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Green Arm Cavity Length Noise Requirement

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1 Introduction

The auxiliary lasers system (ALS) will be used in the lock acquisition of Adv+ to lock the arm cavities away from the infrared laser resonance while the center interferometer (CITF) is locked. When designing the controller filter it is important to know the expected noise sources. For the green arm cavities, the primary noise sources are seismic disturbances and frequency noise introduced by the optical fiber. Further, the maximum allowable noise should be predicted in order for to ensure a proper safety margin for any unforeseen noise.

The green cavity lock is simulated in time-domain by using End-to-end software (e2e)[1]. By simulating the lock, preliminary locking strategies and controller filters may be tested. In Advanced Virgo (AdV), magnet-coil pair actuators reliably control low frequency noise (seismic noise), even up to earthquake level disturbances. For Advanced Virgo Plus (AdV+), additional high frequency noise content is anticipated for the green cavity lock due to optical fiber noise. [2]. A frequency controller is included in ALS and used in e2e to simulate the allowable noise, particularly at high frequencies.

In this note, the control system used in the e2e simulation will be described in detail. Further, loop transfer functions from noise sources will be developed. A filter for the maximum allowable noise will be defined by using the developed loop transfer functions. Finally, a lock acquisition using the maximum allowable noise filter will be simulated to verify the noise limit.

2 Theory

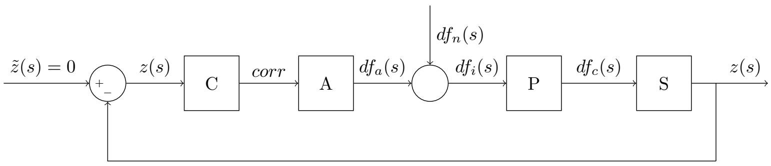


Figure 1: Block diagram for the arm cavity longitudinal control with frequency controller. Variables shown to define loop transfer functions, particularly noise coupling transfer functions.

The block diagram of the control system simulated with e2e is shown in Figure 1. Frequency and length shifts are used interchangeably since they are simply related by the free spectral range,

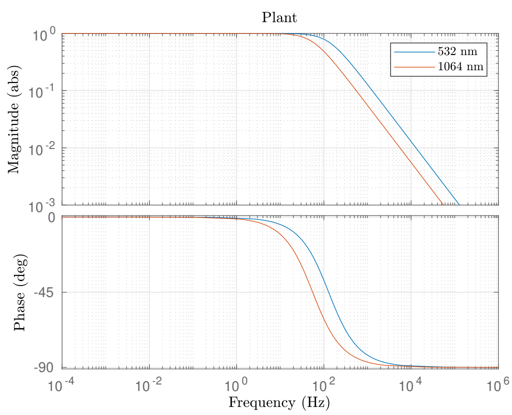
$$\frac{d_f}{c/2L} = \frac{d_l}{\lambda/2}. \tag{2.1}$$

2.1 Component Description

P, Plant Long arm cavity with green laser. Consists of a single pole determined by mirror parameters at $\lambda = 532$ nm. High frequency disturbances are damped by the cavity. Note for ALS, the green light enters through the ETM, and therefore the reflection is of the ETM mirror.

$$g_p = 128.8\,Hz \tag{2.2}$$

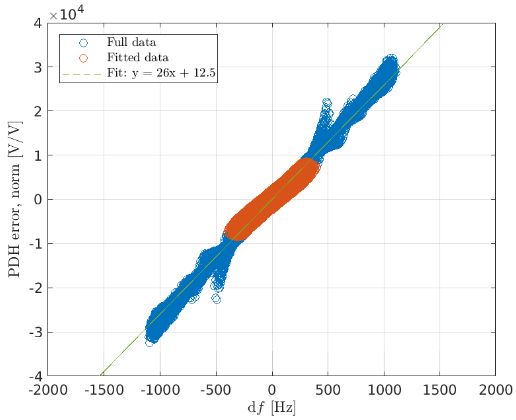
Input Frequency change [Hz]
Output Frequency change transmitted by cavity [Hz]



S, Sensor Normalized Pound Drever Hall (PDH) signal. For this note the PDH signal is the demodulated signal in reflection normalized by transmitted DC power. This component is simply a constant gain which is the slope of the PDH error signal. The slope of the error signal is the linear fit of the normalized error signal to the change in frequency into the arm cavity. The figure to the right shows the calculation of the slope of the error signal. Only the data where the transmitted DC power was over 15% of the maximum transmitted DC power (shown in red) were used to calculate the fit.

