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Defining the arm cavity loss for Advanced Virgo

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1 Introduction

The mirrors constituting the arm cavities of Advanced Virgo are one of the most critical components of the interferometer. The mirrors have to be made with the best available substrate and state of the art techniques for polishing and coating are used to guarantee optimal performance of the interferometer.

One important parameter regarding the mirrors is the flatness specification describing random fluctuations in the height of the mirror surface. Such roughness can introduce extra optical loss inside the cavity as light is scattered at angles such as it can never reach the opposite mirror of the cavity.

At the LMA, we are presently simulating the amount of loss which can be induced by the flatness defects at the mirror surface. The goal is to be able to define the maximum surface defects acceptable for a given optical round trip loss. However as we were using different optical simulation packages (SIESTA[2], OSCAR[3] and SIS[5]), we did not use the same definitions for the round trip loss, which was rather disturbing and generating endless discussions.

The role of this note is to describe different methods to the calculate cavity round trip losses and to understand what each technique really represents.

2 The different definitions of the loss

For the following sections, we define P_{in}, P_{circ}, P_{ref} and P_{trans} as respectively the input power, the circulating power, the reflected power and the transmitted power of the arm cavity.

We also define P^{00}_{circ}, P^{00}_{ref} and P^{00}_{trans} as the circulating power, the reflected power and the transmitted power in the fundamental mode TEM_{00}. The projection of the electric field to the fundamental mode is numerically done with the overlap integral. The input beam of the cavity is always assumed to be in the fundamental mode (i.e. $P^{00}_{in} = P_{in}$).

2.1 Via energy conservation

As the name suggests, this method is based on the conservation of the optical energy by the cavity. What enters in the cavity is equal to what is going out plus what is lost:

$$P_{in} = P_{ref} + P_{trans} + P_{lost} \tag{1}$$

We can divide the previous equation by the circulating power:

$$\frac{P_{in}}{P_{circ}} = \frac{P_{ref}}{P_{circ}} + \frac{P_{trans}}{P_{circ}} + \frac{P_{lost}}{P_{circ}} \tag{2}$$

Introducing $\mathfrak{L}_1$ the round trip power loss and r_1, r_2 and t_1, t_2 the amplitude reflectivities and transmissions of respectively the input and end mirror of the cavities, equation 2 can be written as:

$$\begin{aligned}
\frac{(1 - r_1 r_2 \sqrt{1-\mathfrak{L}_1})^2}{t_1^2} &= \frac{(r_1 - r_2\sqrt{1-\mathfrak{L}_1})^2}{t_1^2} + t_2^2 + \frac{P_{lost}}{P_{circ}} && (3)\\
\frac{1 - r_1^2 - r_2^2 + r_1^2 r_2^2 + (r_2^2 - r_1^2 r_2^2)\mathfrak{L}_1}{t_1^2} &= t_2^2 + \frac{P_{lost}}{P_{circ}} && (4)\\
\frac{1 - r_1^2 - r_2^2 + r_1^2 r_2^2 + (r_2^2 - r_1^2 r_2^2)\mathfrak{L}_1}{t_1^2} &= t_2^2 + \frac{P_{lost}}{P_{circ}} && (5)\\
\frac{(1 - r_1^2)(1 - r_2^2) + r_2^2(1 - r_1^2)\mathfrak{L}_1}{1 - r_1^2} &= t_2^2 + \frac{P_{lost}}{P_{circ}} && (6)\\
(1 - r_2^2) + r_2^2 \mathfrak{L}_1 &= t_2^2 + \frac{P_{lost}}{P_{circ}} && (7)\\
P_{lost} &= r_2^2 \mathfrak{L}_1 P_{circ} && (8)
\end{aligned}$$

By replacing P_{lost} in formula 1, the loss $\mathfrak{L}_1$ can then be defined as:

$$\mathfrak{L}_1 = \frac{P_{in} - (P_{ref} + P_{trans})}{P_{circ} r_2^2} \tag{9}$$

Typically since r_2^2 is close to 1, this coefficient is dropped. This definition is one of the most widely used because it is relatively easy to implement numerically.

2.2 Via the cavity eigenmode

We can also define the clipping loss encountered by the cavity eigen mode. In theory, the eigen mode is the circulating field in the cavity only for cavities with infinite finesse. For high finesse cavities such as in the interferometer arms, the cavity eigenmode can be approximated by the mode circulating inside the cavity when the cavity is set on resonance. The arm cavities of the interferometer are non-degenerate cavities with spherical mirrors, so on resonance, the circulating (or eigen) mode is very close to a perfect fundamental Gaussian mode thanks to the mode cleaning effect.

