Correction of Doppler effect by discrete signal resampling

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Abstract

The detection of continuous gravitational waves has to deal with the Doppler effect induced by the Earth motion with respect to the source [1]. The correction to be applied to the antenna output depends on the source sky direction and on the signal frequency. Since these parameters are, in general, unknown a large computing effort is required to correct for any possible direction and emission frequency. A correction technique independent of the source frequency is discussed in this paper. The moving observer proper time is accelerated (or slowed down) by removing (or doubling) in a timely manner single samples of the detector digitized signal so to keep the synchronization to the rest clock. This technique allows to save a large amount of computing time in data analysis.

1 Introduction

Let us consider a source in the sky emitting a monochromatic wave with amplitude A_0 and frequency ν_0.* The Fourier analysis by an observer at rest with respect to the source, on a 1 year-long data bench, exhibits a peak with amplitude A_0 and width $\delta\nu = \frac{1}{1\mathrm{Year}} \simeq 3.17\ 10^{-8}$ Hz.

*In gravitational waves, A_0 is a tensor, but the argument is valid for fields of any nature.

Let us consider the simple case of a detector moving with constant velocity, covering in 1 year a circular orbit around the Sun and pointing to the source with fixed orientation. The detected signal will be:

$$A(t) = A_0 \cos(2\pi\nu_0 t + \varphi(t)) \qquad \text{where} \qquad \varphi(t) = \epsilon \sin(2\pi\nu_y t) \tag{1}$$

ν_y denotes here the orbital frequency while ϵ is the amplitude of the phase variation due to the Doppler effect (*phase modulation index*). The frequency modulation is thus $\Delta\nu = \nu - \nu_0 = \epsilon\nu_y \cos(2\pi\nu_y t)$.

For a source located on the orbital plane, the amplitude of the frequency modulation ($\epsilon\nu_y$) is given by first order Doppler effect: $\Delta\nu = \nu_0\ (v/c)$, where v is the orbital velocity and c is the speed of light.† As a consequence:

$$\epsilon = \frac{\nu_0 v}{\nu_y c} = \frac{2\pi r}{\lambda} \tag{2}$$

where ($r = v/(2\pi\nu_y)$) is the orbital radius and λ ($= c/\nu_0$) is the source wavelength. The signal energy is not affected by the phase modulation but it is spread on a band $\Delta\nu$ whose halfwidth is given by the frequency modulation amplitude, $\epsilon\nu_y$. In the case of Earth orbital motion (where $v \simeq 30$ km/s, and thus $\beta = v/c \simeq 10^{-4}$), for a 100 Hz wave the square of the spectral amplitude (A_1) is reduced by a factor of the order of $\frac{\delta\nu}{\Delta\nu}$, namely:

$$\frac{A_1^2}{A_0^2} = \frac{\delta\nu}{\Delta\nu} = \frac{\delta\nu}{2\beta\nu_0} \simeq \frac{3.17\ 10^{-8}}{2\ 10^{-4}\ 10^2} \simeq 1.6\ 10^{-6} \tag{3}$$

The ratio of the spectral amplitudes at the source frequency is thus $\simeq 1.25\ 10^{-3}$, corresponding to a reduction of signal to noise ratio $\simeq 58$ dB.

The Doppler effect can be corrected for a given direction by resampling the signal with an average frequency equal to the source one, but with a phase modulation able to correct the one induced by the observer motion. The same monochromatic signal coming from a direction $\overrightarrow{n}$, is detected by a moving observer as follows:

$$A_0\, e^{i\,(\overrightarrow{k}\cdot\overrightarrow{r}(t) - \omega_0 t)} \tag{4}$$

where $\overrightarrow{r}(t)$ is the position of the antenna as measured by the rest observer, while $\overrightarrow{k}$ is the wavelength vector, equal to $\frac{\omega_0}{c}\overrightarrow{n}$, with usual notations. Multiplying the signal by $e^{-i(\overrightarrow{k}\cdot\overrightarrow{r}(t))}$, the rest observer signal ($A_0\, e^{-i\omega_0 t}$) is recovered.

