Robustness of templates for detecting gravitational wave bursts from cosmic strings cusps and kinks.

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Abstract: we show in this note that the templates used for searching GW bursts from cosmic strings cusps and kinks are extremely robust with respect to the choice of the frequency power of the modelled waveform in the frequency domain ($h(f) \propto f^{\alpha}$). That means in practice that we can use "cusps" templates for detecting GW bursts from kinks and vice versa.

I. Introduction.

Simple models predict power laws for the frequency spectrum of amplitudes of gravitational wave (GW) bursts emitted by cosmic strings (CS). For instance Vilenkin et al. [1, 2] predict

$$h(f) = Af^{\alpha} \quad \text{with } f \leq f_u ,$$

with α ~ -5/3 for CS kinks and α ~ -4/3 for CS cusps and f_u is an upper frequency cut related to the cosmic strings parameters.

Having a model for GW amplitudes then allows the use of standard matched filtering techniques for detecting GW emitted by CS, especially cusps, as we have done in the past for VSR1-2 and S5-6 data [3]. However the power law prediction is not so robust and, for example, the exponent coefficient for GW emitted by kinks can be as low as α = - 2 as recently recalled in [4].

As we use templates based on this theoretical prediction, we must question their robustness for detecting GW bursts emitted by CS kinks or cusps.

II. Fitting factors.

In order to study the robustness of templates based on Vilenkin's power laws, we will use such templates but a priori mismatched with the actual waveforms. Let's write the template in the frequency domain as

$$h_1(f) = A_1 f^{\alpha_1}$$

and the amplitude of the actual waveform as

$$h_2(f) = A_2 f^{\alpha_2} ,$$

with a priori $\alpha_1 \neq \alpha_2$. Note that α_1 once is fixed, the only template parameter is the upper frequency cut f_u (related to the CS parameters) and the 1D template bank is in practice built with optimal placement of the frequency cut f_u in the 1D parameter space.

We define the fitting factor between h_1 and h_2 as the scalar product:

$$FF = \langle h_1 | h_2 \rangle = \int_{f_d}^{f_u} \frac{h_1(f) h_2(f)^*}{S_h(f)} df \ ,$$

where $S_h(f)$ denotes the noise power spectral density and the lower frequency cut f_d corresponds to the instrumental seismic cut-off frequency. FF is a measure of how the template can recover the GW signal, and, for instance reaches 1 if the template perfectly matches the GW waveform.

In the following we will suppose that the noise is white (we will see that this crude approximation doesn't change the basic conclusions of this note), so that the fitting factor becomes:

$$FF = \langle h_1 | h_2 \rangle = \int_{f_d}^{f_u} h_1(f) h_2(f)^* df \ ,$$

assuming a proper normalization of the template/waveform.

This normalization reads:

$$\langle h_k | h_k \rangle = 1 = A_k^{\ 2} \int_{f_d}^{f_u} f^{\,2\alpha_k} df \ .$$

Thus

$$A_k^{\ 2} = \frac{1 + 2\alpha_k}{f_u^{\ 1+2\alpha_k} - f_d^{\ 1+2\alpha_k}}$$

Now the (squared) fitting factor can be explicitly derived (after some algebra):

$$FF^2 = \langle h_1 | h_2 \rangle^2 = \frac{(1+2\alpha_1)(1+2\alpha_2)}{(1+\alpha_1+\alpha_2)^2} \frac{\left(1-\gamma^{1+\alpha_1+\alpha_2}\right)^2}{\left(1-\gamma^{1+2\alpha_1}\right)\left(1-\gamma^{1+2\alpha_2}\right)} \qquad (1)$$

where $\gamma = f_d / f_u$. As a check, we find FF =1 when $\alpha_1 = \alpha_2$ as expected.

III. Numerical results and comments.

For the numerical studies, we chose = 10Hz (lower frequency cut due to the seismic "wall") and make α_1 and α_2 vary in the range [-1,-2] for different values of f_u. Some characteristic results are reported in the following tables.

f_u (Hz)	20	50	100	500	1000	2000
FF (%)	99.80	99.23	98.91	98.64	98.62	98.61

Table 1: fitting factor for $\alpha_1 = -5/3$ (usual "kinks" signals) and $\alpha_2 = -4/3$ (usual "cusps" signals).

f_u (Hz)	20	50	100	500	1000	2000
FF	99.42	99.42	99.28	99.21	99.22	99.22

Table 2: fitting factor for $\alpha_1 = -5/3$ (usual "kinks" signals) and $\alpha_2 = -2$ (possible "kinks" signals).

f_u (Hz)	20	50	100	500	1000	2000
FF	99.79	99.04	98.39	97.35	97.13	96.99

Table 3: fitting factor for $\alpha_1 = -4/3$ (usual “cusps” signals) and $\alpha_2 = -1$ (extremal “cusps” signals ?).

The first conclusion is that whatever the set, the fitting factor is always very good and the FF losses are at most a few %. This result is expected to hold for a realistic noise spectral density since the FF losses are kept very small in the white noise case.
A second conclusion (table 1) is that if we use “kinks” templates to detect “cusps” signals (or the reverse), the FF loss remains below 2%. “Kinks” templates can then be used for searching for “cusps” signals (and vice versa).

Finally, the templates we use for usual “kinks” signals (α = -5/3) are robust enough to detect any similar waveforms with any α at least in the range [-2,-1]. This accounts also for a lack of robustness for the theoretical prediction of the exponent α.

Final minor note: this nice robustness can be explained from Eq.(1). If we set $\alpha_1 = \alpha_2 + \varepsilon$ and expand the expression of FF, it can be shown that this expansion is at least at the second order in ε (the lowest non vanishing order of the expansion of each fraction is 2).

References

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