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Constraints on the Advanced Virgo detection bench jitter from OMC alignment: an update

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Contents

1 Introduction

This note is an update of the note [1] using the latest MSRC (Marginally Stable Recycling Cavities) design from October 2011. The main difference with respect to the initial note is the magnification of the telescope on the detection bench which increased from 1.5 to 78.5 since the beam impinging on the suspended detection bench (SDB) is larger in the MSRC configuration (2.1 cm) than in the SVC configuration (Stable Vertical Cavities, 350 μm). A summary of the previous and new parameters is given in table 1.

We estimate limits on the jitter between the output beam of the Advanced Virgo interferometer (ITF) and the suspended detection bench. They are related to the dark fringe power variations introduced by the mis-alignment of the beam onto the output mode-cleaner (OMC).

A simple description of the coupling from the tilt of the bench to a translation of the beam on the OMC has been taken into account in this updated study.

	$\mathcal{F}$	L_{off} (m)	$1-C$	λ (m)	OMC waist (m)	SDB waist (m)
Previous	446 [2]	10^{-11} [3]	10^{-4}	1064×10^{-9}	236×10^{-6} [4]	350×10^{-6} [5]
New	443 [6]	10^{-11} [3]	10^{-4}	1064×10^{-9}	265×10^{-6} [7]	20.8×10^{-3} [8]

	P_{00} (W)	P_{01} (W)	P_{LG01} (W)
Previous	70×10^{-3}	1	280×10^{-3}
New	80×10^{-3} [7]	1	125×10^{-3} [9]

Table 1: Summary of Advanced Virgo parameters used in this note (compared to the parameters from previous note[1]). First table: arm cavity finesse, arm differential offset, contrast defect, laser wavelength and waist sizes. Second table: assumed powers of the beam in the TEM00, TEM01 and LG01 modes, in the case of 125 W input laser power.

2 Constraints on the jitter between the output beam and the detection bench

The variations of OMC output power related to TEM00 losses and high-order modes leakages have been estimated in equation 39 of note [1]. We have defined a parameter X used to described the jitter in translation: $X_a = \frac{a}{x_0}$ (with x_0 the beam waist on the OMC) and in rotation: $X_\alpha = \frac{\alpha}{\theta_D}$ (with $\theta_D = \frac{\lambda}{\pi x_0}$).

2.1 Constraints on the jitter RMS

Choosing the power losses or leakage to be less than $\epsilon = 1\%$ of the nominal power transmitted by the OMC, one can set constraints on the value of X. This value can be used as an estimation of X^{rms}.

2.1.1 Constraints from TEM00 losses

Choosing to loose less than ϵ of the incident TEM00 power due to the X offset, the maximum acceptable value of X is $X_{00}^{rms} = \sqrt{\epsilon} = 0.1$. The corresponding constraints on the translation and rotation are given in the table 3.

2.1.2 Constraints from first order mode power leakages

The TEMmn modes with $m+n = 1$ that are incident onto the OMC are related with misalignments of the ITF. A preliminary estimation of their power in Advanced Virgo is done scaling the powers measured in Virgo+ by the ITF input power.

As in note [1], we assume powers of ∼1 W for the modes TEM01 and TEM10.

From equation 39 from [1], the fraction of first order mode power that leaks onto the TEM00 mode of the mis-aligned OMC is $P_{10}^{in} \times X^2$. Choosing that the contribution from the high-order modes to the OMC transmitted power is less than ϵ, the maximum acceptable value of X^2 is $\epsilon \times \frac{P_{00}}{P_{10}}$. For an input power of 125 W, it yields[1] $X_{01}^{rms} < X_{00}^{rms}\sqrt{\frac{P_{00}}{P_{10}}} = 0.1 \times \sqrt{0.08/1} \sim 0.03$.

The constraints from the leakage of the first order modes to the fundamental mode of the OMC are thus a factor ~ 3.5 stronger than the one coming from the losses of the incident fundamental mode described section 2.1.