As discussed in section 5, the accuracy of this method is limited by the error in the knowledge of the source direction. For continuous wave detection the

†Second order Doppler effect is here neglected.

goal is to make a compensation accurate enough so to restore a large fraction of the signal energy in the original spectral peak and thus to have a signal to noise ratio similar to the one measured by the rest observer. The required accuracy can be computed by developing the phase-modulated signal in Bessel functions:

$$A(t) = A_0 \cos(\omega_0 t + \epsilon \sin(\omega_y t)) = A_0 \sum_{-\infty}^{+\infty} J_n(\epsilon) \cos(\omega_0 + n\omega_y)t \tag{5}$$

where $J_n(\epsilon)$ are the Bessel functions of the first kind and $\omega_y = 2\pi\nu_y$. For $\epsilon = 1$ rad they assume the following values:

$$\begin{aligned} J_0 &= 0.765198 \\ J_1 &= 0.440051 \\ J_2 &= 0.114903 \\ J_3 &= 0.019563 \\ J_4 &= 0.002477 \\ &\ldots\ldots \end{aligned}$$

The term at the source frequency (J_0) keeps thus more than 75% of the signal amplitude (about 60% of the energy). This means that, if the Doppler correction is able to reduce the phase modulation index down below 1 rad ($\epsilon < 1$), the peak spectral amplitude reduction, with respect to the rest frame, will be very small, less than 3 dB. This is taken as our specification, even if the entire argument can be scaled to have a better accuracy in peak amplitude recovering.

To reach 1 rad accuracy, the Earth position has to be known better than $\lambda/2\pi$, that means, for a 100 Hz signal, about 500 km. Since Earth ephemeris are measured with an accuracy of hundreds of meters [2], the correction can be performed up to very high frequency (tens of kHz). Moreover, to keep a dephasing less than 1 rad for a 100 Hz signal detection in a one year-long observation time, the antenna clock has to exhibit a stability of $\frac{1}{2\pi 100 \cdot 1\ year}$ (i.e. better than one part on 2 10^{10}). Gravitational wave antenna clocks, synchronized to GPS time, exhibit a much better stability [3][4].

2 Discrete Resampling Correction

The method discussed here aims to synchronize the moving observer clock to the rest one with an accuracy better than a sampling interval $\Delta t = 1/\nu_s$, where ν_s denotes the sampling frequency. Let us consider an ideal sinusoidal wave with a frequency equal to the sampling one, coming from a given direction in the sky. Its equiphase surfaces are planes perpendicular to the wave vector $\overrightarrow{k}_s$, travelling at the speed of light. In the rest frame the plane equation for a given phase ϕ is given by:

$$\overrightarrow{k}_s \cdot \overrightarrow{r} - \omega_s t = \phi \tag{6}$$

where $\omega_s = 2\pi\ \nu_s$ and $|\overrightarrow{k}_s| = \omega_s/c$.

Let us select the family of planes whose phase ϕ is an integer multiple of 2π. Considering two contiguous planes in Eq.6, it straightforward to understand that these planes travel parallel each other, separated by a time Δt, (i.e. by a distance $c\Delta t$), corresponding to a phase $\omega_s \Delta t$. The rest observer that measures the positions of the family planes at sampling frequency ν_s, i.e. each Δt seconds, will sees at any sample time the $j\ th$ plane takes the place the $(j+1)th$ one had at the previous sample, and to be replaced by the $(j-1)th$ one. In other words, the planes shift each other but the family will fill at any sample the same positions. These fixed planes are used as a reference grid in the rest frame where the moving observer motion is described by a trajectory $\overrightarrow{r}(t)$. Without any loss of generality one can assume that at time $t = 0$ the two clocks (moving and rest one) are synchonized to $\phi = 0$ on the target plane crossing the rest frame origin, coincident with the moving observer starting position. At a time t the phase measured by the moving observer is $\overrightarrow{k}_s \cdot \overrightarrow{r}(t) - \omega_s t$ (Eq.[6]), to be compared with the one detected by the observer at rest in the origin, $\omega_s t$. The dephasing $(\overrightarrow{k}_s \cdot \overrightarrow{r}(t))$ is thus ruled by the moving observer position with respect to the start grid plane where synchronization occurred.

As mentioned above, the goal is to lock the two clocks so to have a time difference not larger than the sampling interval (Δt). When the moving observer crosses one of the two planes nearest to the origin, a dephasing $\omega_s \Delta t$, has been cumulated. This can be compensated slowing down (or accelerating, depending on the dephase sign - see below) the moving observer proper time, by a time Δt. This correction can performed in an easy way, just repeating (or deleting) one of the digitized signal sample. In particular, if the observer motion versus is opposite to the wave one (i.e. $\overrightarrow{v} \cdot \overrightarrow{n} < 0$) a negative dephasing with respect to the rest clock occurs. This means that the moving clock is anticipating the rest one. The moving clock is thus "delayed" by repeating a sample of moving observer signal. Viceversa, when $\overrightarrow{v} \cdot \overrightarrow{n} > 0$, a delay occurs, compensated by removing a sample. The correction just described has to be performed each time the moving observer crosses one of the grid planes.

