# Neural Network Extraction of Quasinormal Modes Frequencies from Gravitational Waveforms

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presenting for

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# Content

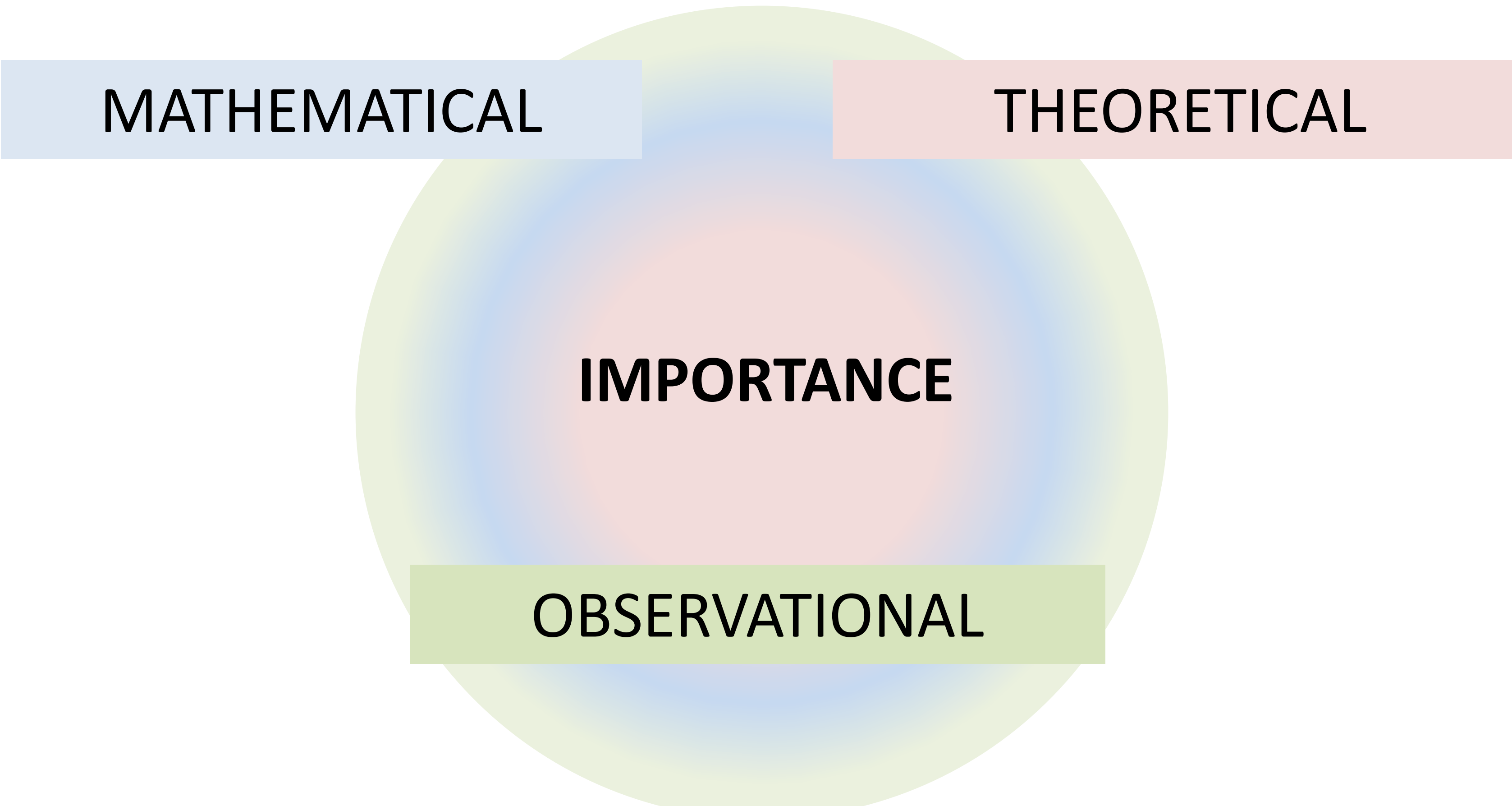
- Quasinormal modes (a few words)

