Mock Data Challenge Generation Package

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with contribution from

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The purpose of the MDC Generation Package is to produce simulated data to prepare the data analysis and science with the future generations of gravitational-wave experiments, such as Advanced LIGO-Virgo or Einstein Telescope. The end product are the time series of the detector outputs, arising both from the instrumental noise and from the GW signal. We follow the procedure developed for the first ET-MDC [1], extending it to a large range of sources and models. In this document, we describe the structure of the MDC Generation Package, and explain how to produce simulated data samples.

I. MDC CODE STRUCTURE

The schematic structure of the MDC Generation Package is presented in Fig. 1. It contains different modules which are shortly described below. Each module can be developed independently in order to make the package more flexible. The package uses the LIGO Algorithm Library (LAL).

- 1. Monte-Carlo Simulations: The core of the program, which simulates the time series, is shown inside the central red box. This module has been implemented for the first ET-MDC [1] and has not changed significantly. As described in [1], the sources are realistically distributed in the parameter space (e.g mass, spin, inclination, polarization), but also in time, for example assuming a Poisson distribution of the arrival times, and redshift, accounting for the star formation history of the Universe (see module "Universe"). The noise is generated from the PSD of the detector (see module "Detector") and tapered to avoid discontinuities at boundaries between data segments.
- 2. Universe: This module contains information on the Universe, locally or at cosmological distances: cosmological parameters, star formation history, initial mass function, metallicity, catalog of galaxies. All these informations are needed to calculate the source rates (and then the average time interval between events), the redshift probability distribution, but also the luminosity distance for high redshift sources. Galaxy catalogs are not yet implemented but will be in the near future.
- 3. Sources: This module is in permanent evolution. It contains the probability distributions

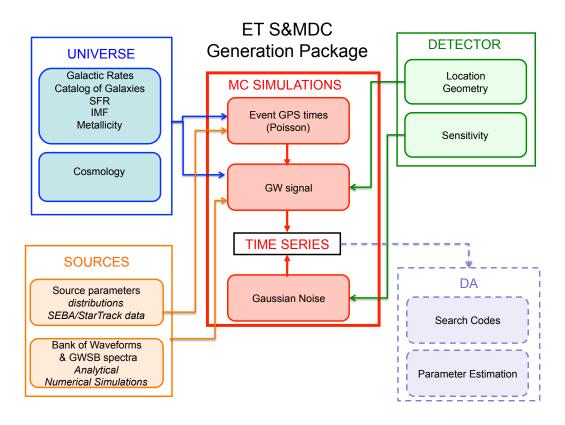


FIG. 1: Structure of the ET Generation package. The different modules are independent and can be developed separately. We also included the module Data Analysis, which is not part of the generation package but uses the simulated time series.

of the source parameters (masses, spins, delays...), needed to simulate realistic populations of sources. These distributions can be analytical models or the results of sophisticated star evolution codes such as StarTrack [2] or SEBA [3] for stellar compact binary systems. It also contains a bank of waveforms that are used to generate the GW signal (others are already implemented in the LAL library), and a catalog of spectra for GW stochastic backgrounds [4] which are generated in the frequency domain, and Fourier transformed into the time domain.

4. **Detectors**: This module contains all the informations related to the detectors: geometry, location, response tensors (needed to calculate the beaming factors) and various models for the sensitivity. Except for Einstein Telescope, the routines are already implemented in LAL. For now, we generate Gaussian colored noise but non Gaussian noise containing glitches will be implemented in the near future.

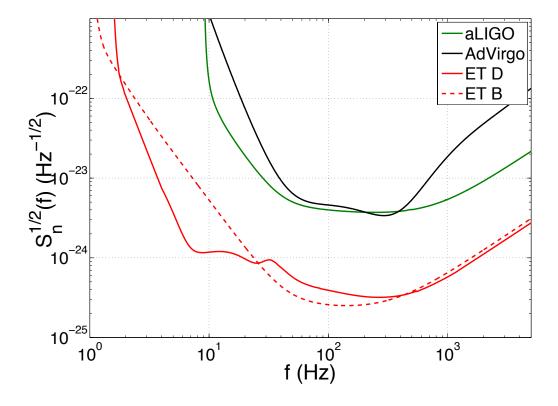


FIG. 2: Sensitivity curves for aLigo, aVirgo, ET-B and ET-D (default)