For a generic wave with frequency ν_0, the achieved time synchronization, implies a phase locking accuracy of $\omega_0 \Delta t$, i.e. $2\pi\nu_0/\nu_s$. To meet the required

1 rad phase accuracy (see sect.1) it thus necessary to operate with a sampling frequency at least 2π times larger the source one. In gravitational wave antennas use of sampling frequencies around 20 kHz is made, and thus the technique can be applied up to several kHz. This widely covers the range where continuous gravitational waves are expected, leaving a margin for better accuracy requirements. It is important to stress that once this time-domain correction is performed for a given direction of the sky, the 1 rad accuracy specification is fullfilled at the same time for all frequencies below $\nu_s/2\pi$. One can thus assume the Doppler effect corrected in all this frequency range, and consider the moving observer at rest with respect to any source coming from the chosen sky direction.

In the usual cases the moving observer velocity does not change too much during the crossing between two grid planes. The time to cross two contiguous planes ($t_{crossing}$) is well approximated by their distance ($c\Delta t$), divided by the amplitude of the observer velocity projection along the wave vector ($|\overrightarrow{v} \cdot \overrightarrow{n}|$):

$$t_{crossing} = 1/(|\overrightarrow{\beta} \cdot \overrightarrow{n}| \; \nu_s) \tag{7}$$

The minimum possible values for $t_{crossing} = 1/(\beta\nu_s)$, occur when the velocity direction is parallel to the wave vector. This means that the correction cannot occur before than $1/\beta$ samples (in the case of Earth, where $\beta \simeq 10^{-4}$, one each about ten thousands). Viceversa, when the velocity of the orbit is almost perpendicular to the wave vector long crossing times take place.

3 Simulation

The technique was tested by developing a C language simulation. A routine computes and writes on disk, at a sampling rate ν_s, the source monochromatic signal with unitary amplitude, namely:

$$A = \sin(-2\pi\nu_0 i \; \Delta t) \tag{8}$$

where i denotes the sample index and Δt is the sampling interval ($= 1/\nu_s$). The same signal as detected by an observer moving in the rest frame with trajectory $\overrightarrow{r}(t) = (x(t), y(t), z(t))$:

$$A = \sin(2\pi\nu_0 \; (\frac{n_x x + n_y y + n_z z}{c} - i\Delta t)) \tag{9}$$

is also computed.

Another routine computes all times t_j the orbit crosses an equiphase plane of the grid. All values of t_j, expressed in term of the rest frame sample index, and a label indicating the corresponding action (-1 for sample suppression and $+1$ for duplication) are stored in a two column ascii file (*mask.dat*). The crossing time computation is made with a good accuracy using the linear approximation mentioned above, and thus with a very small computing effort. Also the computing time required for mask application is negligible: during the data reading it is enough to suppress (or to double) single samples (each a few thousands, in realistic cases) according to what written in *mask.dat*. In our simulation this is made by another routine that applies the mask to the moving observer signal (Eq.9). The analysis was performed in a ROOT-VEGA environment [5], where use of FFTW package [6] is made for spectral analysis.[§]

4 Validation of the technique

Several tests were performed, varying the sampling and signal frequencies, the wave direction and the orbital motion. The first three figures concern a 10 Hz sinusoidal wave with a 100 Hz sampling frequency, travelling along the negative y direction. The antenna, in the origin at time $t = 0$, moves on x-y plane, anticlockwise, along a $3 \cdot 10^8$ m radius circle, with constant speed $\beta = v/c = 10^{-3}$. The chosen numbers allows testing the method on a full orbit with a reasonable amount of data. The corresponding phase modulation index, ruling the spectral peak smearing in the moving reference frame, is now $\epsilon = 2\pi r/\lambda \simeq 62$ rad.

The phase difference between the signal detected by the moving observer and the rest one is plotted in Fig.1 (black points) as a function of the orbital position x-y. The same plot for the corrected signal (red points) is close to zero in any orbital position. The achieved synchronization can be better evaluated in Fig.2, where the same phase difference for the corrected signal is plotted as a function of time, along the entire orbit. The dephasing is contained in a range $\simeq \pm\, 0.62$ rad (as expected by the phase accuracy relation $|\Delta\phi| \leq 2\pi\nu_0/\nu_s$ discussed in sect.2). Around $t = 0$ (see first Fig.2 zoom), when the velocity is almost parallel to the wave vector, short crossing times occur and thus frequent corrections are necessary. At one and three quarters of period (see second zoom), the velocity is almost perpendicular to the wave vector and long crossing times take place.

