Distance between NCals on the North to South axis for O4 VIR-0029A-24

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1 Introduction

During O4 we plan to mainly use pairs of NCals along the North to South axis to monitor the calibration of the NE mirror. Since the distance between the NCals is a key parameter in the NCal calibration uncertainty budget we must ensure that it is precisely measured.

In this note we present the previous and recent measurements on the O4 NCal system that allow us to compute the distance between NCals along the North to South axis.

2 Geometrical survey measurements

In October 2021 the reference plates for the NCal O4 system were installed in the Virgo NEB and measurements were made during their assembly to mechanically check the distances between the reference holes (see VIR-1123A-21).

In November 2021 the EGO infrastructure team made a survey (see logbook entry n° 53883) on the position of the near and far reference holes on the reference plates around the NE tower. The geometrical survey revealed few mm offsets with the mechanical expected values (see table 2 of VIR-0343B-22).

In March 2022 a last mechanical check was made with the template around the NE tower to check the November 2021 survey offsets. Since the uncertainties on the mechanical assembly and the geometrical survey were similar we chose to combine the results of the two measurement methods to determine the position of the reference holes (see table 7 of VIR-0343B-22). Using these measurements and the measurements made on the suspended reference plates we were able to compute the distance of each NCal to its associated reference hole (see VIR-1009C-22).

In December 2023 the EGO infrastructure team made a new survey (see logbook entry n° 62657) on the position of the far reference holes on the reference plates around the NE tower. The near reference holes were no longer reachable due to the presence of the suspended vertical plates. The purpose of this last survey is to ensure that the relative distance between the NCals on the North to South axis remains consistent with the measurements taken two years ago. One concern is whether the setups might have shifted slightly due to the ongoing work on the NCal system. This survey allow us to update the NCal positions and relative distances.

3 Verification of the North to South NCal distance for O4

This section presents the December 2023 measurements from the geometrical survey, comparing them with the 2021 measurements.

3.1 Geometrical survey at Virgo NE - December 5, 2023

As explained in VIR-0343B-22 the geometrical survey was made using reference points around the tower. These points and the reference holes are displayed in table 1 as x and y coordinates in the reference frame of the interferometer (see A GEODETIC REFERENCE FRAME FOR THE VIRGO INTERFEROMETER). Seven stations were used to make the measurements, four more than the first survey, some of the points were measured through several stations. One can notice that the value of the East Far reference hole has the largest fluctuation.

Table 2 shows the measurements of the far reference holes made during both surveys. The difference between the surveys is below the millimeter for the North and South far reference holes but just above 1 mm for the East far reference hole.

Measurement point	Sta	ition 1	Sta	tion 2	Sta	ation 3	Sta	ation 4	Sta	ation 5	Sta	ition 6	Sta	ition 7
measurement point	x [m]	y [m]	x [m]	y [m]	x [m]	y [m]	x [m]	y [m]	x [m]	y [m]	x [m]	y [m]	x [m]	y [m]
N201					2.5117	3000.4421	2.5118	3000.4419						
N202					4.5121	3006.4206	4.5120	3006.4207	4.5118	3006.4204	4.5115	3006.4208		
N203									2.5117	3016.4154	2.5117	3016.4153		
N204	-4.4880	3016.4131	-4.4879	3016.4131							-4.4880	3016.4131	-4.4880	3016.4130
N205	-4.4921	3006.4165	-4.4918	3006.4162									-4.4921	3006.4162
N206	-4.4921	3000.4437	-4.4924	3000.4435										
N208	-5.1469	2999.1258	-5.1473	2999.1256										
N209			-5.7298	3012.4500									-5.7297	3012.4501
N210	4.6479	3012.3953	4.6483	3012.3962	4.6475	3012.3956	4.6476	3012.3956	4.6475	3012.3953	4.6477	3012.3957	4.6483	3012.3960
N211					3.3386	2996.7325	3.3386	2996.7326			3.3369	2996.7327		
North Far	-1.4647	3007.9252	-1.4640	3007.9255									-1.4642	3007.9253
East Far									1.4788	3007.9330	1.4793	3007.9331	1.4809	3007.9330
South Far					1.2609	3003.9769	1.2607	3003.9771						

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Table 1: NCal geometrical survey measurements from logbook entry n° 62657 on December 5, 2023. Positions are in m and in the reference frame of the interferometer.