II. NOISE

The noise generation routine was provided by Carl Rodriguez for the first ET MDC [1]. The noise is simulated by generating a Gaussian time series with a mean of zero and unit variance. This time series are then taken into the frequency domain, colored with the noise PSD, and then inverse Fourier transformed. To alleviate the effects of any potential discontinuities across frame files, the noise curve is gradually tapered away to zero below f_{Min} , and above $f_{Nyquist}/2$. The nominal noise curves for aLIGO, AdVirgo and ET are in the directory **Input** (see Fig. 2). To use other PSDs, just replace these files by your own one or choose one of the noise models that can be found in the directory **Detector**. We noticed strange behavior at frames boundaries when using sensitivity curves with narrow peaks. To avoid this problem we strongly recommend using smooth versions of the noise curves.

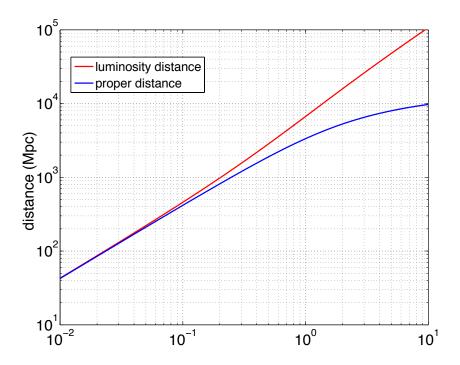


FIG. 3: Proper distance and luminosity distance for a flat Universe with $H_0 = 70$. km/s/Mpc, $\Omega_m = 0.3$ and $\Omega_{\Lambda} = 0.7$

III. THE MODEL OF UNIVERSE AND THE COSMIC RATE

By default, we assume the 737 cosmology: a flat Universe with Hubble parameter $H_0 = 70$. km/s/Mpc, density of matter $\Omega_m = 0.3$, density of dark energy $\Omega_{\Lambda} = 0.7$, but the package also provide files for the WMAP7 cosmology with $H_0 = 70.4$ km/s/Mpc, $\Omega_m = 0.272$ and $\Omega_{\Lambda} = 0.728$. The file dL.txt (or $dL_wmap7.txt$) in the directory **input**, contains the luminosity distance as a function of the redshift up to z = 10 (see Fig. 3). For other cosmologies, the package provides a Mathematica and a Matlab code $cosmo_-$ functions.nb and $cosmo_-$ functions.m in the directory **Cosmology** where the user can change the cosmological parameters and generate a new file. The package contains a few models of star formation rates (see Fig 4) [5] for details and references) in the Mathematica code $cosmic_-rates.nb$ or the Matlab code $cosmic_-rates.m$. They can be used to derive realistic probability distributions of the redshift, as we will see in the next section.

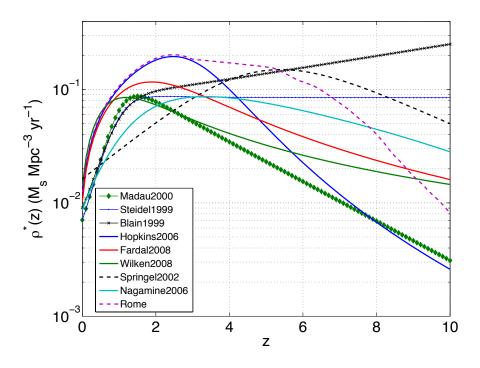


FIG. 4: Different models of the star formation rates. See see [5] for references and details.

IV. THE POPULATION OF COMPACT BINARIES

Compact binaries coalescences (CBC) can be seen by Advanced LIGO/Virgo and Einstein Telescope at very large distances and the background created by the superposition of all the individual sources up to a redshift of $z \sim (5-10)$ has also a very good chance to be detected. The generation code simul.c simulates the GW signal from the entire population taking the source parameters from ascii files $cbc_-0.dat$, $cbc_-1.dat$ etc... in the directory **lists** (one for each data segment). The package provides the code $list_-cbc.c$ to generate lists of parameters for realistic simulated populations.