[§]A Bartlett window is applied to all time signals to avoid truncation effects durign FFT computations. Use of different windows was also made, with negligible differences.

As shown in Fig.3 the correction technique allows to restore a large part of the signal energy, spread by the observer motion on a wide frequency band, in the main spectral bin.

5 Dependence on direction accuracy

The dependence of the results on the accuracy the source direction is known is discussed here. Let us consider the wavevector $\vec{k}$ forming an angle β with the ecliptic plane and another vector, located on the same parallel, inclined with respect to $\overrightarrow{k}$ by an angle $\delta\alpha$. It is straightforward to obtain:

$$(\overrightarrow{k}_1 - \overrightarrow{k}) \cdot \overrightarrow{r} = \frac{\omega_0 r}{c} \cos\beta \left[(\cos\delta\alpha - 1) + \sin\delta\alpha \sin\omega_y t\right] \tag{10}$$

For $\delta\alpha << 1$ rad, this means:

$$(\overrightarrow{k}_1 - \overrightarrow{k}) \cdot \overrightarrow{r} = \frac{\omega_0 r}{c} \delta\alpha \cos\beta \sin\omega_y \tag{11}$$

This represents the residual phase modulation after a correction made using a direction slightly different from the right one (i.e. the one that should cancel completely the orbital Doppler effect by a perfect resampling). In order to meet our specifications the residual modulation index has to be less than 1 rad, that means:

$$\frac{\omega_0 r}{c} \delta\alpha \cos\beta < 1 \Rightarrow \delta\alpha < \frac{c}{\omega_0 r \cos\beta} \tag{12}$$

It is important to stress that the required accuracy in the source direction knowledge increases with the signal frequency. For a 100 Hz source located on the ecliptic plane ($\beta = 0$), and considering an orbital radius equal to the Earth-Sun distance ($\simeq 1.5 \cdot 10^{11} m$), one obtains:

$$\delta\alpha < \frac{3 \cdot 10^8}{1.5 \cdot 10^{11} \cdot 2\pi 100} \simeq 3.18 \cdot 10^{-6} rad = 0.66'' \tag{13}$$

Let us consider the other case: a vector $\overrightarrow{k}_1$, located on the same meridian of $\overrightarrow{k}$, forming an angle $\delta\beta$ along the parallel. It is immediate to obtain:

$$(\overrightarrow{k}_1 - \overrightarrow{k}) \cdot \overrightarrow{r} = \frac{\omega_0 r}{c} \cos\omega_y t \left[\cos\beta(\cos\delta\beta - 1) - \sin\beta \sin\delta\beta\right] \tag{14}$$

For $\delta\beta << 1$ rad, this is:

$$(\overrightarrow{k}_1 - \overrightarrow{k}) \cdot \overrightarrow{r} = -\frac{\omega_0 r}{c} \delta\beta \sin\beta \cos\omega_y t \tag{15}$$

Putting again the residual phase modulation index after the Doppler correction smaller than 1 rad, one obtains the maximum allowed direction mistuning:

$$\frac{\omega_0 r}{c}\delta\beta|\sin\beta| < 1 \Rightarrow \delta\beta < \frac{c}{\omega_0 r|\sin\beta|} \tag{16}$$

For a 100 Hz source located at ecliptic pole ($\sin\beta = 1$), and taking the Earth-Sun distance as orbital radius, one obtains again:

$$\delta\beta < 3.18 \cdot 10^{-6} rad = 0.66'' \tag{17}$$

The number of "independent" direction dN in a solid angle $d\alpha\, d\beta$ is:

$$dN = \frac{d\alpha}{\delta\alpha}\frac{d\beta}{\delta\beta} = \frac{d\alpha}{\frac{c}{\omega_0 r\cos\beta}}\frac{d\beta}{\frac{c}{\omega_0 r|\sin\beta|}} = \left(\frac{\omega_0 r}{c}\right)^2 \cos\beta|\sin\beta| d\alpha d\beta \tag{18}$$

Integrating on the whole solid angle, the number of independent directions to be scanned in order to achieve the required accuracy in Doppler correction is given by:

$$N = \left(\frac{\omega_0 r}{c}\right)^2 \int_0^{2\pi} d\alpha \int_{-\pi/2}^{+\pi/2} \cos\beta|\sin\beta| d\beta = 2\pi\left(\frac{\omega_0 r}{c}\right)^2 \tag{19}$$

that, in the case of Earth orbital motion, means $6.2 \cdot 10^{11}$ directions. A "blind" search of continuous wave in the sky based on a 1 year long FFT is thus prohibitive. Indeed, one can relax direction accuracy by using shorter integration times, at the price to reduce the detection sensitivity. Several techniques, with reduced sensitivity, were developed to work with a reasonable computing time (see, for instance, [7].*

