Gravitational-wave energy, luminosity and angular momentum from numerical relativity simulations of binary neutron stars' mergers

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We collect fitting formulae for gravitational-wave (GW) luminosity, energy and angular momentum derived from the `CoRe` database of numerical relativity simulations of quasicircular binary neutron star mergers. All the fitting formula were developed in [1], to which we refer for a comprehensive presentation.

Simulations. In [1] we use data from about 100 simulations of quasi-circular non-spinnng binaries with the `BAM` and `THC` code. All data are now public at

`http://www.computational-relativity.org/`

A summary of the database is presented in [3]. We do not use the simulations `BAM:0023-BAM:0034`. The data employed span the parameter ranges

$$1 < q < 2 \tag{1}$$
$$40 < \kappa_2^T < 500 \tag{2}$$

and refer to 8 EOS and different input physics, cf. discussion in [1]. Following results robustly describe also binaries with dimensionless spins up to $\chi \sim 0.1$, [1].

Definitions. Main quantities

$$e_{\rm GW} \equiv \frac{E_{\rm GW}}{M\nu} = -\frac{M - M_{\rm ADM}(t=0) - \mathcal{E}_{\rm rad}(t)}{M\nu} \tag{3}$$

The conversion factor of $E_{\rm GW}$ to physical units is $\mathrm{M}_\odot c^2$.

$$j_{\rm binary} \equiv \frac{J_{\rm binary}}{M^2\nu} = \frac{J_{\rm ADM}(t=0) - \mathcal{J}_{\rm rad}(t)}{M^2\nu} \tag{4}$$

The conversion factor to physical units of $J_{\rm binary}$ is $\frac{G{\mathrm{M}_\odot}^2}{c}$.

$$L_{\rm peak} \equiv \max_t \left(\frac{dE_{\rm GW}}{dt}\right) \tag{5}$$

The conversion factor to physical units is $L_{\rm Planck} = c^5/G$.

$M_{\rm ADM}(t=0)$ and $J_{\rm ADM}(t=0)$ are the mass and angular momentum of Arnowitt-Deser-Misner, calculated for the initial binary configuration.

$\mathcal{E}_{\rm rad}(t)$, $\mathcal{J}_{\rm rad}(t)$ are the energy and angular momentum radiated through GW during the simulation [4].

Comments on fits. The behaviour of the quantities above is captured by the symmetric mass ratio ν and the tidal parameters of the binary.

- The merger time $t_{\rm mrg}$ is defined as the peak of the amplitude of the $(2,2)$ mode of the GW.

- The total energy and final angular momentum at taken at the end of the simulation, $t_{\rm pm} \sim t_{\rm mrg} + 20 - 30$ ms. On this timescale the remnant radiate most of the GW energy [5]. At the end of our simulations the GW radiation timescale for angular momentum loss is $\dot{\mathcal{J}}_{\rm rad}/\mathcal{J}_{\rm rad} \sim 0.5$ s and rapidly increasing.

- Luminosity peak [1]. L_0 is the average of the luminosity peaks for binary black hole (BBH) mergers with equivalent parameters [6]. For $\kappa_2^{\rm L} = 0$, it matches nonspinning BBH with $q \sim 1$ in the above sense (L_0). For $\kappa_2^{\rm L} \gtrsim 3600$, it is linearly extended in such a way that the luminosity approaches to 0 for large values of the tidal parameters. The coefficient of determination of the fit is $R^2 = 0.943$. The fit errors are below 30%.

TABLE I. Binary parameters.