2.1.3 Constraints from the second order mode power leakages

The TEMmm modes ($m+n = 2$) are related to the first Laguerre-Gauss mode (LG01, LG10). In Advanced Virgo, the power in the Laguerre-Gauss HOM is expected to be less than 125 mW [9].

[1] We keep two independent analysis: (1) the TEM00 losses with quite robust hypothesis, and (2) the HOM leakages with much more uncertainties on the assumed HOM powers.

In order to have the leakage of these modes to be less than ϵ of the OMC transmitted beam, the maximum acceptable value of X^4 is $2\epsilon\frac{P_{00}^{in}}{P_{20}^{in}}$. For an input power of 125 W, it yields $X_{02}^{rms} < \left(2 \times 0.01 \times \frac{0.08}{0.125}\right)^{\frac{1}{4}} \sim 0.3$, which is looser than the constraints from the TEM00 and TEM01 modes.

2.1.4 Specification on the jitter RMS

The maximum acceptable jitter in translation or rotation, given as X and related to the different modes of the incident beam are summarized in table 2.

	TEM00 losses	TEMmn $(m+n=1)$ leakages	TEMmn $(m+n=2)$ leakages
X^{rms}	0.1	0.03	0.3

Table 2: Maximum value of X estimated in order to have less than 1% of losses of the TEM00 mode and less than 1% leakage of the HOM into the TEM00 mode (in the case of an **input power of 125 W***). These values can be used as estimations of X^{rms}.*

The stronger constraints is coming from the leakage of the 1st TEM modes onto the OMC TEM00 mode. This is the case as soon as the power in the 1st TEM modes is larger than the power is the TEM00 mode.

Due to the way the power of the 1st order modes have been extrapolated from Virgo+ to Advanced Virgo, this contraint might be too tight but it is used in the following to estimate the constraint on the relative jitter between the beam and the detection bench.

2.2 Frequency-dependent constraints on the jitter

The constraints on the RMS of the relative alignment between the OMC and the incoming beam can be extended to estimate constraints as function of frequency. The way this extension is applied is explained in [1].

2.2.1 From jitter RMS to frequency-dependent specifications

It was shown in note [1] that the jitter X induces power fluctuations on the dark fringe, which are equivalent to a differential arm length variations δL^X. In equation 46, the condition $\delta L_-^X < \delta L_-^{design}/\gamma$ is equivalent to:

$$\frac{\delta P(f)}{P_{00}(f)} \quad < \quad \frac{\delta L_-^{design}}{\gamma}\gamma_{RP}\frac{8\mathcal{F}}{\lambda}\frac{C\sin(\phi_{off})}{1-C\cos(\phi_{off})} \tag{1}$$

where $\phi_{off} = 8\mathcal{F}\frac{L_{off}}{\lambda}$, γ_{RP} is used to describe the radiation pressure effect and γ is the safety factor used to find the constraints.

Using the Advanced Virgo design values given in table 1, the constraint is estimated to:

$$\frac{\delta P(f)}{P_{00}(f)} < 1.7 \times 10^{11} \gamma_{RP} \frac{\delta L_{-}^{design}(f)}{\gamma} \quad (2)$$

$$< 5.1 \times 10^{14} \gamma_{RP} \frac{\delta h^{design}(f)}{\gamma} \quad (3)$$

The numbers are the same as in the previous note since it depends only on the change of finesse, from 446 to 443.

Losses of the TEM00 mode - The constraint on $X(f)$ coming from the TEM00 losses can be written as (equation 50 of [1]):

$$X_{00}^{max}(f) = 5.1 \times 10^{14} \gamma_{RP} \frac{\delta h^{design}(f)}{\gamma} \frac{1}{2X_{00}^{rms}} \quad (4)$$

The value of X_{00}^{rms} has been estimated in section 2.1.

Leakage of the TEM01 mode - The constraint on $X(f)$ coming from the TEM01 leakages can be written as (equation 52 of [1]):

$$X_{01}^{max}(f) = \sqrt{\frac{P_{00}}{P_{10}}} X_{00}^{max}(f) \quad (5)$$

The constraints on $X(f)$ from the TEM01 leakage are thus stronger than the constraints from the TEM00 losses if the ITF output power of the 1st HOM is stronger than the power of the fundamental mode.