Data	Station	No	rth far	Ea	ast far	South far		
Date	Station	x [m]	y [m]	x [m]	y [m]	x [m]	y [m]	
	1a					1.2615	3003.9770	
06/11/21	2a	-1.4642	3007.9254	1.4807	3007.9330			
	3a	-1.4640	3007.9255					
	1	-1.4647	3007.9252					
	2	-1.4640	3007.9255					
	3					1.2609	3003.9769	
05/12/23	4					1.2607	3003.9771	
	5			1.4788	3007.9330			
	6			1.4793	3007.9331			
	7	-1.4642	3007.9253	1.4809	3007.9330			

Table 2: NCal measurements on the far reference holes for November 2021 and December 2023 surveys. Positions are in m and in the reference frame of the interferometer.

We point out that the 2021 survey stations were labelled differently in table 2, it is because some measurements points were added or changed compared to the 2023 survey.

In the following sections, for all NCal studies, we will use as usual the reference frame with the x axis being the beam axis instead of y and directed towards the beam emitter.

3.2 North to South far reference holes distance

In this section we will compare this previous North far to South far reference hole distance to the one based on the recent survey measurements.

Since each reference hole has been measured several times, we can combine the measurements to compute a set of distances for each survey as shown in fig. 1. The November 2021 survey made 2 measurements of the far North reference hole and 1 of the far South, we then compute 2 distances. The December 2023 survey made 3 measurements of the far North reference hole and 2 of the far South, we then compute 6 distances. A combination of both surveys can also be performed since the measurements should be compatible resulting in the computation of 15 measurements.

For each set of distance we compute the mean and rms of the distribution shown in table 3.

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Survey	North to South far distance [mm]
2021	4797.83 ± 0.02
2023	4797.45 ± 0.16
2021 + 2023	4797.57 ± 0.23

Table 3: Mean and rms of the distance between the far reference holes along the North to South axis for different surveys.

In table 3 the 2021 measurements show the smallest rms but with only 2 values while the combination of 2021 + 2023 shows a larger error of but with 15 values.



Figure 1: Distance between the North and South far reference holes using the combination of the VRS measurements. From top to bottom the 2021, 2023 and 2021+2023 combined measurements.

The mechanical distance between the near reference holes was 2800 ± 0.5 mm (see table 2 of VIR-0343B-22). Since the distance between the near and far reference holes on a plate is 1000.0 mm \pm 0.1 mm (see table 3 of VIR-0343B-22), the distance between the far reference holes translates to $2800 + 2 * 1000 = 4800 \pm 0.5$ mm.

This results is a 2.4 mm difference on the North to South far reference holes between this value and the combination of 2021+2023 surveys. This difference remains similar to the 2.1/2.2 mm offset computed between the mechanical template and the 2021 survey.

The survey measurements are compatible between 2021 and 2023 within half a millimeter while the mechanical measurements using the metal template was less reliable when assembled around the tower. Therefore we decided to rely on the geometrical survey and use the position of the reference holes based on these measurements. The following section will discuss the method used to determine the new NCal position parameters.

4 Position of the NCal system for O4

4.1 Position of the NCal reference holes

To get the new reference holes position parameters we use the average positions from the 2021 and 2023 surveys. These averaged values are defined relative to the NE tower position center (x=0.0129 m, y=3005.7877 m) provided by the EGO infrastructure (see VIR-0343B-22).

Table 4 shows the resulting average positions and rms of the values. The East near and South near positions were measured only once during the 2021 survey. Therefore, the displayed rms value is selected as the maximum rms among the position data for the reference holes and (if available) the tower measurements points across the surveys. We note that here the x values are along the beam axis as explained in the end of section 3.1.

Object	x [mm]	y [mm]
North near	-1315.15 ± 0.34	$\textbf{-908.40} \pm 0.35$
North far	-2137.68 ± 0.34	-1477.12 ± 0.36
East near	-1319.80 ± 0.45	903.20 ± 0.35
East far	-2145.33 ± 0.45	1467.03 ± 1.10
South near	987.70 ± 0.45	680.60 ± 0.98
South far	1810.70 ± 0.45	1248.13 ± 0.98

Table 4: Average positions of each reference hole according to the values of the 2021 and 2023 geometrical surveys, relative to the tower center. The associated error is the maximum rms among the reference holes and measurement points positions.

