Density of the second batch of PVC used for the O4 NCal rotors VIR-0441A-24

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Contents

1 Introduction

This technical note discusses the density of the second set of PVC cylinders and follows the identical method described in VIR-0160A-22 and VIR-0193A-24 for determining the density of the material used in the O4 rotors.

The technical data sheet of the PVC is provided at the end of VIR-0193A-24.

2 Measurement of the four PVC cylinders

Figure 1 shows the four PVC cylinders machined and labelled R4-14, R4-15, R4-16 and R4-17. The cylinders have been machined to be 210 mm of diameter and 106 mm of height.

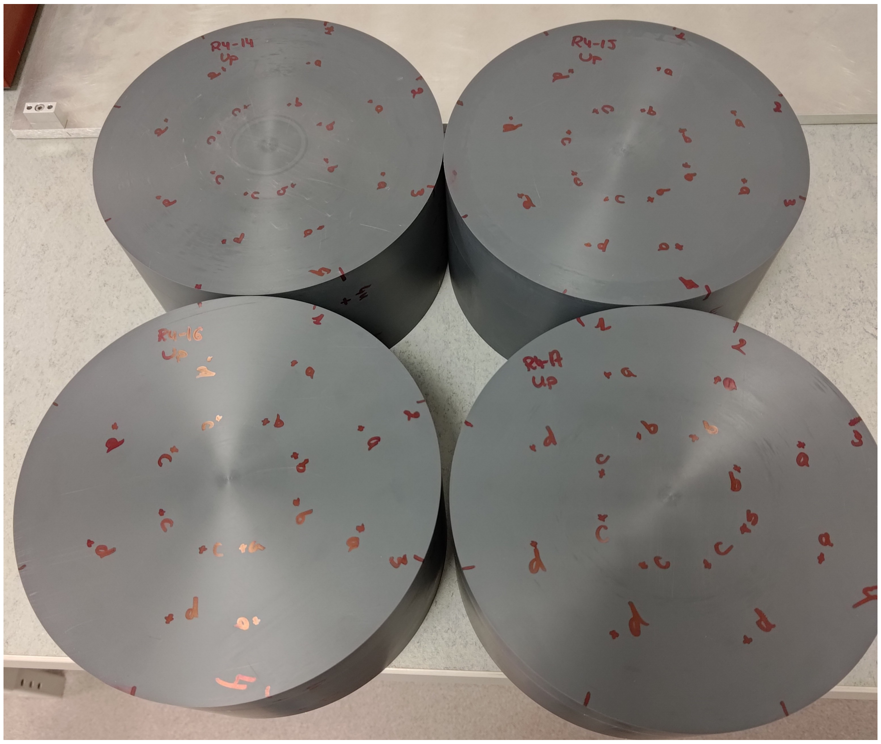

Figure 1: Picture of the four PVC cylinders machined at IPHC. The cylinders have been labelled R4-14, R4-15, R4-16 and R4-17.

As explained in section 1, the measurement method is identical as the aluminum cylinders previously machined at IPHC. Therefore there is a total of 40 measurement points on each cylinder to determine the volume. In addition we measured the mass of each cylinder 5 times (turning back the cylinder upside down between each measurement) and took the average value. Table 1 shows the dispersion of the measurements made on the diameter and height of each cylinder. The dispersion is usually below the hundredth of a millimeter.

Parameter RMS	R4-14	R4-15	R4-16	R4-17
σ_D [μm]	10.9	9.4	13.7	20.6
σ_h [μm]	4.1	1.6	1.7	2.0

Table 1: Dispersion of the measurements made on the cylinders. Top row is the diameter and bottom row is the height.

3 Rescaling the mass

As discussed in section 3 of VIR-0193A-24, the scale used was calibrated with reference weights and the mass of the PVC cylinders was rescaled by an additional 0.47 g (for a mass of about 5.35 kg) . A new set of 5 measurements per reference weight have been performed along with the second set of PVC cylinders, the average values are shown in table 2 at a room temperature of $18.0°$C.

Object	Mean measurement [kg]	RMS [g]	Difference from gauge [g]
RW1	4.99922	<0.1	-0.78
RW2	4.99946	0.1	-0.54
RW1 + RW2	9.99882	0.1	-1.18

Table 2: Measurements made on both reference weights RW1 and RW2 at a room temperature of $18.0°$C. The RW1 + RW2 measurements were made with both reference weights on the scale.

A new scaling value is determined by fitting the mass difference to the gauge value of the reference masses (from table 2) using a linear function and extrapolating to the mass of each PVC cylinder. Figure 2 shows this linear fit. The rescaled mass m_{rescaled} is then expressed as :

$$m_{\text{rescaled}} = 1.04 \times 10^{-1} m + 0.14 \text{ g} \tag{1}$$

with m the average cylinder mass.

