2022, December 22

Contribution of the HR coatings to the thermal detuning of the Virgo FDS Filter Cavity

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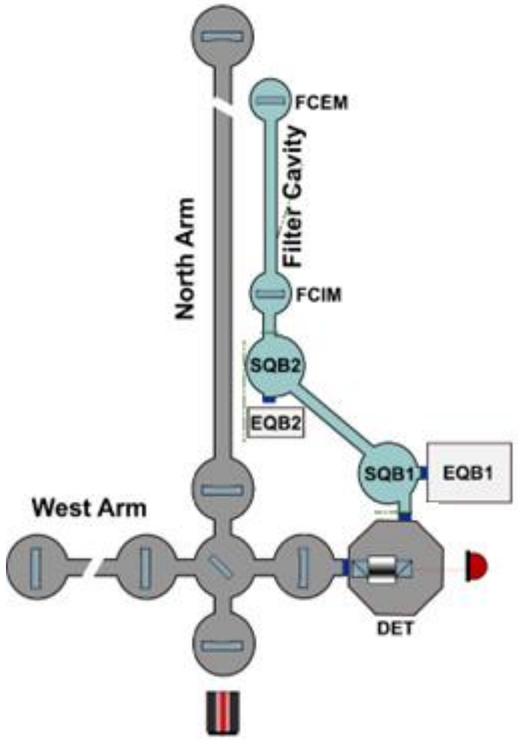
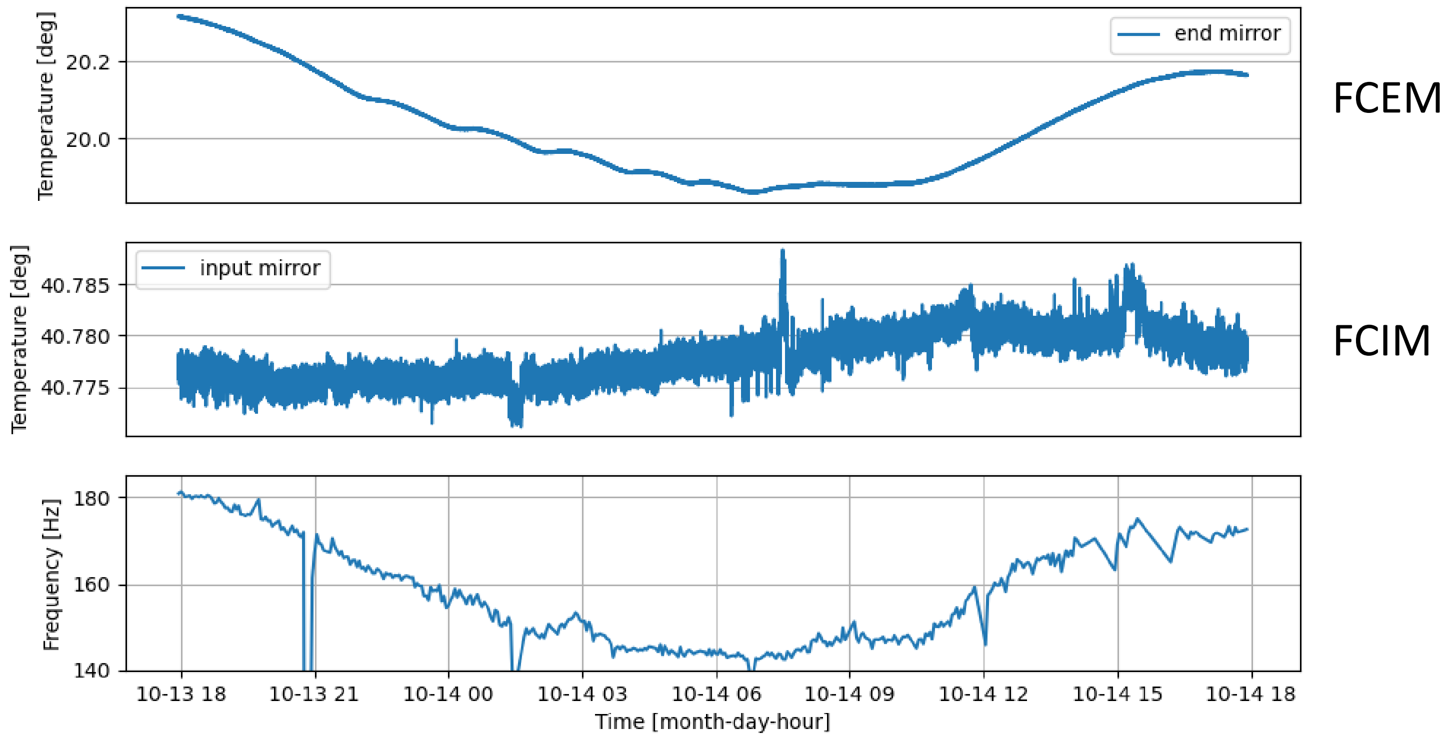
Introduction