Input Frequency change transmitted by cavity [Hz]

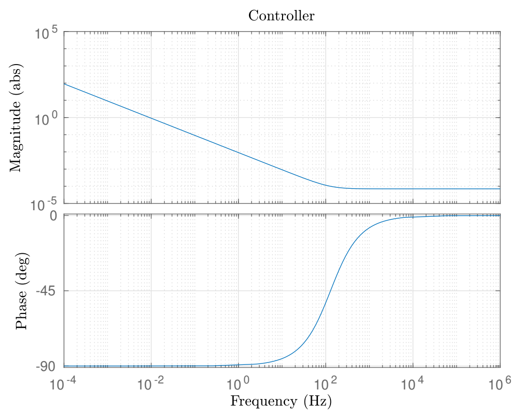
Output Normalized error signal [V/V]



C, Controller Frequency actuator control filter. At this point, the filter is simply an integrator, a zero to cancel the plant pole and a gain calculated so that the open loop transfer function unity gain frequency is 10 kHz with a sampling frequency of >100 kHz.

Input Normalized error signal [V/V]

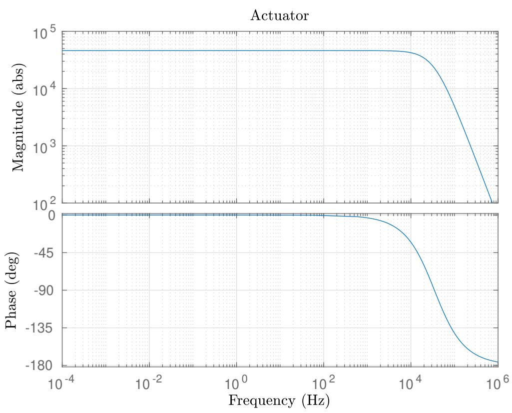
Output Correction signal to actuator [V]



A, Actuator The frequency actuator consists of several components which change the gain and place pole frequencies. 1) a voltage control oscillator (VCO) with a gain of 4,000kHz/V and pole at 30 kHz, 2) A divider to reduce the gain to 23 kHz/V, 3) an acousto-optic modulator with a gain of 1 and a pole at 40 kHz, and 4) a second harmonic generator with a gain of 2 to convert the infrared to green. Note: in e2e the frequency is converted to a length changed enforced on the mirror to simulate a frequency shift.

Input Correction signal to actuator [V]

Output Frequency change inputted to cavity [Hz]



2.2 Loop Transfer Function

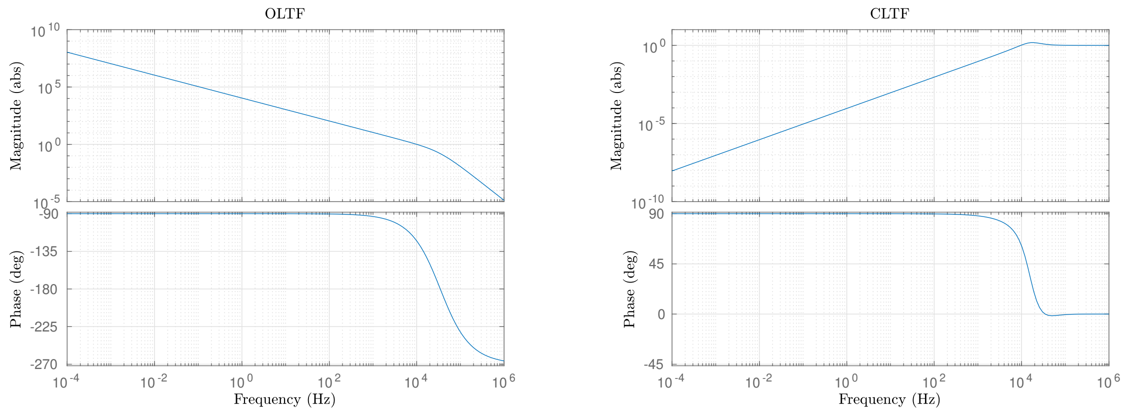
Several useful transfer functions may be derived from the component definitions and the block diagram shown in Figure 1. The most basic is the open loop transfer function (OLTF). This is simply a multiplication of all the components and is equivalent to removing the feedback loop in the bottom of Figure 1,