A. Simple Model

The simple model has been used to generate the extra-galactic populations of double neutron stars (NS-NS) for the first ET-MDC (see[1] for details). It has been extended to include mixed populations of double black holes (BH-BH) and a black hole and a neutron star (BH-NS).

• the time intervals between successive events is drawn from an exponential distribution $P(\tau) = exp(-\tau/\lambda)$, assuming coalescences at the detector is a Poisson process. The average waiting time λ can be calculated by taking the inverse of the merging rate integrated over all redshifts.

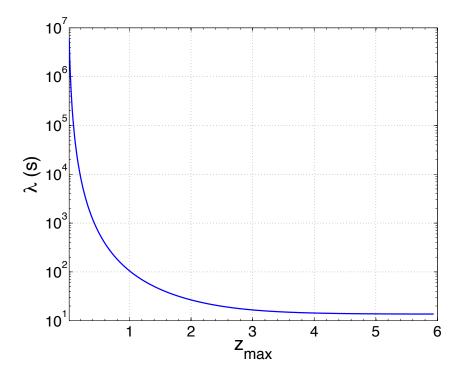


FIG. 5: average waiting time between successive NS-NS coalescences as a function of the maximal redshift, assuming the SFR of [7] and a local rate of $\rho_0 = 1 \text{ Myr}^{-1} \text{ Mpc}^{-3}$.

Fig. 5 shows λ as a function of the maximal redshift $z_{\rm max}$ for a population of NS-NS. For the 'realistic' LIGO local coalescence rate of $\rho_0 = 1~{\rm Myr^{-1}~Mpc^{-3}}$ [6] and the SFR of [7], $\Delta t \sim 13s$. The code $cosmo_-$ functions.nb can be used to calculate Δt for other cosmologies and SFR.

- the user specifies in the command line the fraction of NS-NS, BH-BH and BH-NS and the type of the source is randomly selected accordingly.
- the redshift is drawn from a probability distribution p(z) provided by the ascii files PzNS.txt for NS-NS and PzBH.txt for BH-NS and BH-BH, in the directory **input**, in which the code interpolates. This probability is obtained by normalizing (in the interval 0-10) the coalescence rate $dR_c^o/dz(z)$, which is obtained by multiplying the element of comoving volume $\frac{dV}{dz}$ by the cosmic coalescence rate:

$$\dot{\rho}_c^o(z) \propto \int \frac{\dot{\rho}_*(z_f)}{1 + z_f} P(t_d) dt_d \text{ with } \dot{\rho}_c^o(0) = \dot{\rho}_0$$
 (1)

In this equation, $\dot{\rho}_c$ is the star formation rate, z_f the redshift at the time of formation of the massive binary system and $P(t_d)$ the probability distribution of the delay between the

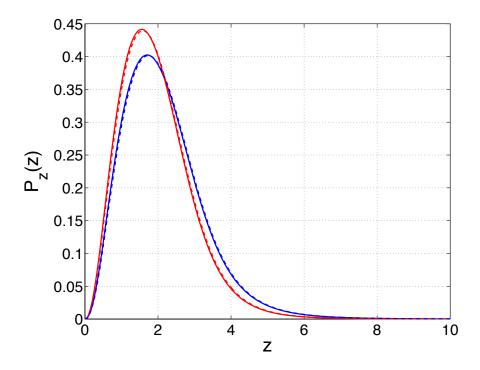
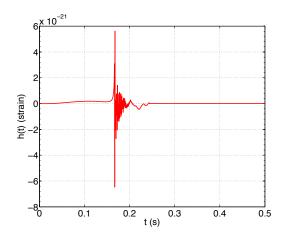


FIG. 6: probability distribution of the redshift for the population of NS-NS (blue) and NS-BH or BH-BH (red), assuming the SFR of [7] and a probability distribution of the delay of the form $P(t_d) \propto 1/t_d$ with a minimal delay of 20 Myr (NS-NS) and 100 Myr (BH-NS or BH-BH). Countinuous lines are for the 737 cosmology and dashed lines for the WMAP7.

formation and the coalescence. We assume a distribution of the form $P(t_d) \propto 1/t_d$ with a minimal delay of 20 Myr as representative of the population of NS-NS, and with minimal delay of 100 Myr as representative of the populations of BH-NS and BH-BH (Dominik M., private communication). The codes $cosmics_rates.m$ and $cosmic_rates.nb$ can be used to produce new files for other SFR and probability distribution of the delay.