Quantity	Definition
Gravitational mass of star A in isolation	$M_{\rm A}$
Total gravitational mass of the binary	$M = M_{\rm A} + M_{\rm B}$
Mass ratio	$q = M_{\rm A}/M_{\rm B} \geq 1$
Symmetric mass ratio	$\nu = q/(1+q)^2$
Compactness of star A	$C_{\rm A} = \frac{GM_{\rm A}}{R_{\rm A}c^2}$
Gravito-Electric Quadrupolar Love number [2] of star A	$k_2^{\rm A}$
Neutron Star's Gravito-Electric Quadrupolar Tidal polarizability [2] of star A [a]	$\kappa_2^{\rm A} = 2\left(\frac{M_{\rm A}}{M}\right)^5 \frac{M_{\rm B}}{M_{\rm A}} \frac{k_2^{\rm A}}{(C_{\rm A})^5}$
Binary's Quadrupolar Tidal polarizability	$\kappa_2^{\rm T} = \kappa_2^{\rm A} + \kappa_2^{\rm B}$
Effective Binary's Quadrupolar Tidal polarizability for Luminosity	$\kappa_2^{\rm L} = 2\left[\left(3 + \frac{M_{\rm A}}{M_{\rm B}}\right)\kappa_2^{\rm A} + ({\rm A} \leftrightarrow {\rm B})\right]$

[a] The relation between this parameter and the more common Λ_2 is given by $\Lambda_2^{\rm A} = \frac{1}{3}\left(\frac{M}{M_{\rm A}}\right)^5 \frac{M_{\rm A}}{M_{\rm B}} \kappa_2^{\rm A}$

- GW Energy at merger and binary angular momentum at merger [1, 7]. The parameter which appears in this formulae is a tidal parameter with a correction depending on ν. a is an empirical parameter needed to amplify this correction. These formulae extend previous results presented in [7] but they are still unpublished in this form. For the energy, $R^2 = 0.992$ and the maximum errors are below 3%; for the angular momentum $R^2 = 0.993$ and the errors are below 1%. This fit can be used to properly estimate the minimum energy emitted by the binary. The fit for the GW energy at merger is shown in Fig. 1, note that for $\kappa_2^{\rm T} = 0$ the fit returns the BBH $q = 1$ value.

- GW total energy as a function of $\kappa_2^{\rm T}$ [1]. The given formula has the form of a piecewise function and this reflect the two possible outcome of the merger: a prompt BH formation or a massive NS. In the case of prompt collapses ($\kappa_2^{\rm T} \lesssim 63$)the emission of GWs stops almost immediately after the moment of merger and the amount of energy does not depend much on the EOS. If the remnant object is a neutron star ($\kappa_2^{\rm T} \gtrsim 73$) the emission of GWs continues after the merger and quantitatively depends on the EOS. For $63 \lesssim \kappa_2^{\rm T} \lesssim 73$ the behaviour is too uncertain and is not possible to give a prediction; the fit returns an average value in those cases.

  A possible application of this formula is an estimate on the maximum energy emitted through GWs in the post merger phase. For an equal-mass binary system with $M = 2.8$, the fit and data predict for the post-merger and total energy resp.

$$E_{\rm GW}^{\rm pm} \lesssim 0.072 \frac{M}{2.8} {\rm M}_\odot c^2 \tag{6}$$

$$E_{\rm GW}^{\rm tot} \lesssim 0.126 \frac{M}{2.8} {\rm M}_\odot c^2 . \tag{7}$$

  Note that this fit has large uncertainties and errors, as shown Fig. 2. Hence, it should be used to provide an estimate or the upper limits. This relation is still unpublished.

- GW total energy as a function of $j_{\rm rem}$ [1]. For this formula, $R^2 = 0.986$ and errors are below 8%.

- Remnant angular momentum: For this formula, $R^2 = 0.982$ and the maximum error is around 2%. This relation is still unpublished.

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TABLE II. Fitting formulae. Note that all the quantities shown are dimensionless.