- Status report about progress on identification by neural network (mock data challenge, KAGRA)

  [current stage: successfully reproduced mock data challenge results for machine learning approach]

# Quasinormal Modes

General context: perturbations of spacetime
(notably, BHs, but also cosmological, astrophysical,…)



# Quasinormal modes

Decomposition of perturbations in modes (scalar, vector, tensor)

## Master equation

DIFFERENTIAL_OPERATOR(mode) = 0

Self adjoint
- complete set
- real eigenvalues
- …

Not Self adjoint
- complete set (?)
- ~~real eigenvalues~~
- …

Quasi normal mode frequencies

$$f_{qnm} = f_R - if_I$$

"DISSIPATIVE" SYSTEMS

# Quasinormal modes

Decomposition of perturbations in modes (scalar, vector, tensor)

## Master equation

DIFFERENTIAL_OPERATOR(mode) = 0

### Black Hole Spacetimes

approximation schemes

*Schwarzschild* ⇒ separation "easier" ⇒ Regge-Wheeler equation, Zerilli equation ⇒ OK!

*Kerr* ⇒ separation "not so easy" ⇒ Teukolsky equation ⇒ Not always accurate

$f_{qnm} \sim V(x_0) - \text{const.} \times (-2V''(x_0))^{1/2}$ ⇐ WKB approaches

# Quasinormal modes

**ONE MOTIVATION**

Test alternative GR theories in the strong field regime

Test that the spacetime is correctly predicted by general relativity as close as possible to the event horizon!

T. Nakamura, H. Nakano, Progr. Theor. Phys, **2016** 041E01

PTEP

Prog. Theor. Exp. Phys. **2016**, 041E01 (8 pages)
DOI: 10.1093/ptep/ptw026

Letter

# How close can we approach the event horizon of the Kerr black hole from the detection of gravitational quasinormal modes?

Takashi Nakamura and Hiroyuki Nakano*

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Using the Wentzel–Kramers–Brillouin method, we show that the peak location ($r_{\rm peak}$) of the potential, which determines the quasinormal mode frequency of the Kerr black hole, obeys an accurate empirical relation as a function of the specific angular momentum $a$ and the gravitational mass $M$. If the quasinormal mode with $a/M \sim 1$ is observed by gravitational wave detectors, we can confirm the black-hole space-time around the event horizon, $r_{\rm peak} = r_+ + O\left(\sqrt{1-q}\right)$, where $r_+$ is the event horizon radius. However, if the quasinormal mode is different from that of general relativity, we are forced to seek the true theory of gravity and/or face the existence of the naked singularity.