- the masses of the two components are drawn from Gaussian distributions. The user can choose the mean, the variance, the minimal and the maximal masses from the command line, for both neutron stars and black hole. For the first ET-MDC (see[1], the NS masses were drawn between $1.2-3~{\rm M}_{\odot}$ from a gaussian distribution of mean $1.4~{\rm M}_{\odot}$ and variance $0.53~{\rm M}_{\odot}$.
- the location in the sky, the orientation, the polarization and the phase at the coalescence are drawn from uniform distributions



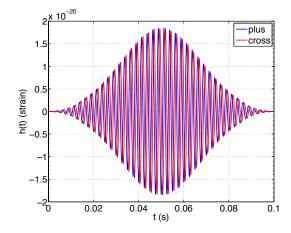


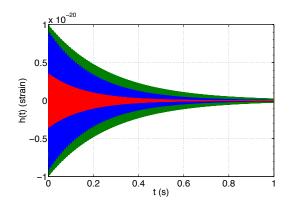
FIG. 7: supernova waveforms included in the package, a model of Dimmelmeier (left) and a long bar (right), both at 10 kpc. For the model of Dimmelmeier we only show the polarization h_+ , the h_\times polarization being 0.

B. Population Synthesis Model

The second model takes as input lists of parameters (the mass of the two components and the redshift), provided by I. Kowalska and derived from the binary evolution code StarTrack [8], assuming the SFR of [7]. Note that only the standard model is provided with the package. In addition, lists of parameters for IMBH or IMRI up to a redshift of $z \sim 3$ have been provided by J. Glair and I. Mandel [9, 10]. All these files, $list_BNS.txt$, $list_BBH.txt$ etc... are stored in the directory Input. The type, the location in the sky, the orientation, the polarization and the phase at the coalescence are drawn from uniform distributions as for the simple model.

V. BURSTS

The directory **Input** contains a few ascii files with burst waveforms (2 supernovas and 6 f-modes, provided by J. Clark), that can be rescaled and injected in the data. All the files contains 3 columns: time, h_+ , h_\times . The first supernova waveform ($bWform_-10.txt$) is a model of Dimmelmeier, the second a model of long bar ($bWform_-11.txt$). Both have optimal orientation (the inclination angle is taken into account later in the generation code). There are 6 signals for f modes, 3 for l = 2, m = 0 ($bWform_-200.txt$, $bWform_-201.txt$ and $bWform_-202.txt$) and 3 for l=m=2 ($bWform_-200.txt$, $bWform_-202.txt$). Those have random inclinations and phases.



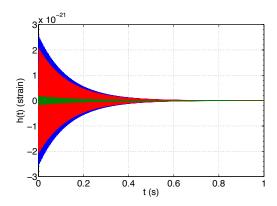


FIG. 8: f-modes waveforms included in the package, 3 models with m=2 and l=0 (left), and 3 models with m=2 and l=0 (right), both at 1 kpc. We show only the polarization h_+ , the h_\times polarization being 0 for l=0 and comparable to the h_+ polarization for l=2.

VI. HOW TO RUN THE CODE TO PRODUCE SIMULATED DATA

In this section we provide instructions to start quickly with the generation code. Please do not hesitate to contact us for further explanations.

A. Installation

First, you have to install and compile the LAL library. Instructions are given here: https://www.lsc-group.phys.uwm.edu/daswg/projects/lalsuite.html. Next, request the package by sending an email to regimbau@oca.eu

The codes can be compiled with the commands in the file *compil*, the path /home/tania/local/master/lib being replaced by the user's path to the lalsuite libraries. Running this file produces three main executables: *list_cbc*, *simul* and *back*. Codes in the directory **Extra** can be compiled by just adding extra lines to the *compil* file or directly from the shell prompt with the command:

gcc -std=gnu99 -Wall -o executable code.c 'pkg-config -cflags -libs lal lalinspiral lal-frame lalsimulation gsl' -Wl,-rpath=/home/tania/local/master/lib.