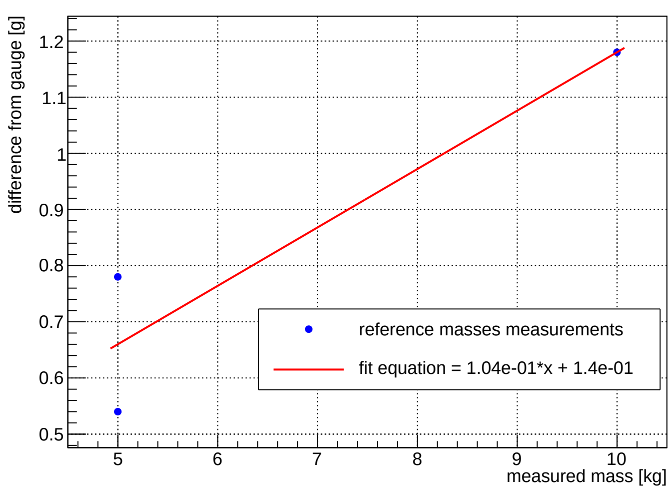

Figure 2: Difference from gauge in g of the measured reference masses in kg.

4 Uncertainties

To determine the mass of the cylinders we used a scale (model KFB 36K0.1) given with an uncertainty of ± 0.3 g. To measure the diameter and the height of the cylinders, we used a measuring column located in the metrology room at IPHC (model Garant 44 5350_600 HC1) with a given error of $1.8{+}\mathrm{L}/600\ \mu$m at 95% CL (L being the measured length in mm).

To compute the relative density uncertainty we assume that the uncertainty on the mass and the lengths are uncorrelated. Therefore we add the relative uncertainties quadratically. However the diameter and height values are correlated since we used the same measuring tool. The systematic uncertainty on the density is then expressed in eq. (2).

$$\left(\frac{\delta_\rho}{\rho}\right)_{\text{syst}} = \sqrt{\left(\frac{\delta_m}{m}\right)^2 + \left(\frac{2\delta_D}{D} + \frac{\delta_h}{h}\right)^2} \tag{2}$$

Additionally, we take into account the maximum dispersion in diameter and height from table 1 to consider the use of average values in the calculation of density. We chose to not divide by the square root of the number of measurement points to remain conservative. The statistical uncertainty on the density is then expressed in eq. (3).

$$\left(\frac{\delta_\rho}{\rho}\right)_{\text{stat}} = \sqrt{\left(\frac{2\sigma_D}{D}\right)^2 + \left(\frac{\sigma_h}{h}\right)^2} \tag{3}$$

Finally to compute the total uncertainty on the density we add quadratically the systematic and statistical uncertainty (eqs. (2) and (3)). Since we consider four cylinders in our analysis we divide the statistical uncertainty by the square root of the number of cylinder. The total uncertainty is then expressed in eq. (4).

$$\left(\frac{\delta_\rho}{\rho}\right)_{\text{tot}} = \sqrt{\left(\frac{\delta_\rho}{\rho}\right)^2_{\text{syst}} + \frac{1}{\sqrt{4}}\left(\frac{\delta_\rho}{\rho}\right)^2_{\text{stat}}} \tag{4}$$

In eqs. (2) and (3), the following parameters are used:

- D the measured diameter of the cylinder : 210 mm.
- h the measured height of the cylinder : 106 mm.
- m the measured mass of the cylinder : 5.3 kg.
- δ_D the uncertainty on the diameter from the column : 2.2×10^{-3} mm.
- σ_D the maximum RMS of the first row of table 1 : 2.06×10^{-2} mm.
- δ_h the uncertainty on the height from the column : 2.0×10^{-3} mm.
- σ_h the maximum RMS of the last row of table 1 : 4.1×10^{-3} mm.
- δ_m the uncertainty on the mass of the cylinder being the uncertainty of the scale : 3×10^{-4} kg.

Table 3 shows the relative uncertainties on the measurements.

Relative uncertainty	Value
δ_m/m	5.66×10^{-5}
δ_D/D	1.05×10^{-5}
δ_h/h	1.89×10^{-5}
σ_D/D	9.81×10^{-5}
σ_h/h	3.87×10^{-5}
$(\delta_\rho/\rho)_{\text{tot}}$	1.33×10^{-4}

Table 3: Relative uncertainties associated to the measurement tools.

5 Density of the PVC

Table 4 shows the average measured values for the diameter, height and rescaled mass (see section 3) as well as the associated computed density for each cylinder. We point out that a weight of 0.5 is applied on the inner points when computing the height. Since the measurements were taken in the metrology room at IPHC at a temperature of $18.3°$C we must take into account the thermal expansion of the material. The material used is labelled as "PVC - U GREY" and the coefficient of linear thermal expansion given is $80\,\mu$m/m/°C. This is taken into account when computing the density at the reference operating temperature of the rotors of $23°$C (see last row of table 4).

Average value		R4-14	R4-15	R4-16	R4-17
D [mm]		209.961	209.979	209.953	209.969
h (weighted) [mm]		106.032	106.038	106.015	106.023
m_{rescaled} from eq. (1) [kg]		5.3004	5.3016	5.2994	5.3009
ρ [kg.m^{-3}]	18.3°C	1443.78	1443.78	1443.88	1443.95
	23°C	1442.15	1442.15	1442.25	1442.32

Table 4: Average measurements made at IPHC on the four PVC cylinders. From top to bottom: the diameter, the height, the mass and the density. A weight of 0.5 is applied on the inner points for the height. The densities at 23°C are highlighted in yellow.

Using table 4 we compute the mean density of the PVC at $23°$C with the associated uncertainty using last row of table 3 (rounded up to 0.2 kg.m^{-3}). The density of the material for the second batch of O4 PVC rotors is then $\rho_{\text{PVC}} = 1442.2 \pm 0.2$ kg.m^{-3}.