The round trip loss $\mathfrak{L}_2$ of the cavity eigenmode is simulated by calculating the round trip loss of the eigenmode when the reflectivity of the cavity mirrors is set to 1. By this way, no other losses are present in the cavity except the clipping loss due to the finite size of the mirrors.

Since the round trip loss is calculated on the cavity eigen mode, the loss is independent of the parameters of the input laser beam. The last statement is true for non-degenerate cavities which is the case for the arm cavities.

2.3 Via energy conservation projected on the TEM_{00} input mode

As the arm cavities are not isolated but part of part of an interferometer, any fraction of the beam not in the fundamental mode may not interfere destructively at the dark port and so be considered as effectively lost. Following this argument, it has been suggested that the loss should also include the fraction of the circulating and reflected beams which are not in the fundamental mode.

So we can define the loss in a fashion similar to that in section 2.1 with the principle of conservation of energy. The loss $\mathfrak{L}_3$ defined on the TEM_{00} is:

$$\mathfrak{L}_3 = \frac{P_{in} - (P_{ref}^{00} + P_{trans}^{00})}{P_{circ}^{00}} \tag{10}$$

2.4 Introducing the coupling coefficient

We can also define the loss on the TEM_{00} using an approach similar to the modal expansion method used to calculate the reflected field from a cavity in case of mode mismatching. Indeed, we can define two different basis for the laser beam. First, the basis of the input beam which is also the basis of the interferometer. Then the basis of the cavity given by the cavity eigenmodes.

Let's call K the coupling coefficient for the fundamental mode to be converted from the input beam basis to the cavity basis. Numerically K is simply the overlap integral from the input fundamental mode to the cavity fundamental eigenmode. For perfect mirrors and ideal mode matching the coefficient K is equal to 1.

Thus, in order to calculate the power in the cavity in the cavity fundamental mode, we first expand the input beam on the cavity basis, that is given by the coefficient K. The power coupling loss by the fundamental mode is defined as $\alpha = 1 - |K|^2$. A power loss of α in the circulating power is equivalent of having a round trip losses increased by $((1-r^1)^2/(r_1(1+r_1)))\alpha$. Then we calculate the loss of the cavity fundamental eigen mode, that is given by the number $\mathfrak{L}_2$ defined in section 2.2. So the total loss $\mathfrak{L}_4$ for the TEM_{00} can be approximated by:

$$\mathfrak{L}_4 = \frac{(1-r_1)^2}{r_1(1+r_1)}\alpha + \mathfrak{L}_2 \tag{11}$$

On a side note, different regions of the surface defects spectrum do not influence the coefficient K and $\mathfrak{L}_2$ the same way. Typically, low spatial frequencies mainly change the shape of the cavity eigen mode, so are directly related to the coefficient K, whereas high spatial frequencies scatter part of the resonant mode outside the cavity which is described by the coefficient $\mathfrak{L}_2$.

3 Relationship between the different losses definitions

As we have seen in the previous section, several methods can be used to define the round trip loss in the cavity. However, all these loss definitions are not totally independent and are often describing the same effects.

For example, with our numerical simulations, we found that the losses $\mathfrak{L}_1$ and $\mathfrak{L}_2$ give almost identical results (within 5%). That is not surprising since the mode circulating inside the cavity (used to define $\mathfrak{L}_1$) is in fact the cavity eigenmode (used to define $\mathfrak{L}_2$) since the cavity has relatively high finesse and is not degenerated. Moreover in the simulations, the input cavity beam is usually very close to the cavity eigen mode to minimise unnecessary mode matching losses, so only the cavity fundamental mode is excited.

We also found a relationship between the loss $\mathfrak{L}_1$ (derived from the energy conservation principle), the loss projected on the TEM_{00} $\mathfrak{L}_3$ and the coupling coefficient K. As a reminder, the loss $\mathfrak{L}_3$ is defined as:

$$\mathfrak{L}_3 = \frac{P_{in} - P^{00}_{ref}}{P^{00}_{circ}} \tag{12}$$

In that case, we ignore the term P^{00}_{trans} which is usually negligible with respect to P^{00}_{ref} since the end mirror only transmits few ppm of light. We have already defined the coupling loss α to expand the input beam (which is our reference for the TEM_{00}) to the cavity eigenmode, in fact the same coefficient can be also used to project the eigenmode resonant field to the TEM_{00}:

$$P^{00}_{circ} = (1-\alpha)P_{circ} \tag{13}$$

In a similar fashion, we found numerically (using the software OSCAR and SIESTA) that:

$$P^{00}_{ref} = (1-4\alpha)P_{ref} \tag{14}$$

Although, we did not manage to explain theoretically the above relationship, it was verified by a wide range of simulations. By inserting equations 13 and 14 in equation 12 we found that:

$$\mathfrak{L}_3 = \frac{P_{in} - (1-4\alpha)P_{ref}}{(1-\alpha)P_{circ}} \tag{15}$$
$$= \mathfrak{L}_1 + \frac{P_{in} + 3P_{ref}}{P_{circ}}\alpha \tag{16}$$
$$\simeq \mathfrak{L}_1 + \frac{4(1-r_1)^2}{t_1^2}\alpha \tag{17}$$