The sensitivity of the proposed correction method on source direction can be studied by the simulation. The mask computed for a given direction is applied to signals coming from slightly different ones. The results turned out in agreement with previous computation, for several sky directions and for different mistuning values. For instance, let us analyze the same configuration used for the previous figures. Just the sampling frequency was increased from 100 Hz to 1 kHz, so to improve the syncronization in the ideal direction case. In this way the phase accuracy (plotted in Fig.2) is improved, and one can better appreciate the effects due to the direction mistuning. The mask used in previous figure, computed for the source direction (0,-1,0), is applied to an identical wave coming from the orbital plane with direction inclined anticlockwise by $\alpha \simeq 0.0157$ rad with

*As shown in this reference, the blind search sensitivity increases with the fourth square of integration time.

respect to the mask direction. According to the previous computation (Eq.12), this deviation should provide with our simulation parameters ($r = 3\ 10^8$ m and $\beta = 2\ 10^{-3}$) a residual phase modulation index of 1 rad in the case of a full resampling Doppler correction. As shown in Fig.4.a, this result is found also with the discrete resampling technique. As expected, with this 1 rad fluctuations, the recovering of peak amplitude (Fig.4.b) is still acceptable, even if at the level of the specification threshold (3 db - see section 1). This proof that the sensitivity of the discrete technique to source direction mistuning is identical to what expected by the complete signal resampling.

6 Effects on the noise

A final test was made by adding a gaussian random noise to the monochromatic signal. The amplitude of the monochromatic signal and the gaussian noise sigma was fixed in our simulation to have a good signal to noise ratio (a few units) at the peak frequency, when the integration time is equal to an orbital period ($\simeq 6284$ s).[†] In Fig.5 the linear spectral density of the three signals (rest, moving and corrected) is reported for the configuration used in the first three figures. The moving detector is not able to distinguish the peak from the noise since the signal energy is spread out on a wide frequency band, making the spectral amplitude smaller than the noise floor. Once the discrete correction is applied the energy in the main bin is almost enterely recovered, and the signal to noise ratio is similar to the one measured by the rest observer.

7 Required computing time

The mask computation and application, used to correct for Doppler effect requires a negligible computing time with respect to other data analysis algorithms. In general, it is not necessary to prepare a mask to be used later on data, as made in our simulation. The next crossing time, and thus the index of the next signal sample to be deleted or duplicated, will be computed inside the routine reading the data. The crucial point is that this computation has to be done not more frequently than a couple of times per second. Indeed for the Earth $\beta \simeq 10^{-4}$ and thus, in the worse case (when the antenna velocity is parallel to the wave vector) a correction each 10^4 samples is necessary (see end

[†] The linear spectral density of the signal peak does not depend on the integration time, while the noise spectral floor decreases with its square root.

of Sect.2). If one considers the typical 20 kHz sampling frequency, this means to compute and operate a correction each 0.5 s. In this period the ground-based detector motion is surely well approximated by a straight line and the linear approximation can be used to compute next crossing time. Once Earth position is computed by ephemeris (an operation necessary for any other Doppler correction techniques), the next crossing time can be calculated by not more than a few tens of operations. Less than a few 10^{-3} operations per sample are thus necessary. This computing cost is very small if compared to what required by other algorithms used for spectral analysis [8] and allows to correct for the first order Doppler effect at all the interesting frequencies.

8 Conclusions

The proposed correction technique is able to synchronize the moving observer clock to the rest one with enough accuracy to make the first order Doppler effect negligible for continuous wave detection. Indeed, the signal peak spectral energy of a monochromatic source is almost enterely recovered, once the correction is applied to the signal detected by the moving reference frame. The tecnique, performed in time-domain and valid for a given direction of a sky, provides a sufficient correction for any source coming from this direction, independently of its frequency. This operation requires a negligible amount of computing time if compared with other algorithms used in continuous gravitational wave data analysis.