Subject Index E01, E02, E31, E38

# Quasinormal modes

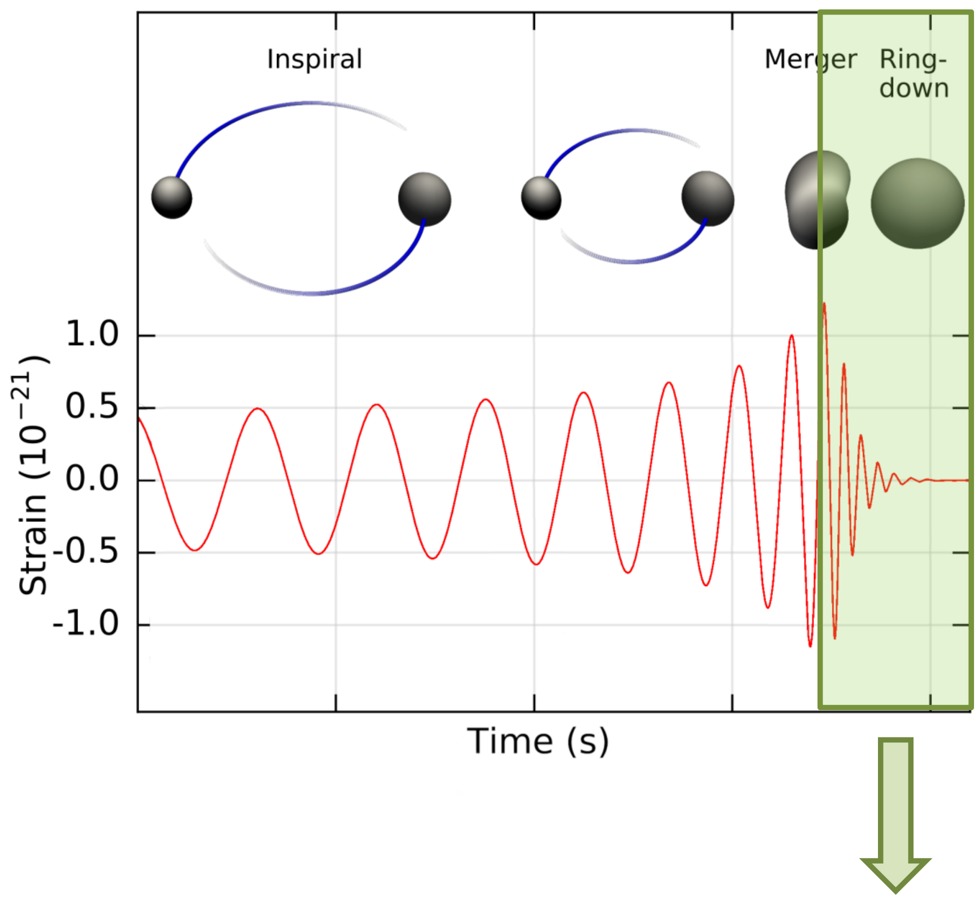
Test alternative GR theories in the strong regime

Test that the spacetime is correctly predicted by general relativity as close as possible to the event horizon!

T. Nakamura, H. Nakano, Progr. Theor. Phys. (**2016**) 041E01

"Then if, e.g., the QNM with *a/M = 0.9999* is observed by GW detectors, we can confirm the space-time around *r = 1.014 45M covering* 99.9996% *of the ergoregion.***"**

# Quasinormal modes




Strong field regime tests of gravity

# Quasinormal modes

**Detection of quasinormal modes in gravitational waveforms and comparison with theoretical predictions**

## Some Challenges

- Calculation of quasi normal modes is affected by theoretical uncertainties
- The signal is the waveform is weak
- Other technical aspects (beginning of the ringdown phase?)
- ...

# Quasinormal modes

## Detection of quasinormal modes in gravitational waveforms and comparison with theoretical predictions

### Pioneering work

H. Nakano, T. Narikawa, K. Oohara, K. Sakai, H. Shinkai, H. Takahashi, T. Tanaka, N. Uchikata, S. Yamamoto, T. S. Yamamoto, Phys. Rev. D **99**, 124032 (2019)

**Goal**

To identify the quasinormal modes frequencies by using only the ringdown part of the gravitational wave signal

PHYSICAL REVIEW D **99,** 124032 (2019)

# Comparison of various methods to extract ringdown frequency from gravitational wave data

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(Received 16 November 2018; published 21 June 2019)

The ringdown part of gravitational waves in the final stage of the merger of compact objects tells us the nature of strong gravity and hence can be used for testing theories of gravity. The ringdown waveform, however, fades out in a very short time with a few cycles, and hence it is challenging to extract the ringdown frequency and its damping timescale. We here propose to build up a suite of mock data of gravitational waves to compare the performance of various approaches developed to detect the dominant quasinormal mode from an excited black hole after merger. In this paper, we present our initial results of comparisons of the following five methods: (1) plain matched filtering with ringdown part method, (2) matched filtering with both merger and ringdown parts method, (3) Hilbert-Huang transformation method, (4) autoregressive modeling method, and (5) neural network method. After comparing the performances of these methods, we discuss our future projects.