Quantity	Fitting formula	Parameters
GW Luminosity peak	$L_{\rm peak}(\kappa_2^{\rm L},\nu) = \begin{cases} \kappa_2^{\rm L} \lesssim 3582 & L_0 \frac{\nu^2}{q^2(\nu)} \frac{1+n_1\kappa_2^{\rm L}+n_2(\kappa_2^{\rm L})^2}{1+d_1\kappa_2^{\rm L}} \\ \kappa_2^{\rm L} \gtrsim 3582 & L_0 \left[a\left(\kappa_2^{\rm L}-b\right)+c\right] \end{cases}$	$L_0 = 2.178\times10^{-2}$ $n_1 = 5.24\times10^{-4}$ $n_2 = -9.36\times10^{-8}$ $d_1 = 2.77\times10^{-2}$ $a = -6.1\times10^{-6}$ $b = 3582$ $c = 1.7\times10^{-2}$
GW Energy at merger	$e^{\rm mrg}_{\rm GW}(\kappa_2^{\rm T},\nu) = e_0 \frac{1+n_1\hat\kappa_2^{\rm T}+n_2(\hat\kappa_2^{\rm T})^2}{1+d_1\hat\kappa_2^{\rm T}+d_2(\hat\kappa_2^{\rm T})^2}$ $\hat\kappa_2^{\rm T} = \kappa_2^{\rm T} + a(1-4\nu)$	$a = 1.2\times10^{3}$ $e_0 = 0.12$ $n_1 = 5.09\times10^{-2}$ $n_2 = 6.44\times10^{-5}$ $d_1 = 9.53\times10^{-2}$ $d_2 = 2.64\times10^{-4}$
Binary's angular momentum at merger	$j^{\rm mrg}(\kappa_2^{\rm T},\nu) = j_0 \frac{1+n_1\hat\kappa_2^{\rm T}+n_2(\hat\kappa_2^{\rm T})^2}{1+d_1\hat\kappa_2^{\rm T}+d_2(\hat\kappa_2^{\rm T})^2}$ $\hat\kappa_2^{\rm T} = \kappa_2^{\rm T} + a(1-4\nu)$	$a = 1.2\times10^{3}$ $j_0 = 2.8$ $n_1 = 7.83\times10^{-2}$ $n_2 = 1.93\times10^{-4}$ $d_1 = 6.63\times10^{-2}$ $d_2 = 1.26\times10^{-4}$
GW post merger energy (as a function of $\kappa_2^{\rm T}$)	$e^{\rm pm}_{\rm GW}(\kappa_2^{\rm T}) = \begin{cases} 0.02 & \kappa_2^{\rm T} \lesssim 63 \\ - & 63 \lesssim \kappa_2^{\rm T} \lesssim 73 \\ a(\kappa_2^{\rm T})^{-\frac{7}{10}}+b & 73 \lesssim \kappa_2^{\rm T} \lesssim 458 \\ c\kappa_2^{\rm T}+d & \kappa_2^{\rm T} \gtrsim 458 \end{cases}$	$a = 2.44$ $b = -0.019$ $c = -5.1\times10^{-5}$ $d = 0.038$
GW total energy (as a function of $j_{\rm rem}$)	$e^{\rm tot}_{\rm GW}(j_{\rm rem}) = c_0 + c_1 j_{\rm rem} + c_2 (j_{\rm rem})^2$	$c_0 = 0.94$ $c_1 = -0.43$ $c_2 = 0.053$
Remnant angular momentum	$j_{\rm rem}(e^{\rm tot}_{\rm GW}) = c_0 + c_1 e^{\rm tot}_{\rm GW} + c_2 (e^{\rm tot}_{\rm GW})^2$	$c_0 = 4.39$ $c_1 = -17.2$ $c_2 = 38.5$