From the estimation of the TEM01 power given above, the constraints are thus stronger by a factor ~ 3.5 than the one from the TEM00 losses.

Leakage of the TEM02 mode - It was shown that leakages from high order modes can be neglected [1].

2.2.2 Estimation of the beam specifications at the level of the OMC

From previous section, the higher constraints come from the leakage of the TEM01 mode into the TEM00 mode. From its RMS value X_{01}^{rms} and equation 51 from [1], one can derive the maximum amplitude spectrum of $X(f)$.

The Advanced Virgo reference sensitivity curve from the baseline design [10] (figure 2, p.11) has been used. The values at 10 Hz, 20 Hz and 100 Hz to derive the first constraint on $X(f)$ are given in tables 3 and 4.

To estimate the constraints on a and α, the waist of the beam at the OMC has to be known. The value of the OMC beam waist is given in table 1. It implies $\theta_D = 1.3\,\mathrm{mrad}$.

The constraints have been derived for different input powers (different values of the factor γ_{PR}). They are summarized in the tables.

Concerning the estimation of the factor γ_{RP} above 10 Hz, a value of 1 has been used for $P_{in} < 25\,\mathrm{W}$. It has been approximated by $\gamma_{RP} = 10^{-3} \times f^{2.3}$ for $P_{in} = 50\,\mathrm{W}$ and $\gamma_{RP} = 10^{-4} \times f^{2.7}$ for $P_{in} = 125\,\mathrm{W}$, using 1 as maximum value.

2.3 Estimation of the beam specifications at the level of the suspended detection bench

The constraint on the beam jitter a and beam angular stability α at the level of the OMC waist can be translated into constraints at the input of the suspended detection bench.

The telescope (and following optics that make the beam waist match for the OMC) can be described by a matrix that transforms the beam at the level of the bench to the beam at the level of the OMC, using the magnification $M = \frac{x_{SDB}}{x_{OMC}}$ (see table 1):

$$\begin{pmatrix} a_{OMC} \\ \alpha_{OMC} \end{pmatrix} = \begin{pmatrix} -\frac{1}{M} & \beta \\ 0 & -M \end{pmatrix} \begin{pmatrix} a_{SDB} \\ \alpha_{SDB} \end{pmatrix} \tag{6}$$

where β, the coupling factor from bench tilts to translations on the OMC, is expected to be of the order of 1 [12].

The constraints on the translation and tilts at the level of the detection bench are thus:

$$a^{SDB}_{max} \sim a^{OMC}_{max} \times M \tag{7}$$

$$\tag{8}$$

$$\alpha^{SDB}_{max} \sim Min \begin{matrix} \alpha^{OMC}_{max}/M \\ a^{OMC}_{max}/\beta \end{matrix} \tag{9}$$

$$\tag{10}$$

The estimated constraints are reported in table 3 and 4, assuming $\beta = 1$. Note that the tighter tilt constraints come from the direct coupling from tilt of the bench to tilt on the OMC.

3 Summary

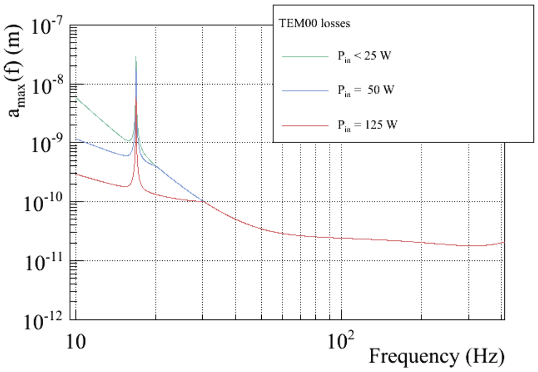
The dark fringe power variations introduced by mis-alignment of the ITF beam with respect to the suspended detection bench and the OMC have been estimated and converted into equivalent strain noise. Jitter specifications were extracted from the comparison of this level with the Advanced Virgo baseline sensitivity curve.