$$\begin{aligned} z_2(s) =& CAPz_1(s) \\ OLTF = \frac{z_2(s)}{z_1(s)} =& CAPS. \end{aligned} \tag{2.3}$$

The open loop transfer function shows how an initial disturbance is operated on by the system. Figure 2a shows the Bode diagram of the OLTF. Note that the UGF is at 10kHz as designed.

The closed loop transfer function (CLTF) is also derived from Figure 1 by considering the feedback loop and the setpoint, $\tilde{z}(s)$. The set point for this system $\tilde{z} = 0$ since zero error indicates the cavity is on resonance. The system function for a the CLTF with no noise may be derived as

$$\begin{aligned} z(s) =& \tilde{z}(s) - CAPSz(s) \\ CLTF = \frac{z(s)}{\tilde{z}(s)} =& \frac{1}{1 + CAPS} = \frac{1}{1 + OLTF}. \end{aligned} \tag{2.4}$$



(a) OLTF   (b) CLTF

Figure 2: Open and closed loop transfer functions. Note that the UGF is at 10kHz.

Figure 2b shows the CLTF response for the system. The plot shows that for low frequencies, any error in the system is reduced by many orders of magnitude, increasing as f until the UGF. Above the UGF the controller is noneffective and disturbances are transferred directly.

Another useful transfer function is from frequency change due to noise $df_n(s)$ to frequency change transmitted by the plant $df_c(s)$, which may be derived as

$$\begin{aligned} df_c(s) =& Pdfn(s) - CAPSz(s) \\ \frac{df_c(s)}{dfn(s)} =& \frac{P}{1 + CAPS} = P * CLTF \end{aligned} \tag{2.5}$$

This transfer function describes how much noise is transmitted through the cavity under control.

Finally, the transfer function from frequency change due to noise $df_n(s)$ to correction voltage *corr* is useful to determine the controller effort due to noise disturbances. This transfer function is defined as

$$\begin{aligned} corr(s) =& - PSCdfn(s) - APSCcorr(s) \\ \frac{corr(s)}{dfn(s)} =& \frac{-PSC}{1 + APSC} = -PSC * CLTF. \end{aligned} \tag{2.6}$$

2.3 Maximum Allowable Noise

The maximum allowable noise allowed for the frequency controller is determined by four constraints:

1. **avoid actuator saturation** the frequency actuator saturates at 10V. The maximum allowable frequency noise is calculated by substituting 10 V for $corr$ in Equation 2.6 and solving for $df_n(s)$.
2. **achieve control accuracy to maintain 99% of intracavity power** the transmitted DC power is used as a measurement of cavity power. In order to maintain 99% of the transmitted DC power, the cavity must be within 0.07 nm of the resonant length (according to the transmitted DC power vs. cavity length Gaussian). The maximum allowable noise is calculated by substituting 0.07 nm (as a frequency by using Equation 2.1) for $df_c(s)$ in Equation 2.5 and solving for $df_n(s)$.
3. **avoid IR resonance in CARM** when the cavity is 7 Hz away from the green laser resonance, the IR laser also becomes resonant which adds the arm cavity back as an additional degree of freedom in the lock acquisition of the CITF [2]. An additional factor of safety of 3 is added to the 7 Hz requirement. The maximum allowable noise is calculated by substituting 7/3 Hz for $df_c(s)$ in Equation 2.5 and solving for $df_n(s)$
4. **allow time power to build in cavity** the lock of the arm cavity is not attempted until the PDH error signal is in the linear region which is guaranteed when the transmitted DC power reaches 50% of the maximum power. The frequency noise must be low enough so that the cavity power has time to build to the 50% threshold, putting a limit on the cavity speed. The rise time was determined to be $t_r = 1.53ms$ by simulating a stationary cavity in e2e. So long as the cavity stays within the full width at half maximum ($fwhm$ = 257 Hz) the power will build to at least 50% within the rise time. Since this requirement is a speed, the magnitude of the noise is dependent on the frequency. Therefore this constraint adds a integrator filter instead of a simple gain. Assuming that the cavity is moving at a constant velocity, the noise gain is the full width at half maximum multiplied by the sampling frequency (100 kHz) applied at a frequency of $1/t_r$. Because the simulation time step is equivalent to the ITF sensor sampling time, the noise limit is equivalent. The maximum allowable noise is calculated by dividing this filter by the transfer function in Equation 2.5.

Figure 3 plots the noise level constrains in the relevant frequency and magnitude range. The maximum allowable noise is calculated as the minimum of these curves. The filter is defined by using the fourth constraint transfer function as this dominates at high frequencies. This transfer function has a double pole at 0. One pole at 0 was replaced by 0.6 Hz, the crossover frequency with the CARM offset requirement to match the CARM offset transfer function curve. The second pole at 0 was replaced by 36 mHz, the crossover frequency with the correction saturation curve to finish the low frequency requirements. The allowable noise filter is then defined as

$$H_n(s) = \frac{3.7787e-14(s+809.4)^2(s+3.246e05)(s^2+1.153e05s+9.965e09)}{(s+0.2262)(s+3.77)(s+809.4)(s+1.885e05)(s+2.513e05)}. \tag{2.7}$$

Figure 3 hows that this filter accurately reproduces the minimum of all noise constraints. White noise was used as an input to the allowable noise limit filter in order to verify the correct definition of the transfer function. An amplitude of 0.3 was required so that the high frequency fluctuations remained below the noise limit.

Also shown in Figure 3 are the noise magnitude from known sources: seismic disturbances and noise fluctuations due to the fiber optic. The ground noise is close to the allowable noise limit. However, this is not a concern as there is also the magnetic coil pair actuators (whose transfer function is not included in the plot) which can supplement the frequency actuator if needed. The frequency noise remains at least an order of magnitude below the allowable noise limit for all frequencies.

3 e2e Lock Acquisition

The maximum allowable noise filter was verified by using e2e to perform a lock acquisition on the cavity. All noise sources were turned off except for the noise injection using the maximum allowable noise filter defined by Equation 2.7. The amplitude was 0.3 which is the same which is used for the white noise in Figure 3. Note, the

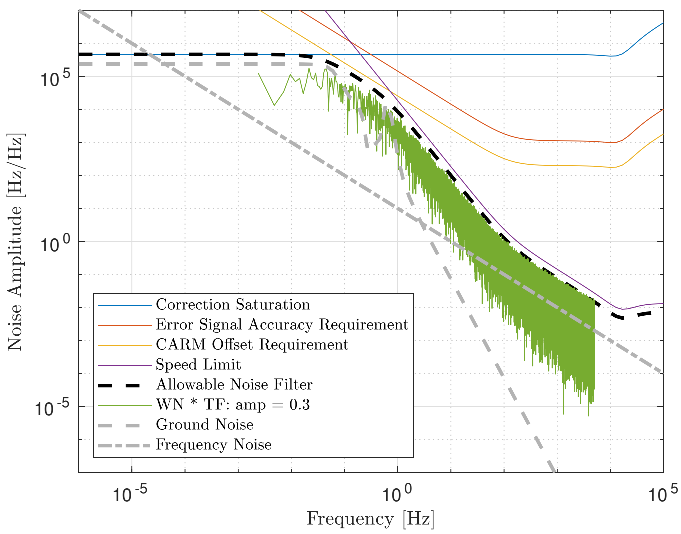


Figure 3: Maximum allowable noise filter defined as the minimum magnitude of anticipated constraining performance parameters. The filter is applied to white noise with an amplitude of $0.3/\sqrt{2}$ to show that the noise fluctuations remain below the maximum allowable curve. Also shown are the known ground and frequency noise levels.