Figure 2 shows a top view of the positions of the reference holes according to the mechanical center of the NE tower O(0,0). Each setup is characterized as an axis with the angle ϕ relative to the beam axis.



Figure 2: Top view of the position of the reference holes according to the average surveys measurements. The axis of each setup is taken to be the line drawn between the near and far reference holes of the setup, they are shown as colored dashed lines. The center O is the mechanical center of the NE building. The beam axis is labelled as x.

The setup axes are not pointing to the center O but are slightly offsetted as seen on fig. 3. This results in a disalignment of the setup relative to the center O, we compute the associated disalignment or "twist" angle ψ for each setup, this angle is determined from the middle of each setup to the mechanical center O:

- $\psi_N = 0.02^\circ$ towards -x.
- $\psi_E = 0.04^\circ$ towards +x.
- $\psi_S = 0.01^\circ$ towards +x.

These angles are well below the 0.1° mechanical twist uncertainty (see section 4 of VIR-0530A-23) and will not be taken into account.



Figure 3: Zoomed in top view of the center of the mechanical center of the NE tower where the axis of each setup is shown as colored dashed lines. The beam axis is labelled as x.

4.2 Vertical offset of the reference holes

The difference of elevation of the setups increases the distance between NCals. We will determine this difference using the survey measurements and also estimate the effect on the injected signal.

The surveys also provided the vertical coordinate z of the measured points. Table 5 shows the measurements made with the 2021 and 2023 surveys along the z axis in the reference frame of the interferometer.

Measurement Doint	16/	/11/2021 z	[m]			05/12/2023 z [m]				
	Station 1	Station 2	Station 3	Station 1	Station 2	Station 3	Station 4	Station 5	Station 6	Station 7
N201	-1.9632					-1.9631	-1.9631			
N202	-1.9995					-1.9991	-1.9992	-1.9992	-1.9992	
N203								-1.9979	-1.9979	
N204			-2.0023	-2.0023	-2.0024				-2.0023	-2.0024
N205		-1.9948	-1.9948	-1.9947	-1.9948					-1.9949
N206			-1.9924	-1.9926	-1.9925					
N208			-1.9916	-1.9915	-1.9915					
N209		-1.9938	-1.9938		-1.9937					-1.9937
N210	-1.9909	-1.9911	-1.9911	-1.9911	-1.9911	-1.9912	-1.9911	-1.9911	-1.9911	-1.9911
N211	-1.9989					-1.9992	-1.9991		-1.9976	
North Near		-1.1301	-1.1310							
North Far		-1.1303	-1.1314	-1.1298	-1.1298					-1.1299
East Near		-1.1310								
East Far		-1.1314						-1.1292	-1.1292	-1.1290
South Near	-1.1347									
South Far	-1.1344					-1.1363	-1.1363			

Table 5: NCal geometrical survey measurements from logbook entry n° 53883 and 62657 on November 11, 2021 and December 5, 2023. Positions are in m and along the z axis in the reference frame of the interferometer.

Using the measurements of table 5 we compute the average elevation of each reference hole taking the maximum rms of the measurement points as the uncertainty. The distance between the center of the reflector used by the survey and the reference plate is 25 + 10 = 35 mm as shown in fig. 4. The dimensions of this reflector are shown in the drawing at the end of this note.



Figure 4: Picture of the reflector used to make the survey measurements on the reference plates with the support and the IPHC machined 10 mm spacer.

The previous value is then subtracted of the NE tower elevation (z=-0.902 m) provided by the EGO infrastructure. The final result is shown in table 6.

Object	z [mm]
North Near	-263.55 ± 0.64
North Far	$\textbf{-263.24} \pm 0.78$
East Near	$\textbf{-264.00}\pm0.12$
East Far	$\textbf{-262.70}\pm0.12$
South Near	$\textbf{-267.70}\pm0.90$
South Far	$\textbf{-268.67} \pm 0.90$

Table 6: Average elevations of each reference hole according to the values of the 2021 and 2023 geometrical surveys, relative to the tower center. The associated error is the maximum rms among the reference holes and measurement points positions.