B. Produce parameter lists

For each data segment, you need to provide files with the parameters of the sources you want to inject (a file for CBC *cbc_j.dat* and for bursts *burst_j.dat*, *j* being the segment number).

For CBC, the parameters are : source number, starting time (GPS), mass1 and mass2 (in M_{\odot}), luminosity distance (Mpc), right ascension, decl, polarization, inclination and initial phase (in rad), type (1 for NS-NS, 2 for BH-NS, 3 for BH-BH, 4 for IMRI, 5 for IMBHB in GC, and 6 for IMBHB seed). You can provide your own lists of parameters or use the code $list_cbc.c$ described in the previous section. You can also use this code as a template for your own models.

For the bursts, the parameters are: source number, starting time (GPS), distance (kpc), right ascension, declination, polarization and inclination (in rad), model. Note that for the supernova waveforms, the polarization and the inclination will not be used as they are already taken into account in the waveform file. These waveforms in the files are sampled at 8192 Hz but are resampled in the case another sampling rate is used to generate the data. The model codes are: Dimmelmeier waveform (code 10), long Bar model (code 11), f modes m = 2 and l = 0 (code 200, code 201, code 202), f modes m = 2 and l = 2 (code 220, code 221, code 222).

The command menu can be output with the option -h:

```
./list_cbc -h
```

Usage: pipeline [options]

Options:

-h	print this message
-s	seed for random generation
-t	starting GPS time
-d	average time interval between events
-r	sampling rate
-f	minimal frequency
-1	segment duration
-S	number of data segments
-p	fraction of BNS
-P	fraction of BHNS
-q	fraction of BBH
-Q	fraction of IMRI
-j	fraction of IMBHB_GC

```
-J
                      fraction of IMBHB_seed
                      remove BBH, BHNS and BNS
--no-scbc
--no-imbh
                      remove IMRI and IMBHB
                      minimal redshift
-z
-Z
                      maximal redshift
                      randomly select z, m1, m2 from distibutions for BBH, BHNS and BNS
--proba
                      minimal mass of NS
-m
-M
                      maximal mass of NS
                      mean of the mass distribution of NS
-a
-b
                      standart deviation of the mass distribution of NS
                      minimal mass of BH
-n
                      maximal mass of BH
-N
                      mean of the mass distribution of BH
-11
                      standart deviation of the mass distribution of BH
```

select ET or ALV (but not both) for sustraction of the detectable events(useful for SGWB studies)

beaming angle in degrees for GRB (BNS and BHNS)

--ET subtract sources detectable by ET
--ALV subtract sources detectable by ALV

-R SNR detection threshold

for Condor use only, split the data into intervals, each containing -S \$value data segments
-I number of intervals (by default 1)
-i # of the interval (by default 0)

Typical commands look like:

-G

./list_cbc -S 1000 -l 2048 -r 8192 - f 10 -t 800000000 -d 20 -p 0.85 -P 0.02 -q 0.13 -Q 0 -j 0 -J 0 -z 0 -Z 5 -proba -m 1 -M 3 -a 1.4 -b 0.1 -n 5 -N 13 -u 9 -v 1 -G 20

This generates lists for 1000 segments of duration 2048 s, sampled at 8192 Hz, with minimal frequency 10 Hz, the first segment starting at GPS 800000000. The average waiting time between coalescences is 20 s. The population contains 85% of NS-NS, 2% of NH-NS and 13% of BH-BH, with redshift between 0-5. The masses (in unit of solar mass) of NS and BH are randomly selected from Gaussian probability distributions of mean 1.4 for NS (9 for BH) and standard deviation 0.1

for NS (1 for BH). The NS masses are selected between 1-4 and the BH masses between 5-9. The half beaming angle of GRB is assumed to be 20 degrees.