amplitude is divided by $\sqrt{2}$ because e2e uses white noise such that the two-sided power spectral density (PSD) is equal to 1 whereas the filter was designed using a one-sided PSD.

Figure 4 shows the results of the simulation. The top left plot shows that the injected noise is just below the maximum allowable noise filter, as expected. The top right plot shows that the actuator frequency change PSD matches the noise PSD except above the UGF where the actuator tapers off as $1/f$. The bottom left plot shows that the achieved control accuracy is well above the required 99% based on the transmitted DC power. The 3σ value is within 0.1% of the maximum transmitted DC power. Finally, the bottom right plot shows that the correction voltage is well below saturation.

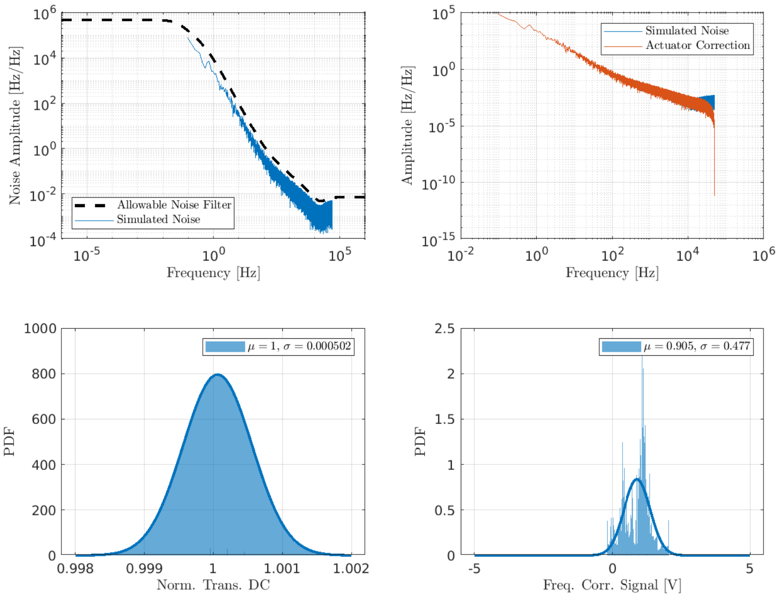


Figure 4: Performance of the lock acquisition simulated by using e2e using the allowable noise filter with an amplitude of 0.3.

4 Conclusion

The maximum allowable noise filter has been identified and defined by Equation 2.7. The cavity lock acquisition using the maximum injected noise has been verified by using e2e. The accuracy and correction voltage are within the required design values. The maximum allowable noise limit may be increased with a more complex control filter. However, the current level of noise is satisfactory below the maximum allowable limit.

A Data and Filter Location

The following file locations are valid at the time of writing. The maximum allowable filter defined in this note may be located in the file
`/data/dev/optics/Simulation/e2e/greenCavity/allowableNoiseFilter.mat`.
The simulation data for the verification data may be located at
`/data/dev/optics/Simulation/e2e/greenCavity/allowableNoise/`.

References

[1] Biplab Bhawal, Matt Evans, Ed Maros, Malik Rahman, and Hiro Yamamoto. Getting started with end-to-end model. *Internal Working Note of the LIGO Project*, LIGO-T980051-00-E, 1998. 2

[2] J. Casanueva and N. Leroy. Auxiliary laser system: study of the lock acquisition strategy. *Virgo Note*, VIR-0327A-19, 2019. 2, 5