The elevation of the NCals are computed using the drawing shown at the end of VIR-1009C-22. The NCal axis is elevated of 700 - 440 = 260 mm from the reference plate. For each NCal we take the linear interpolation at the NCal position of the Near and Far reference hole elevation to which we add 260 mm. The NCals axes elevation are shown in table 7, the uncertainties are taken as the same as in table 6.

NCal	axis elevation [mm]
NNN (1.7 m)	$\textbf{-3.52}\pm0.64$
NNF (2.1 m)	$\textbf{-3.40}\pm0.78$
NEN (1.7 m)	$\textbf{-3.87}\pm0.12$
NEF (2.1 m)	$\textbf{-3.35}\pm0.12$
NSN (1.7 m)	$\textbf{-8.18}\pm0.90$
NSF (2.1 m)	$\textbf{-8.57}\pm0.90$

Table 7: Axis elevation of each NCal.

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Between a pair of NCal along the North to South axis, the worst case scenario (3.52 mm on North and 8.57 mm on South) the elevation offset is below 6 mm. For a $\delta z = 6$ mm vertical offset, the North to South NCal distance d = 1700 + 1700 is changed by $d' - d = \sqrt{\delta z^2 + d^2} - d = 5 \times 10^{-3}$ mm.

The elevation z of the NCals relative to the center of mass of the mirror induces a deviation in the injected signal that propagates as $(5/2) * (z/d)^2$ (see table 7 of Newtonian calibrator tests during the Virgo O3 data taking). In the worst case scenario (8.57 mm with a 0.90 mm error) the elevation offset for a NCal is below 10 mm. This offset translates to a signal variation of $9 \times 10^{-3}\%$ at 1.7 m and $6 \times 10^{-3}\%$ at 2.1 m, using the simple analytic formula. The results of a FROMAGE computation are similar: $8 \times 10^{-3}\%$ at 1.7 m and $5 \times 10^{-3}\%$ at 2.1 m.

4.3 Checking the setup to beam axis angle ϕ

In the previous O4 NCal parametrization (see table 4 of VIR-0530A-23) we computed an average setup to beam angle of $\phi = 34.5^{\circ}$ for each setup with an uncertainty of 0.95 mrad.

Considering the new angles shown in figs. 2 and 3 and the values from table 4 we take the worst case scenario being the South setup. Figure 5 shows a top view representation of the South near reference hole. The scale has been enlarged for the representation. In this configuration $\delta y = 0.98$ mm is the uncertainty on the y coordinate and produces the largest ϕ deviation $\delta \phi_1 = 0.06^\circ$ or 1.05 mrad. We take this value as the uncertainty on the angle ϕ between a NCal and the beam axis.



Figure 5: Non-scaled top view of the South near reference hole at $\phi = 34.59^{\circ}$ from the beam axis x. The mechanical center of the tower is labelled as O. The uncertainties on x and y are respectively labelled δx and δy . The uncertainties on the angle ϕ are labelled $\delta \phi_1$ and $\delta \phi_2$.

4.4 Checking the rotor twist ψ

Following the discussion made in the previous section we computed an optimal misalignment or "twist" angle ψ for a rotor to beam angle $\phi = 34.5^{\circ}$ (see table 4 and section 3.2 of VIR-0530A-23). Considering the new angles shown in figs. 2 and 3 we must check if the chosen optimal twist angle is still compatible with negligible signal uncertainties.

Figures 6 to 8 shows the maximum signal obtained using FROMAGE for a rotor on the North, East and South setup at respectively $\phi_N = 34.66^\circ$, $\phi_E = 34.33^\circ$ and $\phi_S = 34.59^\circ$ with nominal distances from the mirror of 1.7 m and 2.1 m.



Figure 6: Top plots show the maximum signals for North NCal to mirror distances of (left to right) 1.7 m and 2.1 m at $\phi = 34.66^{\circ}$, the red curves show the FROMAGE simulations and the black curve show quadratic fits. Bottom plots show the fit residuals which are usually the order of $10^{-4}\%$.



Figure 7: Top plots show the maximum signals for East NCal to mirror distances of (left to right) 1.7 m and 2.1 m at $\phi = 34.33^{\circ}$, the red curves show the FROMAGE simulations and the black curve show quadratic fits. Bottom plots show the fit residuals which are usually the order of $10^{-4}\%$.