./list_cbc -S 1000 -l 2048 -r 8192 - f 10 -t 800000000 -d 200 -p 0.8 -P 0.02 -q 0.12 -Q 0.03 -j 0.02 -J 0.05 -G 20

This generates lists for 1000 segments of duration 2048 s, sampled at 8192 Hz, with minimal frequency 10 Hz, the first segment starting at GPS 800000000. The average waiting time between coalescences is 200 s. The population contains 85% of BH-NS, 2% of BH-NS, 13% of BH-BH, 3% of IMRI, 2% of seed IMBHB and 5% of IMBHB is globular cluster. The range of redshift is 0-10 (by default) for stellar CBC and up to 4 for IMBH populations. The masses of the two components and the redshift are read from ascii files. The half beaming angle of GRB is assumed to be 20 degrees.

One can use condor to produce lists for a long observation time. In this case the data will be split into intervals, each interval being run on a different node. In order to produce 10×1000 segments using 10 nodes, on would do (\$job =0,1...9):

./list_cbc -i \$job -I 10 -S 1000 -l 2048 -r 8192 - f 10 -t 800000000 -d 200 -p 0.8 -P 0.02 -q 0.12 -Q 0.03 -j 0.02 -J 0.05 -G 20

There are also a few extra commands that can be useful for specific projects:

- The flags -ALV or -ET can be used for stochastic MDC, to remove from the lists all the sources detected by the network Ad. LIGO/Virgo or by ET with a signal-to-noise ratio larger than the value set by the command -R.
- The flags -no-scbc and -no-imbh can be used to isolate stellar CBC or IMBHB/IMRI populations, by removing either the stellar CBCs or IMBHBs/IMRIs, but keeping exactly the same parameters for the remaining sources.

C. Generate data

The user can specify whether noise should be added to the data (option -noise), and what gravitational signal should be included (options -scbc for the population of stellar compact binaries, -imbh for IMBHB and IMRI, -burst for occasional supernovas and f modes, -sgwb for a gaussian stochastic background or -agwb for a population of sine-gaussian or ring down signals mimicking a (non Gaussian) background of identical sources (type -h for help).

For now, we use TaylorT4 waveforms for NS-NS and BH-NS (soon with tidal corrections), and EOBNv2 waveforms for BH-BH, IMBHB and IMRI, but it is possible to edit the codes and choose any of the templates included in the LAL library.

The generation code menu can be output with the option -h:

Mac-TRegimbau-Artemis-MJ-noa-2:trunk regimbau\$./simul -h

Usage: pipeline [options]

Options:

-h print this message --verbose verbose mode --ascii write to ascii files --ET select ET detectors (cannot select both ET and ALV) --ALV select ALV detectors --virgo include Virgo in ALV --noise generate noise simulate sgwb --sgwb --scbc simulate stellar cbc simulate IMBBHs and IMRIs --imbh

--burst simulate burst

--agwb simulate background of sine-gaussians or ringdown

-s seed for noise generation

-j job number

-J start job number

-t start time of the serie

-l segment duration

-r sampling rate of the time serie

-f minimal frequency

-F reference frequency for sgwb

-o reference omega for sgwb

-I power index sgwb
-H Hubble parameter

-a amplitude factor for CBC

-b sine-gaussian or ringdown frequency for agwb.

Set to 0 for uniform between 10-500 Hz

-B sine-gaussian or ringdown decay time for agwb

-m duty cycle for agwb

-w waveform for agwb : (0: sine-Gaussian(default) 1: ringdown)

-R working directory

Typical commands are:

./simul -t 800000000 -j 0 -l 2048 -r 8192 -f 5 -ET-noise -scbc -imbh-burst

This generates 3 frames in the directory **output** (E1-E1:STRAIN-800000000-2048.gwf, E2-E2:STRAIN-800000000-2048.gwf and E3-E3:STRAIN-800000000-2048.gwf,) starting at GPS time 800000000, of length 2048 s, sampling rate 8192 Hz, with minimal frequency 5 Hz, containing ET noise, the CBC population (stellar CBCs, IMBHBs and IMRIs) and a burst.