DOI: 10.1103/PhysRevD.99.124032

# Taking the challenge: mock data

Set A

Only the ringdown phase was modified

Set B

The entire template was modified

252 files for each set + 90 from the Kagra group study

3 SNR realizations: 60 - 30 - 20

# Mock data construction specifications

Random total mass in the range $50M_\odot - 70M_\odot$

$f_R$ modified in range $\pm 30\%$

$f_I$ modified in range $\pm 50\%$

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We produce a mock strain of the form

$$h_{mod}(x) = A(x)\cos(M\omega(x)\,x) \quad , \text{ with } \quad x = \frac{t - t_p}{M}$$

where $A(x)$ and $\omega(x)$ are obtained by fitting the original template with two suitable functions, and by knowing the theoretical QNM[1].

[1]Physical Review D, 99(12), Jun 2019

## Fitting functions for Set A

*for x>0*

$$A(x) = \frac{A^{GR}(0) + a_0 x + a_1 x^2}{1 - \left(M\omega_I - a_0/A^{GR}(0)\right) x + a_2 x^2} e^{-M\omega_I x}$$

$$M\omega(x) = \left(M\omega^{GR}(0) - M\omega_R + b_0 x + b_1 x^2 + b_2 x^3\right) \times \exp\left[\frac{(b_0 + b_3)x}{M\left(\omega_R - \omega^{GR}(0)\right)}\right] + M\omega_R$$

## Fitting functions for Set B

$$A(x) = \frac{A^{GR}(x)}{1 + e^{4M\omega_I^{GR} x}} + \frac{A^{RD}(x)}{1 + e^{-4M\omega_I^{GR} x}}$$

$$\omega(x) = \frac{\omega^{GR}(x)}{1 + e^{4M\omega_I^{GR} x}} + \frac{\omega^{RD}(x)}{1 + e^{-4M\omega_I^{GR} x}}$$

with

$$A^{RD}(x) = \frac{\mathscr{A}}{1 + e^{-M\omega_I^{GR} x} + e^{M\omega_I x}}$$

$$\omega^{RD}(x) = \omega^{GR}(0) + \frac{\omega_R - \omega^{GR}(0)}{1 + e^{-2M\omega_I x}}$$

# QNM estimation method(s)

## CNN

1D convolutional neural network developed with Keras with Tensorflow as back-end.[7,8]

## Matched filtering-RD

A simple matched filtering algorithm to compare with the performance of our CNN and that of the original study.[9]

[7] https://keras.io/
[8] https://www.tensorflow.org/
[9] Physical Review D, 68(10), Nov 2003