9 Acknowledgments

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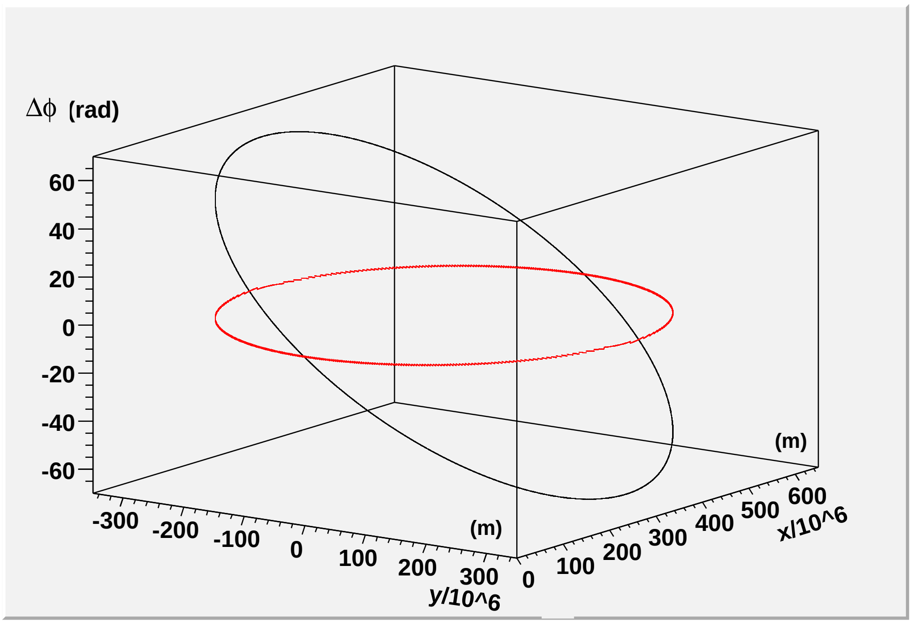

Figure 1: Phase difference between the sinuisodal wave as detected by the moving observer with respect to rest observer as a function of the antenna orbital position x-y (*black points*). The same plot for the dephasing between the corrected signal and the one detected by the rest observer is displayed (*red points*). INPUT PARAMETERS: source frequency = 10 Hz, sampling frequency = 100 Hz, wave direction = (0,-1,0), orbital radius = 3 10^8 m, $\beta = 10^{-3}$.