Experimental measurement



$$\frac{\partial f_2}{\partial T} = 84 \text{ Hz/°C}$$

Filter Cavity

- **Resonance conditions**

$$\begin{cases} \varphi_{\mathrm{EM}}(\lambda_1) + \frac{4\pi L}{\lambda_1} + \varphi_{\mathrm{IM}}(\lambda_1) = 2p_1\pi \\ \varphi_{\mathrm{EM}}(\lambda_2) + \frac{4\pi L}{\lambda_2} + \varphi_{\mathrm{IM}}(\lambda_2) = 2p_2\pi \end{cases} \qquad p_1, p_2 \in \mathbb{N}$$

- **Theoretical data**
  - λ_1= 532 nm ; $\varphi_{\mathrm{EM}}(\lambda_1)$ = 0,6931 rads ; $\varphi_{\mathrm{IM}}(\lambda_1)$ = -0,7959 rads
  - λ_2= 1064 nm ; $\varphi_{\mathrm{EM}}(\lambda_2)$ = 0,1970 rads ; $\varphi_{\mathrm{IM}}(\lambda_2)$ = -2,7323 rads
- **Experimental measurement**
  - L = 284.8994(51) $\pm$ 2.8×10^{-5} m

Filter Cavity HR coatings

- **Effect of a temperature variation on the optical properties of a multilayer stack**
  - ***Hypothesis: perfect adhesion between the layers (f) and the substrate (s)***
  - ***Direct effects***
    - ✓ *Modification of the physical thickness of each layer (coefficient of thermal expansion CTE, α_f)*
    - ✓ *Change in the refractive index of each of the layers (thermo-optic coefficient, β_f)*
  - ***Indirect effects***
    - ✓ *Change in thickness of each layer due to the thermo-mechanical expansion of the substrate (coefficient of thermal expansion α_s, Poisson ratio ν_f)*
    - ✓ *Change in the refractive index of each layer due to the thermo-mechanical expansion of the substrate (coefficient of thermal expansion α_s, Poisson ratio ν_f, elasto-optic coefficients p_{11} et p_{12})*

R. Parmentier and M. Lequime, "Substrate-Strain-Induced Tunability of Dense Wavelength-Division Multiplexing Thin-Film Filters," Opt. Letters **28**, 728-730 (2003) and **28**, 1279 (2003)
M. Lequime, "Tunable Thin-Film Filters: Review and Perspectives," in *Advances in Optical Thin Films*, C. Amra, N. Kaiser, and H.A. Macleod, eds., Proc. SPIE **5250**, 302-311 (2003)

Filter Cavity HR coatings

- **Thermal sensitivity: general formula applicable to each layer**

$$\frac{1}{\Delta T}\cdot\frac{\Delta(ne)_f}{(ne)_f} = \alpha_f + \beta_f - \frac{\alpha_s - \alpha_f}{1-\nu_f}\left\{2\nu_f + \frac{n_f^2}{2}\left[(1-\nu_f)p_{11} + (1-3\nu_f)p_{12}\right]\right\}$$

$$\alpha_f = \frac{1}{e_f}\frac{\partial e_f}{\partial T}$$

Thermal expansion of the layer

Change in thickness of each layer due to the thermo-mechanical expansion of the substrate

$$\beta_f = \frac{1}{n_f}\frac{\partial n_f}{\partial T}$$

Relative change in the refractive index of each layer induced by a temperature change

Change in the refractive index of each layer due to the thermo-mechanical expansion of the substrate (elasto-optic effect)

R. Parmentier and M. Lequime, "Substrate-Strain-Induced Tunability of Dense Wavelength-Division Multiplexing Thin-Film Filters," Opt. Letters **28**, 728-730 (2003) and **28**, 1279 (2003)
M. Lequime, "Tunable Thin-Film Filters: Review and Perspectives," in *Advances in Optical Thin Films*, C. Amra, N. Kaiser, and H.A. Macleod, eds., Proc. SPIE **5250**, 302-311 (2003)

Thin-Film properties

Quantity	S. Michel	E. Çetinörgü	G. H. Ogin	GWINC	Model
α_f SiO2 (°C^{-1})		2.1×10^{-6}	5.5×10^{-6}	5.1×10^{-7}	2.1×10^{-6}
ν_f SiO2	0.17	0.11		0.17	0.11
β_f SiO2 (°C^{-1})	6.9×10^{-6}		1.9×10^{-6}	5.9×10^{-6}	3.9×10^{-6}
p_{11}/p_{12} SiO2	0.121/0.270				0.121/0.270
α_f Ta2O5 (°C^{-1})	2.4×10^{-6}	4.4×10^{-6}	8.9×10^{-6}	3.6×10^{-6}	3.4×10^{-6}
ν_f Ta2O5	0.23	0.27		0.23	0.24
β_f Ta2O5 (°C^{-1})	1.1×10^{-6}		5.8×10^{-5}	6.8×10^{-6}	4.0×10^{-6}
p_{11}/p_{12} Ta2O5	0.068/0.164				0.068/0.164