# CNN architecture

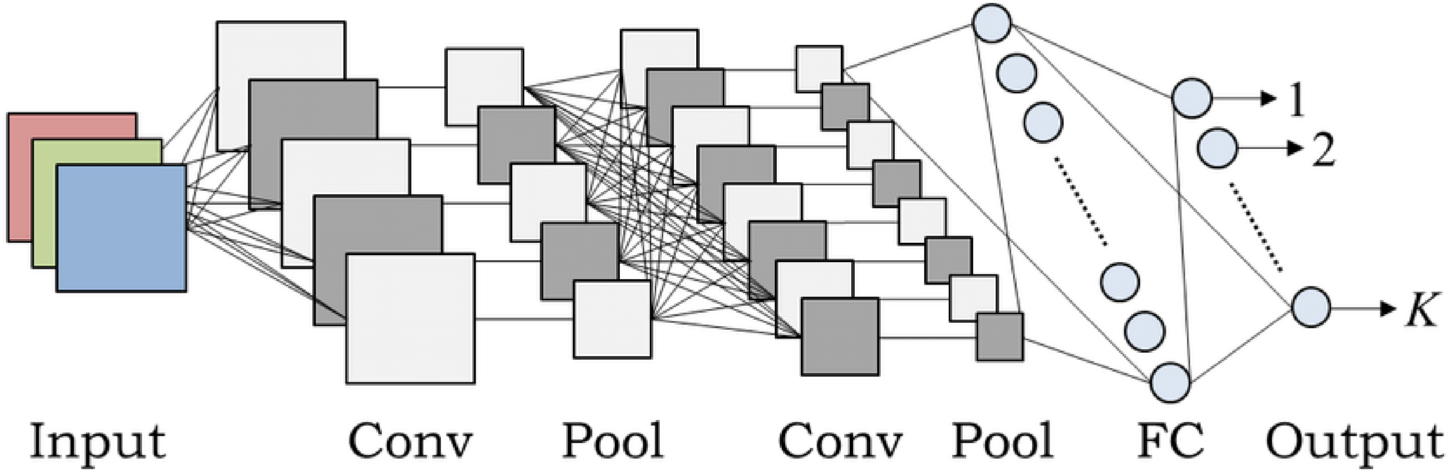
**Input**

512 points time series of the mock data

**Output**

QNM frequency $(f_R, f_I)$

Structured with multiple layers[10]



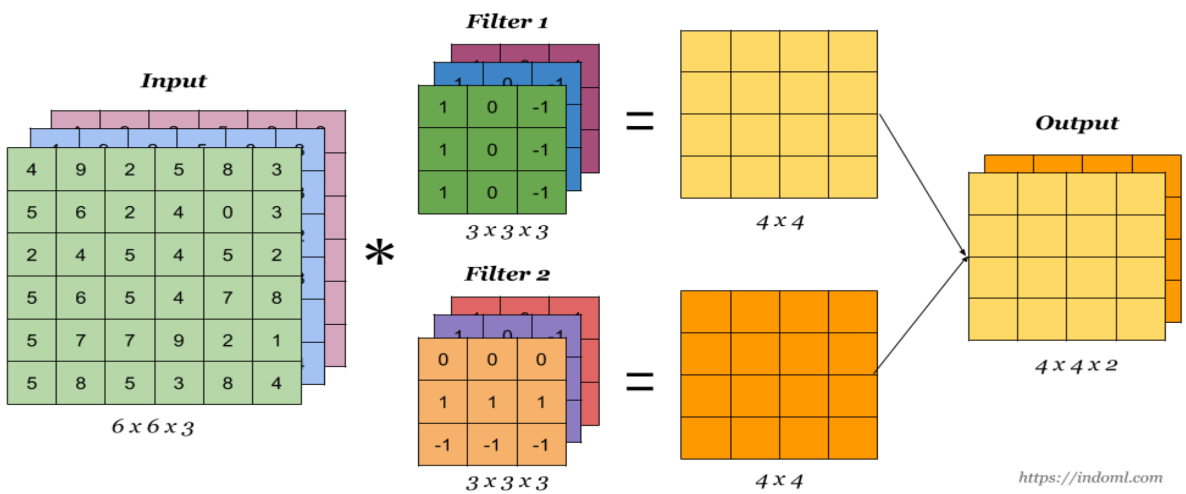

[10] I. J. Goodfellow, Y. Bengio, A. Courville. *Deep Learning*. MIT Press, 2016

# Convolutional layer

L filters cross-correlate the input applying a linear transformation on their receptive field. The coefficients and biases of the transformations are learnt via the training process.

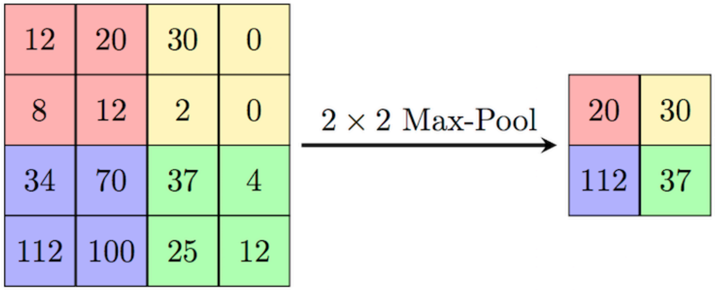
# Activation layer

Applies a rectifying function on each point of the output of a convolutional layer. Introduces non-linearity in the network.



# Pooling layer

Reduces the spacial resolution of the input layer by merging together clusters of points.