Figure 8: Top plots show the maximum signals for South NCal to mirror distances of (left to right) 1.7 m and 2.1 m at $\phi = 34.59^{\circ}$, the red curves show the FROMAGE simulations and the black curve show quadratic fits. Bottom plots show the fit residuals which are usually the order of $10^{-4}\%$.

Using the fit results from figs. 6 to 8 we compute the twist ψ_{max} associated to the maximum signal for both NCal to mirror distance presented in table 8. One can notice that due to the difference of angle ϕ for each setup the optimal twist angle ψ , which maximizes the injected signal, varies from 11.45° to 12.15° .

Setup	$\psi_{ m max}$ at 1.7 m	$\psi_{ m max}$ at 2.1 m
North	12.03°	11.68°
East	11.45°	11.55°
South	12.05°	12.15°

Table 8: Twist ψ_{max} associated to the maximum signal for each setup distances.

Since the current O4 configuration is so that the NCals are twisted 12° towards the beam axis, table 9 shows the small relative amplitude deviation for various NCal positions. The current 12° twist of the O4 NCals is therefore compatible within the uncertainty budget of $10^{-3}\%$ on the injected signal.

Setup with $\psi=12^\circ$	Amplitude deviation [%] at 1.7 m	Amplitude deviation [%] 2.1 m
North	5.2×10^{-4}	2.4×10^{-3}
East	$3.8 imes 10^{-3}$	3.2×10^{-3}
South	6.6×10^{-4}	1.3×10^{-3}

Table 9: Relative amplitude deviations for different NCal to mirror distances at a fixed $\psi = 12^{\circ} \pm \delta \psi = 0.1^{\circ}$.

4.5 Optimal rotor twist ψ to study parasitic coupling

4.5.1 With mirror position at the tower mechanical center

Since July 2023 we started investigating possible sources of parasitic coupling between the NCals and the mirror. This parasitic coupling can be studied using a setup producing a minimal signal in the interferometer. This can be achieved by twisting the NCals of a theoretical angle ψ_{min} (theo) = 90°.

The NCals are fixed on the suspended plates by their bottom part where a metal plate is screwed on the suspended plate. This metal plate has been machined to that the NCals could be twisted by $\pm 12^{\circ}$, $\pm 24^{\circ}$ and $\pm 90^{\circ}$ with a mechanical precision of about 0.1° .

But using FROMAGE we noticed that the extended mirror and rotor geometry affects the value of ψ_{min} . Figure 9 shows the computed twist around the minimum strain for NCals on the East setup at $\phi_E = 34.33^{\circ}$ and distances of 1.7 m and 2.1 m from the mirror. The signal cancellation twist angle has shifted from 90° to 89.7° at 1.7 m and 89.8° at 2.1 m. At $\psi = 90^{\circ}$ and a distance of 1.7 m the signal is 0.23% while at 2.1 m it is about 0.15%. Considering the 0.1° mechanical uncertainty on ψ the variation of the signal is within [0.15, 0.30] % at 1.7 m and [0.07, 0.22] % at 2.1 m.



Figure 9: Twist ψ around the minimum normalized strain h/h_0 for NCals on the East setup at $\phi_E = 34.33^{\circ}$ and distances of 1.7 m and 2.1 m from the mirror. Here h_0 is the strain of the NCal considered at $\psi = 12^{\circ}$. The minimum strain is achieved at 1.7 m for $\psi_{min} = 89.7^{\circ}$ and at 2.1 m for $\psi_{min} = 89.8^{\circ}$.

4.5.2 With mirror position from NCal measurements

Recent measurements of the mirror position using a pair of opposite NCals on the North to South axis revealed that the NE mirror might suffer a 5 mm offset from the mechanical center of the tower towards the North setup (see VIR-0955A-23). This effect combined with the misalignment of the setups displayed in figs. 2 and 3 can shift the position of the minimum strain.

Figure 10 shows a top view representation of the East setup. The scale has been enlarged for the representation. The mechanical center of the tower is labelled as O and centered on (0,0) and the offsetted mirror position is labelled as O' and is 5 mm away on the positive x values along the North to South axis $(\phi_{\text{NS}} = 34.5^{\circ})$. We consider the Near and Far NCals on the East axis at their nominal distances of 1.7 m and 2.1 m from the mechanical center.