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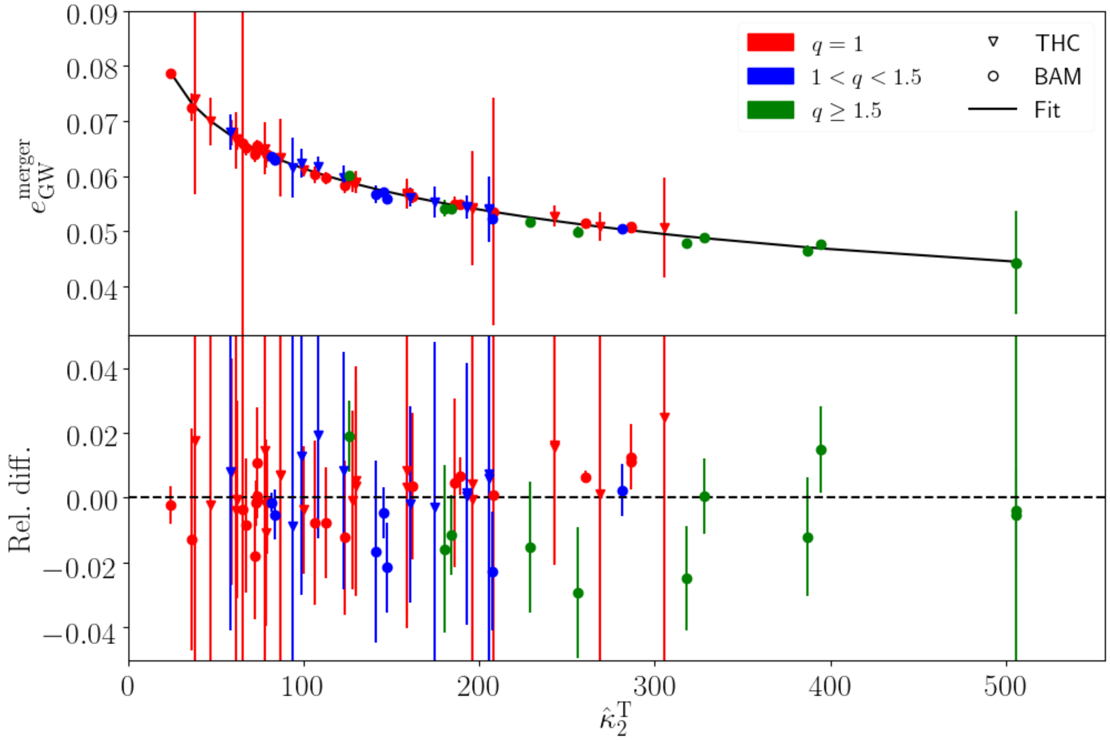

FIG. 1. Fit of the energy per unit mass radiated up to merger.

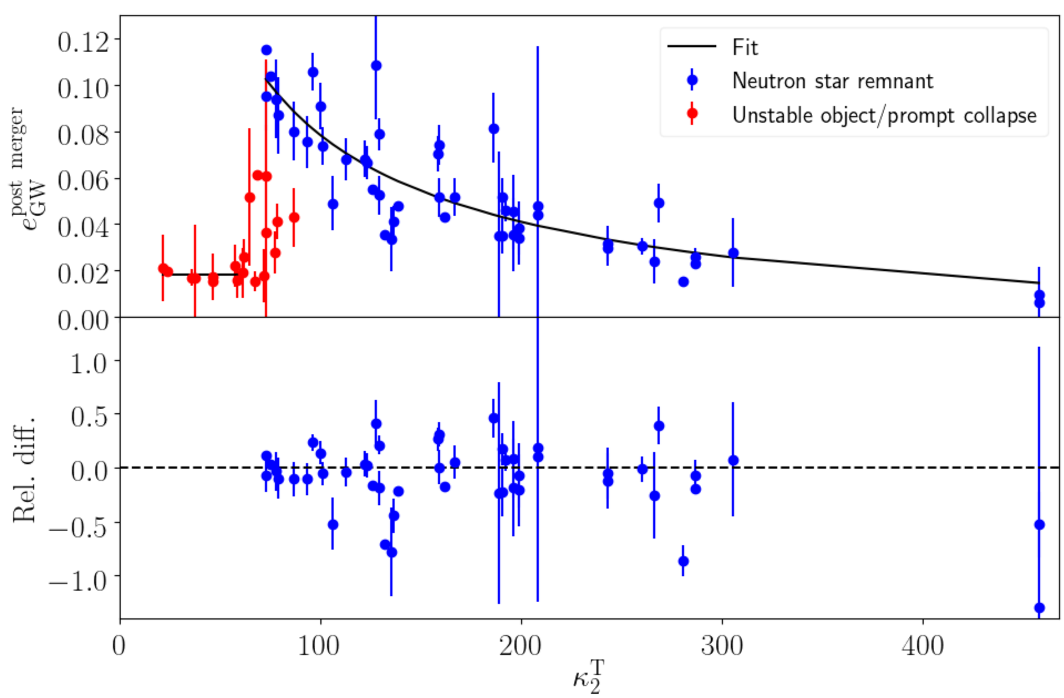

FIG. 2. Fit of the energy per unit mass radiated in the post-merger phase.