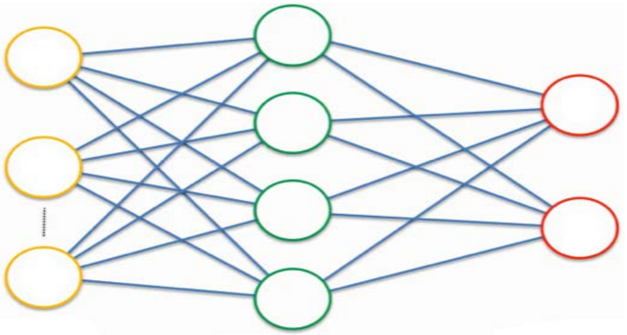



# Flatten layer

Unrolls the input layer in a 1D vector to be processed by a dense layer.

# Dense layer

Connects each point of the input layer to each point of the output, acting as a multi-layer perceptron.



# Matched Filtering-ringdown[9]

We convolve the mock data time series $s(t)$ with templates of the form

$$\hat{h}(t) = \frac{1}{N} e^{-\omega_I (t - t_0)} \cos\left[\omega_R (t - t_0) - \phi_0\right]$$

The frequencies $\omega_R$, $\omega_I$ are selected in order to maximize the integral $\left(s|\hat{h}\right)$, where

$$(a|b) = \int_{-\infty}^{\infty} \tilde{a}^*(f)\tilde{b}(f)\mathrm{d}f$$

We can find the best initial phase $\phi_0$ analytically, but the initial time $t_0$ of the QNM had to be approximated by averaging the set $[\omega_R(t_0), \omega_I(t_0)]$ over the interval $t_0 = (t_p, t_p + 20\Delta t)$, where $t_p$ is the time of the maximum of the mock data.

