 (resp. 1) meter for a ring (resp. triangular) cavity.

2- The higher order modes of the SB2 can also be located between the TEM_{00} modes of the carrier and of SB2. It should then be required that the transmission is smaller than few% for $f_2 - (n+m)f_{sep}$ at least for $(n+m) < 3$. This is less safe than the solution 1 since the higher order mode content of the sidebands is difficult to anticipate. This solution requires to use a larger waist: $w_0 > 540$ μm. This is compatible with a good filtering of the carrier higher order modes (see Section 4.1) only if the length of the cavity is increased by about 30% (which will anyway ensure a better filtering of f_2, see Section 3).

The choice between these two solutions should be made when the Advanced Virgo optical design and modulation frequencies are better known.

5 Material and geometry

In this section we investigate if it is possible to use a monolithic cavity for the OMC as was done in Virgo and which geometry can be used.

5.1 Monolithic cavity

The advantages of a monolithic cavity is to keep the mechanical resonances at high frequency and to use a simple thermal control for the lock. The possible drawbacks of a monolithic cavity are the absorption losses and the thermal effects.

The losses due to absorption in the substrate are given by:

$$L = \frac{2F_{OMC}}{\pi} \frac{2l_{opt}}{n} p_{abs} \tag{10}$$

where p_{abs} is the absorption per length unit. For Suprasil 311 SV the absorption is $p_{abs} = 0.25$ ppm/cm. Using the value of $F_{OMC} l_{opt}$ derived in Section 3 gives $L = 0.05\%$ which is safe and shows that a larger cavity can also be used.

The focal length induced by thermal lensing is given by

$$f_{th} = \frac{w_0^2}{2\delta s} \tag{11}$$

where δs is the induced increase of optical path given by [9]:

$$\delta s = 1.3 \frac{\beta}{4\pi\kappa} p_{abs} l \times P \tag{12}$$

where $\beta = dn/dT = 1.1 \times 10^{-5} K^{-1}$ for silica, $\kappa = 1 W m^{-1} K^{-1}$ is the thermal conductivity, l is the geometrical path length in the substrate and $P = 2F_{OMC}/\pi P_{Loff}$ is the power stored in the cavity. For all cases discussed in Section 4 the induced focal length is always well above the focal length of the curved faces (for example, f_{th} =1700 m for solution 1 in Section 4.2 to be compared to $f = \rho/2 \simeq 0.5$ m).

5.2 Geometry

Concerning the geometry, the main decision to be made is about the angle of incidence on the mirrors. Large angle of incidence are prefered to lower the back-scattering: the diffusion is known to vary with $1/\theta^2$ for angles below typically 5 degrees and constant above. Unfortunately large angle of incidence introduce an astigmatism and therefore losses. For an angle of incidence θ_i the difference of focal length in x and y is $\delta f = f \frac{sin^2\theta_i}{cos\theta_i}$. This results in a difference of the waist location in the two planes: $\delta z = \delta f \simeq f\theta_i^2$ for small angles. The losses due to this mismatching are given by $L = (\delta z / 2 z_R)^2$ with $z_R = \pi w_0^2/\lambda \simeq \sqrt{\rho l}/n$. To keep the losses below 1% implies $\theta_i^2 < 0.1 \frac{4}{n}\sqrt{\frac{l}{\rho}}$, and, using the values derived from solution 1 (resp. 2) of Section 4.2: $\theta_i < 13$ degrees ($\theta_i < 5$ degrees).

6 Back-scattered light

The fraction of light back-scattered by the OMC is given by [6, 8]:

$$f_{sc} = BRDF \left(\frac{4F_{OMC}}{\pi} \right)^2 \frac{\lambda^2}{\pi w_0^2} \tag{13}$$

where BRDF is the fraction of scattered light per unit angle and per face, and the factor 4 accounts for the number of faces (to be replaced by 3 for a triangular cavity). The above formula assumes the worst case where the scattered field from each surface adds up in phase. Assuming that the incident angle is large enough to be in the region where the diffusion is uniform gives: $BRDF = P/2\pi$ where P is the diffusion losses per face as defined in Section 3. With $F_{OMC} = 200$, $w_0 = 220$ μm (lowest value considered in Section 4) , $P = 30$ ppm the fraction of diffused light is $f_{sc} = 2 \times 10^{-6}$. This is above the specification $f_{sc} < 4 \times 10^{-7}$ given in Section 4.2.2 of [4]. Installing a Faraday isolator before the OMC should nevertheless give a good safety margin (the present Faraday isolator placed after the OMC provides an isolation of 1000) even in the case the diffusion is higher due to the small incident angle on the OMC faces.