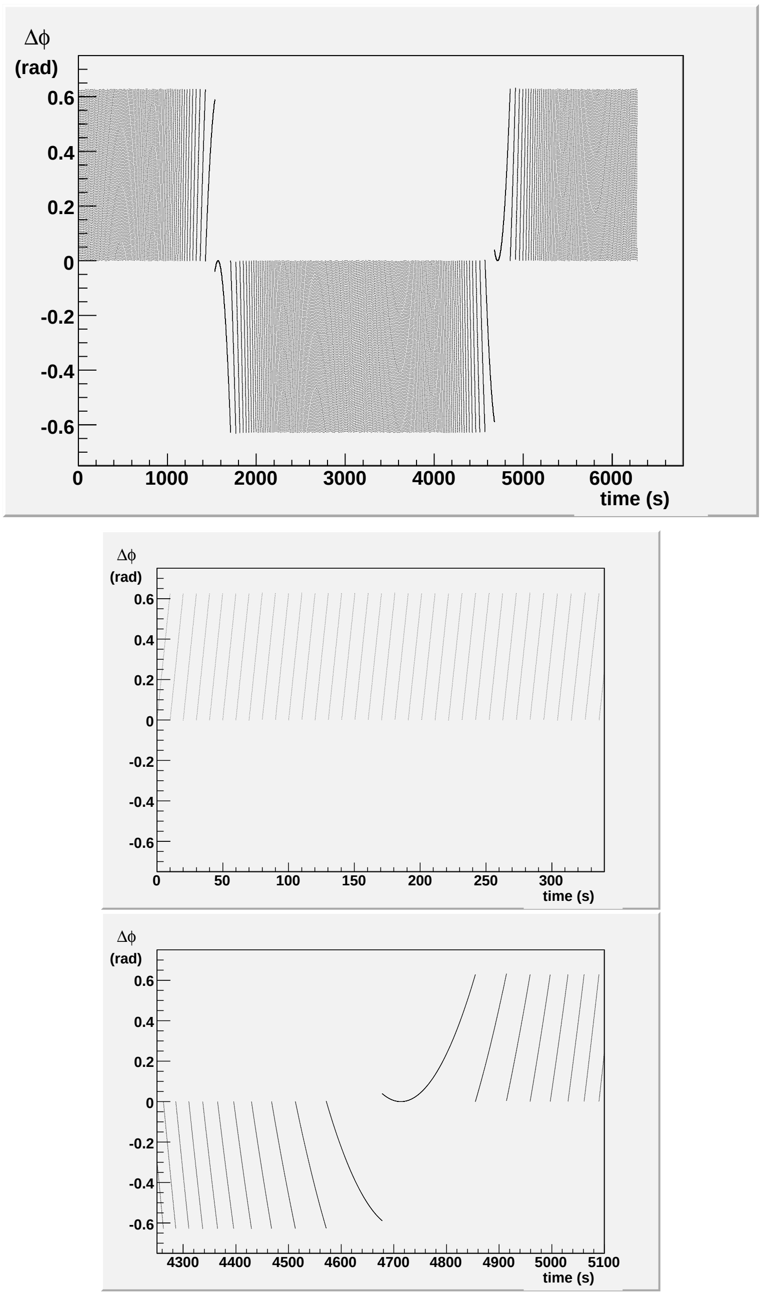

Figure 2: Same phase difference in red points of Fig.1, plotted as a function of time for the entire orbit ($\simeq 6,283$ s). Two zooms of the previous plot around t=0 and t=3/4 of the orbital period are reported.

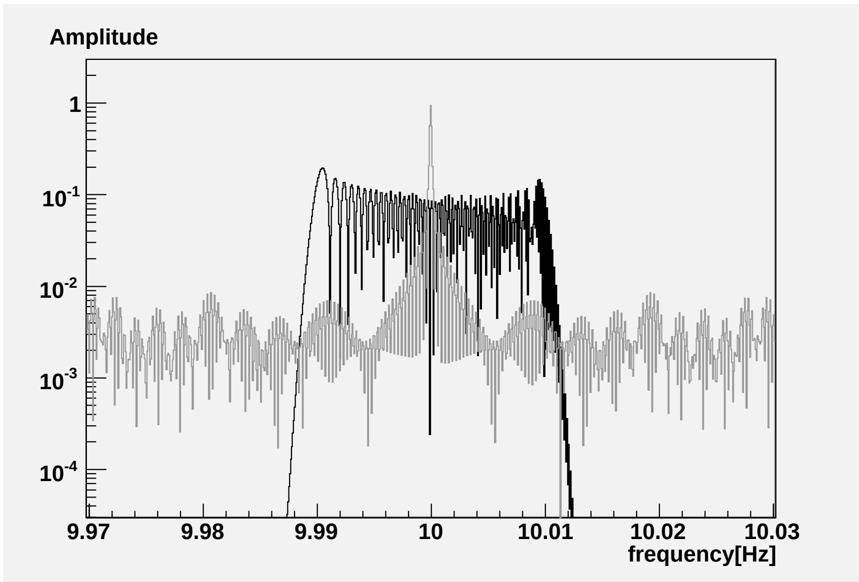

Figure 3: Linear spectral density of the signal as measured by moving observer (*black*) and of the corrected signal (*gray*). The configuration is the same of the two previous figures. The energy of the signal detected in moving observer is spread out on many bins. The spectral peak amplitude measured by the rest observer (equal to 1) is almost entirely recovered by the correction techique (peak linear spectral density $\simeq 0.91$). An integration time of two orbital periods (about 12,566 s) was considered. Use of log scale was done to emphasize the differences in the tails.