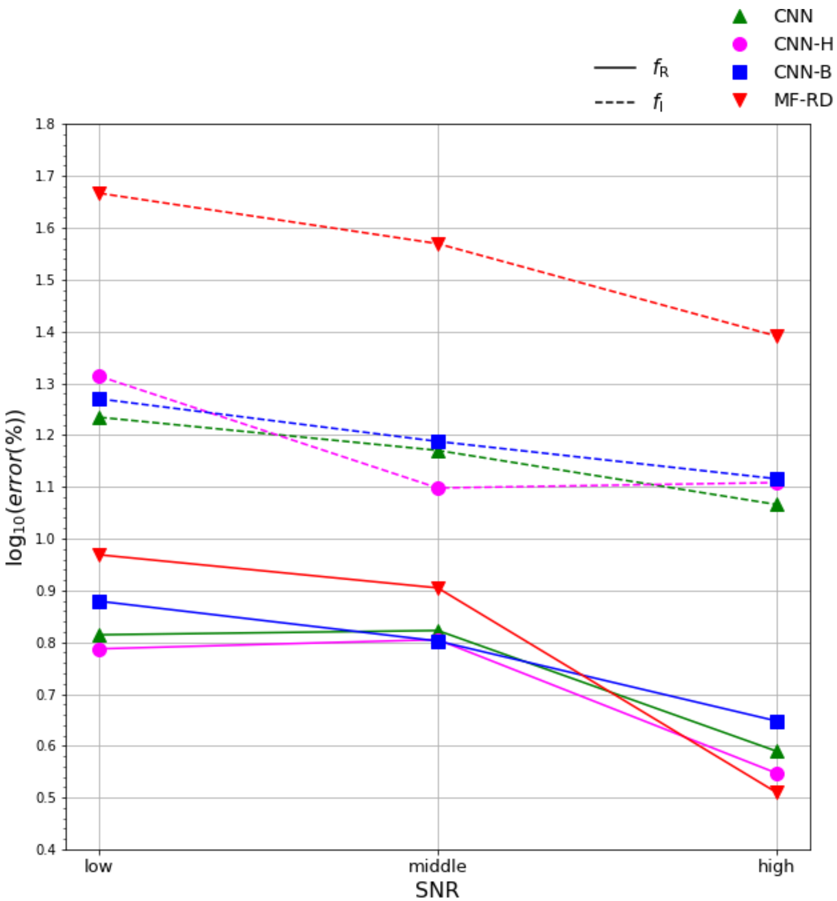
[9]Physical Review D, 68(10), Nov 2003

# CNN Training

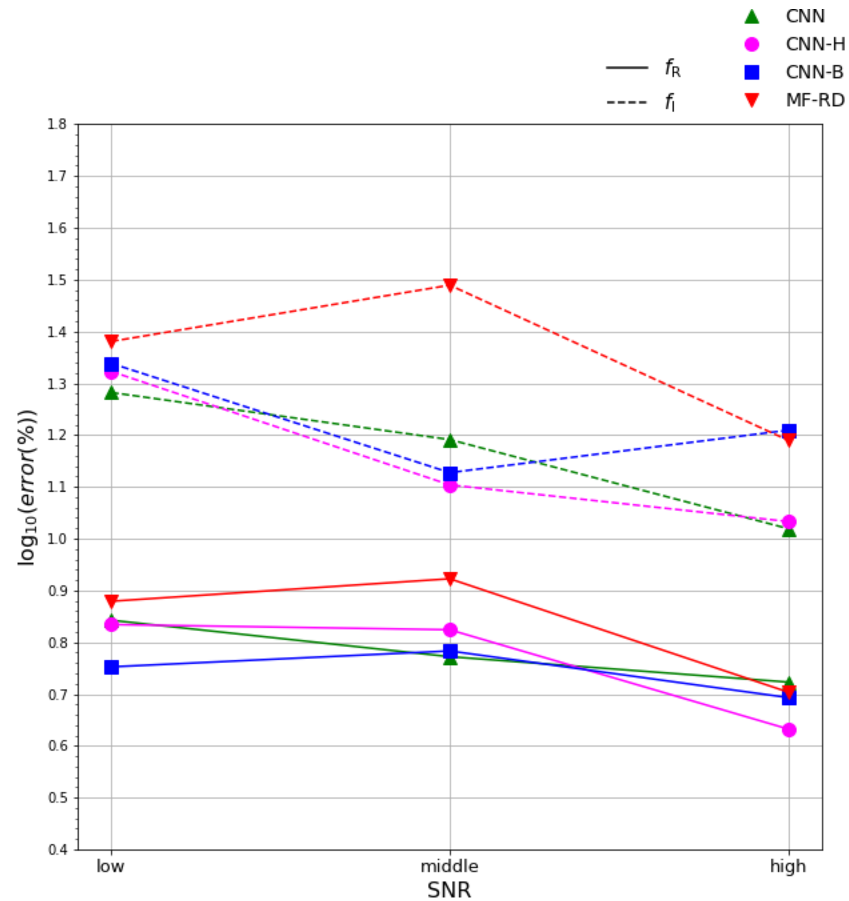
CNNs learns to perform tasks through an algorithm called backpropagation, in which the network parameters are optimized by training the CNN with examples of the task (supervised training)

- **CNN** – Trained with examples from both set A and B
- **CCNB –** Trained with examples from set B only
- **CNNH –** Trained with higher SNR examples from both set A and B

Total of 120960 training examples




Set A+B




Set A2+B2

Error $\approx 4-6\%$ for $f_R$, $\approx 12-15\%$ for $f_I$ for high SNR

# Summary

- Quasinormal Modes are messengers of the strong gravity regime
- An **independent** detection of quasinormal modes from the ringdown phase only, while challenging, allows us to extract substantial physical information without relying on the `weak gravity regime' inspiral phase

- Neural networks are competitive with other approaches in the identifcation of quasinormal mode frequencies (mock data challenge), and can potentially soften some open issues, like the dependence from the determination of the merger time

# Possible improvements

- Focus on challenges related to future applications on "real life" data
- Better estimation of uncertainties in the final results
- Further experiments with a variety of mock data building techniques (higher computational time, so move the implementation to a better infrastructure)
- …