When the mirror position is located on the mechanical center O of the tower, if the East NCals are twisted by 90° relative to the setup axis, then they are in fact twisted by $\psi_{\text{NEN}} = 89.95^{\circ}$ (for 1.7 m) and $\psi_{\text{NEF}} = 89.96^{\circ}$ (for 2.1 m). Using fig. 10 and the 0.1° uncertainty, the residual signal is within [0.12, 0.26] % at 1.7 m and [0.04, 0.19] % at 2.1 m.

Now if we consider the offseted mirror position O', then the NCals at 1.7 m and 2.1 m will be twisted by respectively $\psi_{\text{NEN}} = 89.79^{\circ}$ and $\psi_{\text{NEF}} = 89.83^{\circ}$ instead of the nominal 90°. Using fig. 10 and the 0.1° uncertainty, the residual signal is within [0.01, 0.15] % at 1.7 m and [0.05, 0.10] % at 2.1 m. We then expect a reduced gravitational coupling in this configuration allowing us to better study the parasitic coupling between the NCals and the mirror.



Figure 10: Non-scaled top view of the center of the mechanical center of the NE tower. The mechanical center of the tower is labelled as O and the offsetted mirror position of 5 mm along the NS axis ($\phi = 34.5^{\circ}$) is labelled as O'. The Near and Far East NCals are located along the East axis at $\phi = 34.33^{\circ}$. Twist angles of both NCals relative to the center O or O' are labelled respectively η and λ . The beam axis is labelled as x.

4.6 Distance between the NCals on the North to South axis

In order to compute the distance between two NCals on the North to South axis we will use the distance between the North and South far reference holes of 4797.57 ± 0.23 mm from the last row of table 3. To this value we will subtract the distance between the considered NCals rods (slot of the NCal on the suspended plate) and their associated far reference hole.

The distance between an NCal rod and a far reference hole includes first the distance between the far reference hole and the closest NCal rod 100.00 ± 0.05 mm (see section 2.1 of VIR-1009C-22). Then if the

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NCal is several slot away from the far reference hole we use the distance between the NCal rods on the corresponding suspended plate (see table 1 of VIR-1009C-22) with an uncertainty of 0.05 mm. In table 3 of VIR-1009C-22 this value is shown for a specific set of NCals. Because the NCals suffer a slight axis offset when mounted in their box varying from 0.03 to 0.30 mm depending on the NCal (see table 2 of VIR-1009C-22), since we plan to move the NCals throughout the O4 run, we chose to take 0.30 mm as the NCal axis offset uncertainty. Table 10 shows the distances of each NCal from the far reference hole and their uncertainty. For East and North setups the uncertainty is taken as the quadratic sum of all previous uncertainties with the setup motion uncertainty of 0.20 mm for a total value of 0.37 mm.

Plate	NCal	Distance to reference hole [mm]
NINI	Near	900.24 ± 0.37
ININ	Far	499.76 ± 0.37
NE	Near	899.96 ± 0.37
INE	Far	500.18 ± 0.37
NC	Near	499.94 ± 0.37
IND	Far	100.00 ± 0.37

Table 10: Distances between each NCal to the far reference hole of the associated setup.

The distance between two NCals on the same setup is also computed using table 1 of VIR-1009C-22 and shown in table 11. The uncertainty is taken as the quadratic sum of 0.05 mm and twice the maximum axis offset of a NCal (0.30 mm) since two NCals are considered making a total of 0.43 mm.

Plate	Near to Far NCal distance [mm]
NN	400.48 ± 0.43
NE	399.78 ± 0.43
NS	399.94 ± 0.43

Table 11: Distances between two NCals on a setup.

Using the results of table 10 we can compute the distance between two NCals on the North to South axis shown in table 12. The uncertainty on these distances is taken as the quadratic sum of the North to South far reference hole distance uncertainty (0.23 mm from last row of table 3), twice the table 10 uncertainty (0.37 mm) due to the two NCals, and the plane uncertainty of 0.067 mm (see section 3 of VIR-1009C-22 on a possible horizontal tilt defect) for a total uncertainty of 0.58 mm.

NCals	Distance [mm]
NNN-NSN	3397.39 ± 0.58
NNF-NSF	4197.81 ± 0.58
NNN-NSF	3797.33 ± 0.58
NNF-NSN	3797.87 ± 0.58

Table 12: Distances between two NCals on the North to South axis.