If you want to output the time series as ASCII files rather than frames, add the option -ascii. It will produce a single file $serie_{-}0.dat$ in textbfoutput, the first column being the GPS time and the next three columns h(t) for the three ET detectors.

./simul -t 800000000 -j 2 -l 2048 -r 4096 -f 10 -noise -scbc -a 3

This will generate 2 frames in the directory **output** (H1-H1:STRAIN-800004096-2048.gwf and L1-L1:STRAIN-800004096-2048.gwf,) starting at GPS time 800004096, of length 2048 s, sampling rate 4096 Hz, with minimal frequency 10 Hz, containing aLIGO noise and the stellar CBC population. The option -a 3 says the code to multiply the GW signal h(t) by a factor of 3.

It's possible to generate data for Virgo (-virgo). This will generate an extra frame in the directory **output** (V1:STRAIN-800004096-2048.gwf).

Remark It's important to use the same duration, minimal frequency and starting time used to produce the list files.

VII. STOCHASTIC STUDIES

A few options have been recently added to the generation codes to facilitate stochastic studies and the development of new data analysis methods.

./simul -t 800000000 -j 2 -l 2048 -r 4096 -f 10 –noise –sgwb -i 3 -o 1e-6 -F 100 -H 0.7

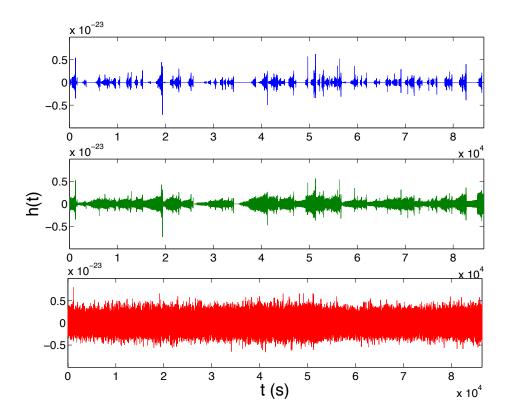


FIG. 9: Time series of length 8192 s, containing the population of NS-NS (simple model) up to a maximal redshift of $z_{\text{max}} = 0.5$, with minimal frequency 10 Hz (top), 5 Hz (middle) and 1Hz (bottom).

The option -sgwb will generate frames with a stochastic GW background of the form $\Omega_{gw}(f) = \Omega_{ref}(f/f_{ref})^{\alpha}$, generated in the frequency domain, with a power index $\alpha = 3$ and a reference value of $\Omega_{ref} = 10^{-6}$ at the reference frequency $f_{ref} = 100$ Hz. The Hubble parameter will be assumed to be $h_0 = 0.7$.

./simul -t 800000000 -j 2 -l 2048 -r 4096 -f 10 –noise –agwb -w 0 -b 100 -B 0.1 -m $0.1\,$

This will generate frames with a GW background of sine-gaussian waveforms with frequency $f_0 = 100$ Hz, damping time $\tau = 0.1$ s and duty cycle $\mu = 0.1$. As for CBC, the event times are selected form a Poisson distribution. The average time interval is defined by the duty cycle and the damping time as $\Delta t = \tau/\mu$.

We have also a few useful codes to study the background formed by all the CBCs (most of them are not included in the package but can be sent separately). The code background.c estimates analytically the spectrum $\Omega_{gw}(f)$ of the background created by all the CBC sources in the data

set. It reads the parameters of the sources in the file list_cbc.txt created by list_cbc.c.

The menu can be displayed with the option -h:

Mac-TRegimbau-Artemis-MJ-noa-2:MDC_Generation regimbau\$./back -h

print this message

Usage: pipeline [options]

Options:

print	display omega for the reference frequency
average	uses expression of omega averaged over orientation
ET	calculate GW PSD for the 3 ET detectors
ALV	calculate PSD for the 3 ALV detectors
noIM	subtract IMBBH and IMRI
-f	minimal frequency
-F	reference frequency
-t	CBC type (0: all 1:BNS 2:BHNS 3:BBH 4:IMRI 5:IMBBH_GC 6:IMBBH_seed,
-T	duration
-a	amplitude factor
-Н	Hubble parameter
_	

The commands:

-h

```
./list_cbc -S 1000 -l 2048 -r 4096 - f 10 -t 800000000 -d 20 -p 0.85 -P 0.02 -q 0.13 -Q 0 -j 0 -J 0 -z 0 -Z 5 -proba -m 1 -M 3 -a 1.4 -b 0.1 -n 5 -N 13 -u 9 -v 1 -G 20 ./back -T 2048000 -f 10 -t 1 -a 3 -print F 100
```

first produce the list of CBC parameters $list_cbc.txt$ in **input** which is then read by **background.c** to calculate the spectrum $\Omega_{gw}(f)$. It is important to use the same observation time used to produce the list (here 1000×2048) and the same amplitude factor we want to inject.

With the options -print -F 100 the value of $\Omega_{gw}(f)$ at the reference frequency 100 Hz is displayed. If the option -average is selected, the code uses an expression of Ω_{gw} averaged over source orientation, otherwise, it uses the values of the inclination in $list_cbc.txt$. The output file omega.txt contains the frequency, $\Omega_{gw}(f)$ and also the spectral energy density $S_h(f)$. The option -ET or -ALV calculates also the observed PSD taking into account the geometry and the location of the detectors. The extra file $psd_gw.txt$ contains the frequency, the "isotropic" PSD (simply $(sin^2\alpha/5)Sh(f)$ where α is the arm separation) and the PSDs calculated from the positions in $list_cbc.txt$ for all the detectors.

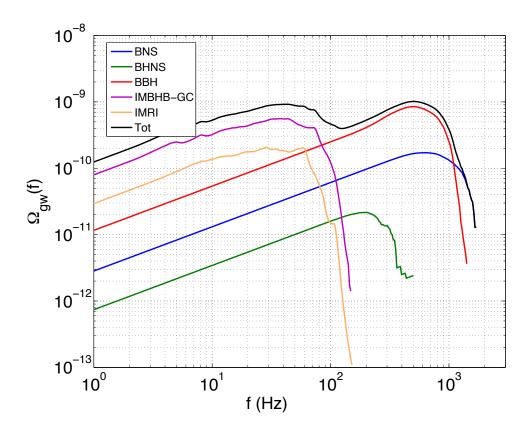


FIG. 10: Energy density of the population of CBC injected in the second ET MDC

Fig. 10 show the spectrum injected in the first 2048000 s of the second ET-MDC.

VIII. DEVELOPMENT

This package is still being developed. At that time it is complete enough to produce simple data for mock data challenges in view of the preparation for the Advanced detector era and ET. In the next few month, the package will be made more user friendly, and will include a few more options like glitchy noise, the use of galaxy catalogs to model the close Universe, tidal effects for BNS and BHNS, cosmological stochastic backgrounds, pulsars etc...

^[1] A Mock Data Challenge for the Einstein Gravitational-Wave Telescope Regimbau T., Dent T. and ET Science and MDC team, http://arxiv.org/abs/1201.3563

^[2] A Comprehensive Study of Binary Compact Objects as Gravitational Wave Sources: Evolutionary Channels, K. Belczynski, V. Kalogera, and T. Bulik, , Rates, and Physical Properties, Astrop. J. 572, 407

(2002);

- [3] http://www.sns.ias.edu/~starlab/
- [4] http://homepages.spa.umn.edu/~cwu/
- [5] The Astrophysical Background Regimbau T., in "The astrophysical gravitational wave stochastic background"; arXiv:1101.2762
- [6] The LIGO Collaboration, Predictions for the rates of compact binary coalescences observable by ground-based gravitational-wave detectors; arXiv:1003.2480
- [7] A. M. Hopkins and J. Beacom, Astrophys. J., 651, 142 (2006).
- [8] Dominik M. et al., Double Compact Objects I: The Significance Of The Common Envelope On Merger Rates; arXiv:1202.4901
- [9] Mandel, I. and Brown, D. A. and Gair, J. R. and Miller, M. C., ApJ 681, 1431 (2008)
- [10] Gair, J. R. and Mandel, I. and Miller, M. C. and Volonteri, M., General Relativity and Gravitation 43, 485 (2011)