Backscattering noise from cryotraps

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0.1 Notation and parameters

We assume a perfectly cylindrical cryotrap of internal diameter $2a$ with $a =$ 45cm. We assume it to extend from z_1 =2.75m to z_2 =4.25m. The assumed material is a stainless steel of the same kind as the vacuum pipe itself, because its BRDF has been extensively measured in the 90', so that we have actual figures. Namely, the backscattering rate for incidence ϑ is (see [1]) :

$$b(\vartheta) \ = \ 0.83\, \mathrm{e}^{-5.5\vartheta} \tag{1}$$

If we consider the complementary angle $\theta = \pi/2 - \vartheta$, we get :

$$b(\theta) \ = \ 1.47\, 10^{-4}\ \mathrm{e}^{5.5\theta} \tag{2}$$

The material actually used for the cryotrap may be different, but this will give an order of magnitude. We call $n(f)$ the phase noise caused by the seismic excitation of the backscattering material along the horizontal direction (along the optical axis). For a weak modulation depth, we have simply :

$$n(f) \ = \ \frac{2\sqrt{2}\pi}{\lambda}\xi(f)$$

where $\xi(f) \ll \lambda$ is the spectral density of **longitudinal** random motion of the cryotrap. If the modulation is strong, due to some resonance at f_0, we can define $f_{\mathrm{max}} \sim 8\pi\xi(f_0)/\lambda$, and adopt the following crude model, based on the properties of the Bessel functions (roughly, $J_n(z) \sim 0$ for $n > 2|z|$) :

$$\begin{cases} n(f) \ = \ (2f_{\mathrm{max}})^{-1/2} \ \text{ if } f \leq f_{\mathrm{max}} \\ n(f) \ = \ 0 \ \text{ if } f > f_{\mathrm{max}} \end{cases}$$

We take usually for the ground motion, a spectral density measured at Cascina during a quiet period :

$$\xi(f) \ \sim \ 10^{-8} \left[\frac{10\,\mathrm{Hz}}{f}\right]^2 \ \ \mathrm{m\ Hz}^{-1/2} \tag{3}$$

This may be multiplied by a resonance factor if necessary. The angular distribution of light scattered by the mirrors, in the angular region of interest may be modelized by :

$$p(\theta) \ = \ \frac{\kappa}{\theta^2} \tag{4}$$

and the integrated scattering being denoted by ϵ, we have for 10 ppm losses $\epsilon \times \kappa \sim 10^{-7}$.

0.2 Backscattering

Let us call $h(f)$ the PSD of gravitational signal equivalent to the backscattering noise. If we follow the ideas of [1] (p.6089), we can compute the contribution to $h(f)^2$ of the element of cylinder localized at z, of angular width $d\theta$, seen under angle θ from the mirror :

$$dh^2(f) \;=\; \frac{\lambda^4\epsilon^2}{64\pi^4 L^2 z^2}\, p(\theta)^2\, d\Omega\, b(\theta)\, n(f)^2 \tag{5}$$

where L is the length (3km) of the Virgo cavities. In (5), the solid angle $d\Omega$ represents the angular region occupied by the element of cylinder seen from the mirror :

$$d\Omega \;=\; 2\pi \sin\theta\, d\theta$$

we also take $z = a/\tan\theta$. After substituting the function $p(\theta)$, we get :

$$dh^2(f) \;=\; \frac{\lambda^4\epsilon^2\kappa^2}{32\pi^3 L^2 a^2}\, \frac{b(\theta)\sin^3\theta\, d\theta}{\theta^4\cos^2\theta}\, n(f)^2 \tag{6}$$

but, with the parameters taken in the preceding section, with $\theta_i \equiv \arctan(a/z_i)$,

$$\int_{\theta_2}^{\theta_1} \frac{\sin^3\theta\, \mathrm{e}^{5.5\theta}}{\theta^4\cos^2\theta}\, d\theta \;\simeq\; \int_{\theta_2}^{\theta_1} \frac{\mathrm{e}^{5.5\theta}}{\theta}\, d\theta \simeq\; 0.9$$

so that :

$$h(f) \;=\; \frac{\lambda^2\epsilon\kappa}{L\, a}\sqrt{\frac{0.9 B_0}{32\pi^3}}\, n(f) \tag{7}$$

with $B_0 \equiv 1.47\, 10^{-4}$.

0.3 Numerical result

After substituting the parameters, we get :

$$h(f) \;\sim\; 3\, 10^{-26}\, n(f) \tag{8}$$

In the low excitation regime, we have :

$$n(f) \;\sim\; 0.085 \left[\frac{10\,\mathrm{Hz}}{f}\right]^2 \;\mathrm{Rd\,Hz}^{-1/2}$$

so that

$$h(f) \;\sim\; 2.5\, 10^{-27} \;\left[\frac{10\,\mathrm{Hz}}{f}\right]^2 \;\mathrm{Hz}^{-1/2} \tag{9}$$

Bibliography

[1] JYV et al. Phys. Rev. D 56 N.10 (1997) p.6085