7 Length control and noise

Any power fluctuation at the output of the mode cleaner is interpreted as a gravitational wave signal. In this section the power fluctuation due to the mode cleaner length variation are considered. The power transmitted by the output mode cleaner is given by:

$$P_t = \frac{P_0}{1 + (\frac{2F_{OMC}}{\pi})^2 \sin^2(kl)} \tag{14}$$

where l is the cavity length. If the cavity is away from resonance by a constant offset Δl_0, a length noise δl will give a variation of the output power:

$$\frac{\delta P}{P} = 2(4F_{OMC})^2 \frac{\Delta l_0}{\lambda} \frac{\delta l}{\lambda} \tag{15}$$

The shot noise on the dark fringe is given by:

$$\frac{\delta P_{sn}}{P} = \frac{\lambda}{2FL_{off}} \sqrt{\frac{2h\nu}{P_0 G_{rec} T_{SR}}}. \tag{16}$$

Imposing that the noise due to the OMC length noise is a factor 10 below the shot noise gives (this condition can be relaxed for frequencies below 100 Hz since the sensitivity will not be shot noise limited):

$$\Delta l_0 \delta l < 3 \times 10^{-28} m^2/\sqrt{\text{Hz}} \tag{17}$$

An upper limit on the length noise of the Virgo output mode cleaner (monolitic cavity of 2.5 cm) was measured [10] and typically $\delta l < 2 \times 10^{-15} m/\sqrt{Hz}$ for frequencies above 70 Hz. This will have to be measured with the new cavity but taking this value as reference would impose to reach $\Delta l_0 < 1.5 \times 10^{-12} m$ in order to meet the requirement. The Pound-Drever signal which will be used for the length control is linear in the range $l_{lin} = \lambda/(2 \times \sqrt{3} F_{OMC}) = 1.4 \times 10^{-9}\ m$ for $F_{OMC} = 200$. Therefore, to obtain $\Delta l_0 < 1.5 \times 10^{-12} m$ the electronic offset of this signal should be tuned and constant within about 1 permille of the signal dynamic range which is feasible.

8 Summary

The specifications for the Advanced Virgo output mode cleaner have been detailed with the Advanced Virgo conceptual design as baseline to set values. The characteristics of a cavity that would meet all the requirements is given in Table 1 (first line). This assumes that solution 1 of Section 4.2, considered as the safest solution is chosen. The finesse of the FP cavities and the transmission of SR mirror are not yet frozen (the finesse could be lowered while SR transmission could be increased). Increasing the SR transmission would

	F_{OMC}	l_{opt} (cm)	l (cm)	ρ (cm)	w_0 (μm)	f_{sc}	$\theta_{i,max}$ (deg)
AdV baseline	200	11	3.7	110	220	2×10^{-6}	13
F=400	200	18	6.2	70	220	2×10^{-6}	16
$f_2 = 50$ MHz	200	17	6.0	180	280	1.5×10^{-6}	13
m=0.2	200	21.4	7.4	60	220	2×10^{-6}	18

Table 1: Characteristics of a monolitic cavity meeting the requirements developped in this note for the Advanced Virgo baseline and in the cases $F = 400$, $f_2 = 50$ MHz and $m = 0.2$. These numbers refer to the solution 1 of Section 4.2 and to a constant finesse $F_{OMC} = 200$.

	F_{OMC}	l_{opt} (cm)	l (cm)	ρ (cm)	w_0 (μm)	f_{sc}	$\theta_{i,max}$ (deg)
AdV baseline	100	21.4	7.4	60	220	0.6×10^{-6}	18
F=400	170	21.4	7.4	60	220	1.7×10^{-6}	18
$f_2 = 50$ MHz	160	21.4	7.4	150	280	0.9×10^{-6}	14
m=0.2	200	21.4	7.4	60	220	2×10^{-6}	18

Table 2: Characteristics of a monolitic cavity meeting the requirements developped in this note for the Advanced Virgo baseline and in the cases $F = 400$, $f_2 = 50$ MHz and $m = 0.2$. These numbers refer to the solution 1 of Section 4.2 and to a constant geometrical length $l = 7.4$ cm.

lead to less stringent constraints while decreasing the finesse (only) would imply a higher filtering at f_2 leading to a longer cavity. Decreasing the value of f_2 would also lead to a longer cavity. It should also be underlined that the modulation index was arbitrarily chosen to $m = 0.1$ and a better filtering (i.e. longer cavity) could be needed in case a larger index is needed for locking purposes. As an example, the characteristics of a cavity with a finesse $F_{OMC} = 200$ and meeting these requirements are given in Table 1 for F=400, $f_2 = 50$ MHz or $m = 0.2$, keeping all other parameters as in the AdV baseline (except for F=400 where the recycling gain is 2 times higher since the total losses in the FP cavities are smaller). In all cases the cavity is small enough so that a monolitic cavity can be used. Table 2 gives these characteristics for a cavity of constant geometrical length.

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