The **looser estimated constraints** that need to be fulfilled are summarized in table 3 and in figure 1. They are estimated from the TEM00 losses.
Some **tighter conservative constraints** are given in table 4 and in figure 2 They are estimated from the TEM01 leakages (assuming 1 W in the first order TEM modes for an input power of 125 W).

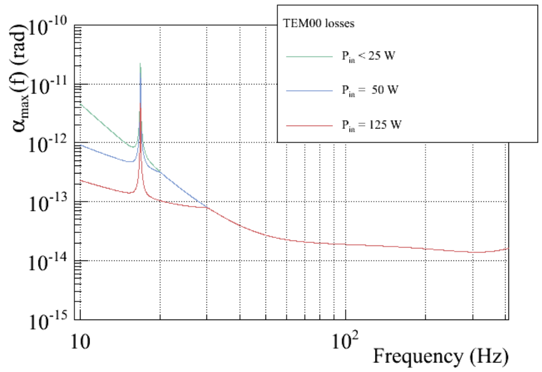
Note that these constraints depend on the **waist of the ITF output beam**: for a larger beam waist, the translation constraints will be looser while the rotation constraints will be tighter.

Different **differential offsets** L_{off} might be used during AdV commissioning phases when using lower input powers (such that the power of the carrier field is around 80 mW in all configurations). The values are within a factor 2 of the value of 10^{-11} m used in this note. The derived constraints on bench jitter would also vary by a factor ~ 2 with respect to what is given in this note (following equation 1).

The specifications apply both to the jitter of the AdV detection bench and to alignment issues through the jitter of the ITF output beam.

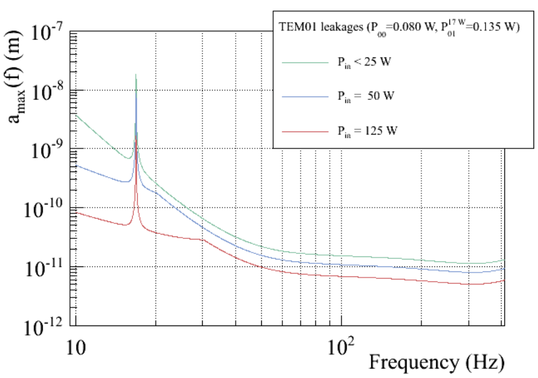


(a) Translation

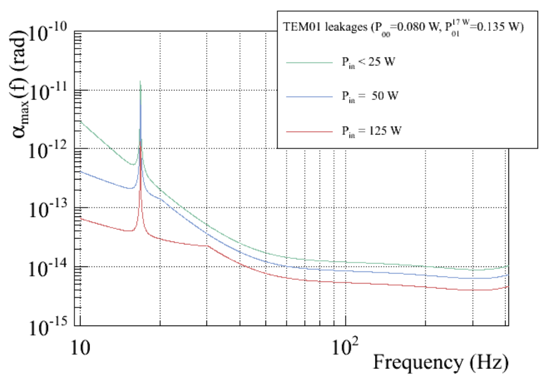


(b) Rotation

Figure 1: ***Jitter constraints derived from TEM00 mode losses*** *estimated for different input ITF powers.*



(a) Translation



(b) Rotation

Figure 2: ***Jitter (conservative) constraints derived from TEM01 mode leakages*** *estimated for different input ITF powers.*