S. Michel, "Vers une determination optique directe des coefficients opto-mécaniques et thermo-optiques des couches minces optiques, Thèse de Doctorat, Université Aix-Marseille (2008)
E. Çetinörgü, et al., "Mechanical and thermoelastic characteristics of optical thin films deposited by dual ion beam sputtering," Appl. Opt. **48**, 4536-4544 (2009)
G. H. Ogin, "Measurement of Thermo-Optic Properties of Thin Film Dielectric Coatings," PhD Thesis, California Institute of Technology (2013)
LIGO's Gravitational Wave Interferometer Noise Calculator – GWINC Version 3
M. Evans, et al., "Thermo-optic noise in coated mirrors for high-precision optical measurements," Phys. Rev. D **78**, 102003 (2008)
C.-L. Tien, et al., "Simultaneous determination of the thermal expansion coefficient and the elastic modulus of Ta2O5 thin film using PSI," J. Modern Optics **47**, 1681-1691 (2000)
A. K. Chu, et al., "Temperature dependence of refractive index of Ta2O5 Dielectric Films," J. Electro. Mater. **26**, 889-892 (1997)

Thin-Film properties

- **Three important remarks**
  - ***The properties of thin film materials are highly dependent on the deposition process and the machine parameters***
  - ***It is not clear whether the elasto-optical coefficients have to be taken into account separately or whether they are already included in the thermo-optical coefficients***
  - ***The thermal sensitivity can be almost cancelled out by an appropriate choice of the thermal expansion coefficient of the substrate (DWDM application)***

H. Takashashi, "Temperature stability of thin-film narrow-bandpass filters produced by ion-assisted deposition," Appl. Opt. **34**, 667-675 (1995)
M. Lequime, "Tunable Thin-Film Filters: Review and Perspectives," in *Advances in Optical Thin Films*, C. Amra, N. Kaiser, and H.A. Macleod, eds., Proc. SPIE **5250**, 302-311 (2003)

Filter Cavity HR coatings

- **Application to FC EM and FC IM mirrors**
  - ***Silica substrate (Suprasil):*** $\alpha_s = 5.1 \times 10^{-7}\ °\mathrm{C}^{-1}$
  - ***SiO2 layers:*** $\frac{1}{\Delta T} \cdot \frac{\Delta(ne)}{ne} = 6.4 \times 10^{-6}\ °\mathrm{C}^{-1}$ ($7.5 \times 10^{-6}\ °C^{-1}$)
  - ***Ta2O5 layers:*** $\frac{1}{\Delta T} \cdot \frac{\Delta(ne)}{ne} = 9.2 \times 10^{-6}\ °\mathrm{C}^{-1}$ ($10.8 \times 10^{-6}\ °C^{-1}$)

Material	Thermo-mechanical effect	Thermo-optic effect	Elastic effect	*Elasto-optic effect*
SiO2	2.1×10^{-6}	3.9×10^{-6}	0.4×10^{-6}	1.1×10^{-6}
Ta2O5	3.4×10^{-6}	4.0×10^{-6}	1.8×10^{-6}	1.6×10^{-6}

Filter Cavity

- **Application to FC EM and FC IM mirrors**
  - ***Thermal sensitivity of reflection phase shifts***

$$\frac{\partial \varphi_{\mathrm{IM}}}{\partial T}(\lambda_1) = 1.2 \times 10^{-3}\ \text{rads/°C} \qquad \frac{\partial \varphi_{\mathrm{IM}}}{\partial T}(\lambda_2) = 4.9 \times 10^{-5}\ \text{rads/°C}$$

$$\frac{\partial \varphi_{\mathrm{EM}}}{\partial T}(\lambda_1) = 2.7 \times 10^{-3}\ \text{rads/°C} \qquad \frac{\partial \varphi_{\mathrm{EM}}}{\partial T}(\lambda_2) = 1.3 \times 10^{-4}\ \text{rads/°C}$$

  - ***IM mirror kept at a constant temperature***
  - ***Resonance condition at λ_1 maintained by a cavity length adjustment***

$$\frac{4\pi f_1}{c}\frac{\partial L}{\partial T} = -\left.\frac{\partial \varphi_{\mathrm{EM}}}{\partial T}\right|_{f_1}$$

Filter Cavity

- **Consequence**

$$\frac{4\pi}{c}\left\{L\frac{\partial f_2}{\partial T}+f_2\frac{\partial L}{\partial T}\right\}=-\left.\frac{\partial \varphi_{\mathrm{EM}}}{\partial T}\right|_{f_2}$$

$$\frac{\partial f_2}{\partial T}=\frac{c}{4\pi L}\left\{\frac{1}{2}\left.\frac{\partial \varphi_{\mathrm{EM}}}{\partial T}\right|_{f_1}-\left.\frac{\partial \varphi_{\mathrm{EM}}}{\partial T}\right|_{f_2}\right\}$$

- **FC thermal detuning**

$$\frac{\partial f_2}{\partial T}=102\ \mathrm{Hz/°C}$$

$$\frac{\partial f_2}{\partial T}=121\ \mathrm{Hz/°C}$$