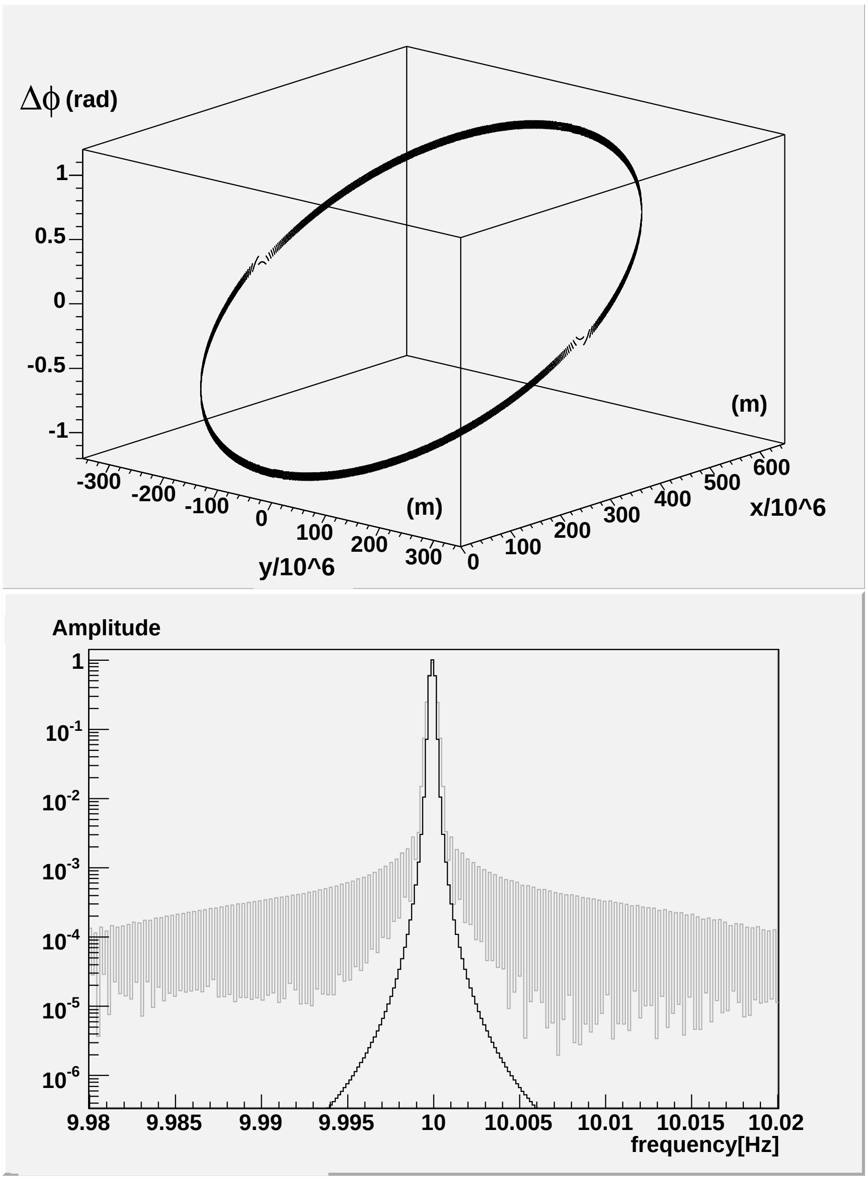

Figure 4: *Top*: Same plot of Fig.1 when the mask computed to correct for direction (0,-1,0) is applied to a signal coming from the same orbital plane, but inclined by 0.0157 rad anticlockwise with respect to the negative y direction. *Bottom*: Linear spectral density of the corrected signal (gray curve) and of the rest one (black) computed on one orbital period (about 6,283 s). The ratio between corrected and rest linear spectral densities at the peak frequency is around 0.75.

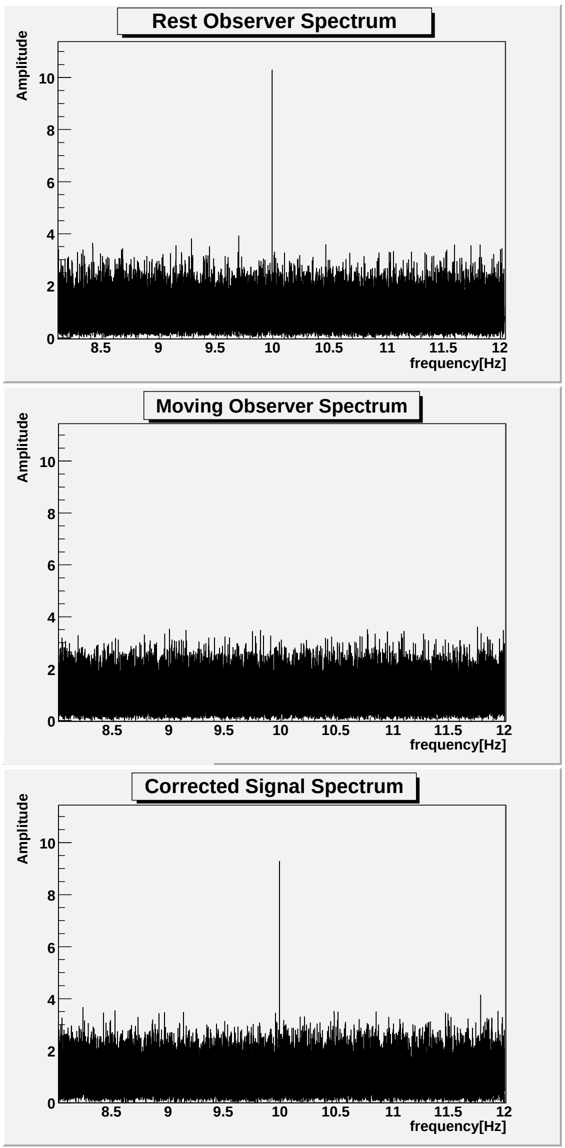

Figure 5: Linear spectral density of the signal as measured by rest (*top plot*), moving (*middle plot*) and corrected (*bottom plot*) observer when a gaussian random noise is added to the monochromatic wave. The configuration is the same of Fig.1. The integration time is two orbital periods (about 12,566 s).