			RMS	10 Hz	20 Hz	100 Hz
		δh^{design}	-	1.1×10^{-21}	7.7×10^{-23}	4.5×10^{-24}
$P < 25$ W	γ_{RP}		-	1.00	1.00	1.00
	OMC input	$X_{max}(f)$	0.1	2.8×10^{-07}	2.0×10^{-08}	1.2×10^{-09}
		$a_{max}(f)$ (m)	26×10^{-6}	7.4×10^{-11}	5.2×10^{-12}	3.1×10^{-13}
		$\alpha_{max}(f)$ (rad)	128×10^{-6}	3.6×10^{-10}	2.5×10^{-11}	1.5×10^{-12}
	SDB input	$a_{max}(f)$ (m)	2080×10^{-6}	5.8×10^{-09}	4.1×10^{-10}	2.4×10^{-11}
		$\alpha_{max}(f)$ (rad)	2×10^{-6}	4.6×10^{-12}	3.2×10^{-13}	1.9×10^{-14}
$P < 125$ W	γ_{RP}		-	0.05	0.33	1.00
	OMC input	$X_{max}(f)$	0.1	1.4×10^{-08}	6.4×10^{-09}	1.2×10^{-09}
		$a_{max}(f)$ (m)	26×10^{-6}	3.7×10^{-12}	1.7×10^{-12}	3.1×10^{-13}
		$\alpha_{max}(f)$ (rad)	128×10^{-6}	1.8×10^{-11}	8.1×10^{-12}	1.5×10^{-12}
	SDB input	$a_{max}(f)$ (m)	2080×10^{-6}	2.9×10^{-10}	1.3×10^{-10}	2.4×10^{-11}
		$\alpha_{max}(f)$ (rad)	2×10^{-6}	2.3×10^{-13}	1.0×10^{-13}	1.9×10^{-14}

Table 3: **(Minimal) jitter constraints from the TEM00 losses.** *RMS in given units, assuming the losses represent less than 1% of the power in the TEM00 mode. Maximum values at 10 Hz, 20 Hz and 100 Hz are given in units/*$\sqrt{Hz}$*, with a margin factor* $\gamma = 10$*. Different values of* γ_{RP} *have been used depending on the input power P (derived from [11], p.7).*

			RMS	10 Hz	20 Hz	100 Hz
		δh^{design}	-	1.1×10^{-21}	7.7×10^{-23}	4.5×10^{-24}

$P < 25$ W	γ_{RP}		-	1.00	1.00	1.00
	OMC input	$X_{max}(f)$	0.06	1.8×10^{-07}	1.2×10^{-08}	7.3×10^{-10}
		$a_{max}(f)$ (m)	16.8×10^{-6}	4.7×10^{-11}	3.3×10^{-12}	1.9×10^{-13}
		$\alpha_{max}(f)$ (rad)	81.1×10^{-6}	2.3×10^{-10}	1.6×10^{-11}	9.4×10^{-13}
	SDB input	$a_{max}(f)$ (m)	1320.4×10^{-6}	3.7×10^{-09}	2.6×10^{-10}	1.5×10^{-11}
		$\alpha_{max}(f)$ (rad)	1.0×10^{-6}	2.9×10^{-12}	2.0×10^{-13}	1.2×10^{-14}

$P < 125$ W	γ_{RP}		-	0.05	0.33	1.00
	OMC input	$X_{max}(f)$	0.03	4.0×10^{-09}	1.8×10^{-09}	3.3×10^{-10}
		$a_{max}(f)$ (m)	7.5×10^{-6}	1.1×10^{-12}	4.8×10^{-13}	8.7×10^{-14}
		$\alpha_{max}(f)$ (rad)	36.3×10^{-6}	5.1×10^{-12}	2.3×10^{-12}	4.2×10^{-13}
	SDB input	$a_{max}(f)$ (m)	590.5×10^{-6}	8.3×10^{-11}	3.8×10^{-11}	6.8×10^{-12}
		$\alpha_{max}(f)$ (rad)	0.5×10^{-6}	6.5×10^{-14}	2.9×10^{-14}	5.3×10^{-15}

Table 4: **(Conservative) jitter constraints from the TEM01 mode leakages**. *RMS in given units, assuming the leakages represent less than 1% of the power in the TEM00 mode. Maximum values at 10 Hz, 20 Hz and 100 Hz are given in units/$\sqrt{Hz}$, with a margin factor $\gamma = 10$. Different values of γ_{RP} have been used depending on the input power P (derived from [11], p.7).*

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