So in summary, we can simply note that:

$$\mathfrak{L}_1 \approx \mathfrak{L}_2 < \mathfrak{L}_4 < \mathfrak{L}_3 \tag{18}$$

4 Loss applied as additional mirror transmission

M. Laval *et al.* [1] have shown that cavity losses due to mirror defects can be numerically equivalent to an increase in the end mirror transmission. That equivalence is of great interest for modal expansion simulation codes, where we can simulate the effects of realistic mirrors without having to include the actual mirror maps themselves (which can slow down significantly the execution of the code).

Below, we remind briefly the method described in Laval's note. First we calculate G_0, the cavity gain relative to the input mode TEM_{00}:

$$G_0 = \frac{P_{circ}^{00}}{P_{in}} \tag{19}$$

Second, we compute the cavity reflectivity R_0 also relative to the input mode:

$$R_0 = \frac{P_{ref}^{00}}{P_{in}} \tag{20}$$

Then, we can calculate the "effective" amplitude reflectivities $\widetilde{r}_1$, $\widetilde{r}_2$ of the cavity mirrors by solving the following system of equations:

$$\begin{cases} G_0 &= \frac{1-\widetilde{r}_1^2}{(1-\widetilde{r}_1\widetilde{r}_2)^2} \\ R_0 &= \left(\frac{\widetilde{r}_1-\widetilde{r}_2}{1-\widetilde{r}_1\widetilde{r}_2}\right)^2 \end{cases} \tag{21}$$

Finally, we compute the effective transmittance $\widetilde{T}_2 = 1 - \widetilde{r}_2^2$ for the end mirror. In this case the additional loss $\mathfrak{L}_5$ is defined as the difference between the effective transmittance of the mirror $\widetilde{T}_2$ and the physical one t_2^2.

In fact the loss $\mathfrak{L}_5$ applied to the cavity end mirror is exactly equivalent to the loss $\mathfrak{L}_3$ as we will demonstrate next. For the distracted reader, the loss $\mathfrak{L}_3$ was defined in section 2.3 as:

$$\mathfrak{L}_3 = \frac{P_{in} - (P_{ref}^{00} + P_{trans}^{00})}{P_{circ}^{00}} \tag{22}$$
$$= \frac{P_{in}}{P_{circ}^{00}} - \frac{P_{ref}^{00}}{P_{circ}^{00}} - \frac{P_{trans}^{00}}{P_{circ}^{00}} \tag{23}$$

Which can be rewritten as:

$$\mathfrak{L}_3 = \frac{1}{G_0} - \frac{R_0}{G_0} - t_2^2 \tag{24}$$
$$= \frac{1-R_0}{G_0} - t_2^2 \tag{25}$$

$$= \frac{(1-\widetilde{r}_1\widetilde{r}_2)^2 - (\widetilde{r}_1 - \widetilde{r}_2)^2}{1-\widetilde{r}_1^2} - t_2^2 \tag{26}$$

$$= (1-\widetilde{r}_2^2) - t_2^2 \tag{27}$$

$$= \widetilde{T}_2 - t_2^2 \tag{28}$$

$$= \mathfrak{L}_5 \tag{29}$$

5 Remarks and open question

In the Advanced Virgo Baseline Design[4], it is mentioned that the round trip loss for the arm cavities must be 75 ppm. From the present document, we can see that it is essential to specify which definition must be used to compute the losses. In particular, are the 75 ppm losses defined as the round trip loss for the cavity circulating field or for the eigen mode projected to the TEM_{00}.

A second point must be kept in mind. On the Advanced Virgo Baseline document, only the total round trip loss is defined, how much the loss from the surface defects contributes to the total loss ? For reference, for Advanced LIGO, the loss from surface defects (at low and high spatial frequencies) represents 60 ppm per round trip.

It should be noted that not all the losses have the same effect on the interferometer. The diffraction loss due to the finite size of the mirrors scattered light in the vacuum tank and the light is definitively lost, whereas the coupling loss generates higher order modes which degrades the contrast and encourages the installation of an output mode cleaner.

6 Acknowledgment

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