

Inferring core-collapse supernova physics with gravitational waves

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Outline

★ Introduction: Core-collapse supernovae

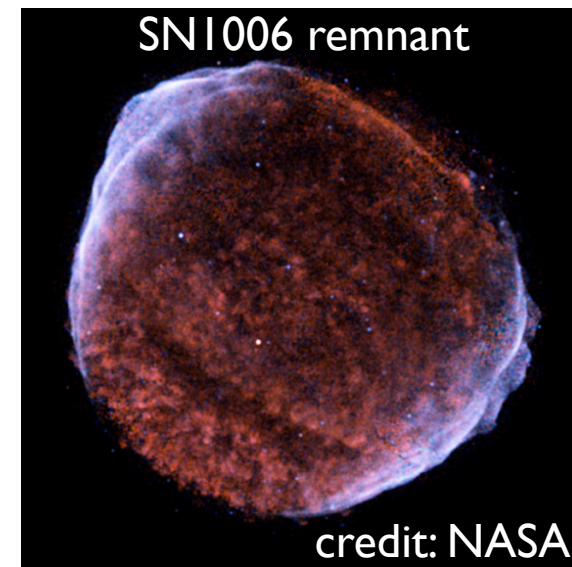
★ Supernova Model Evidence Extractor

- Singular Value Decomposition
- Bayesian model selection

★ Results

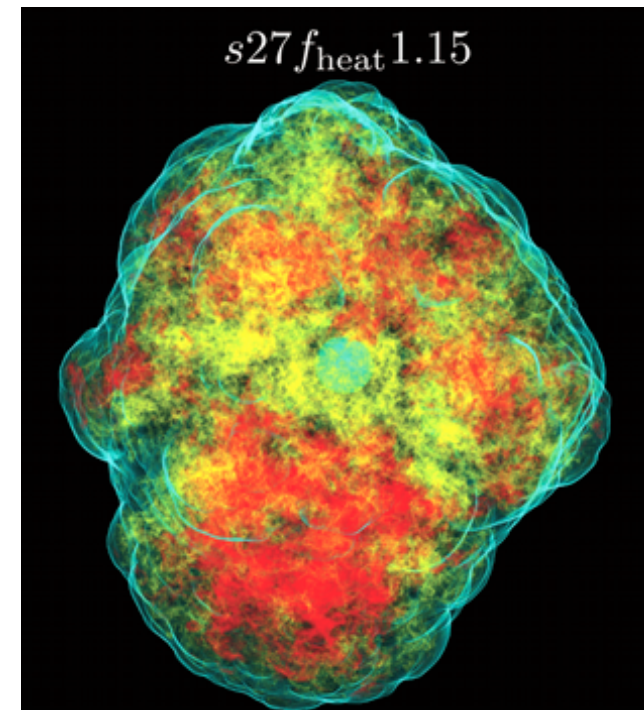
- Signal injections in Advanced LIGO noise
- Signal injections in ET noise

★ Summary and future work



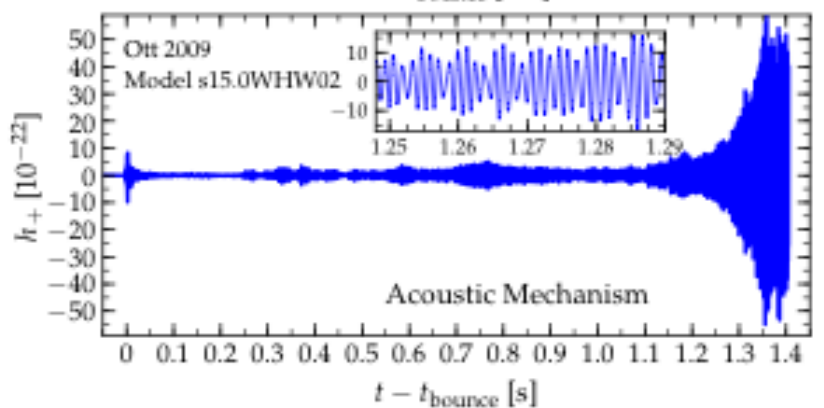
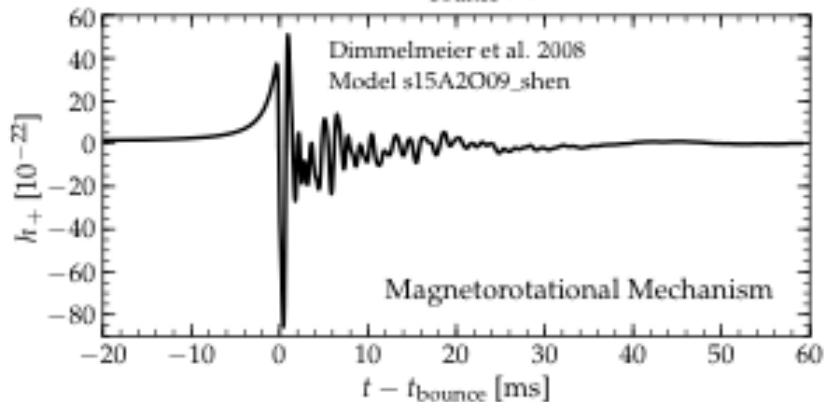
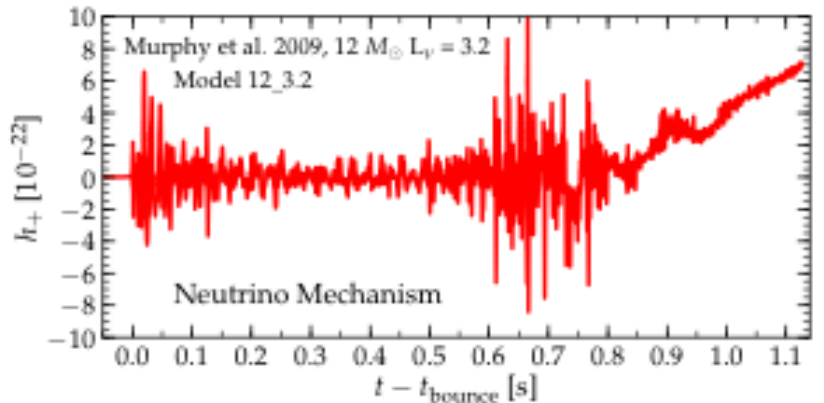
Core-collapse supernovae

- ★ Several models have been proposed to explain the processes behind core-collapse supernovae
- ★ These models lead to different gravitational wave emission mechanisms
- ★ Numerical simulations of the various mechanisms have produced catalogues of waveforms for the different mechanisms
- ★ Can we distinguish between the waveforms from the different catalogues and, thus, learn about the astrophysics behind core-collapse supernovae?



<http://www.stellarcollapse.org/>

Core-collapse supernova

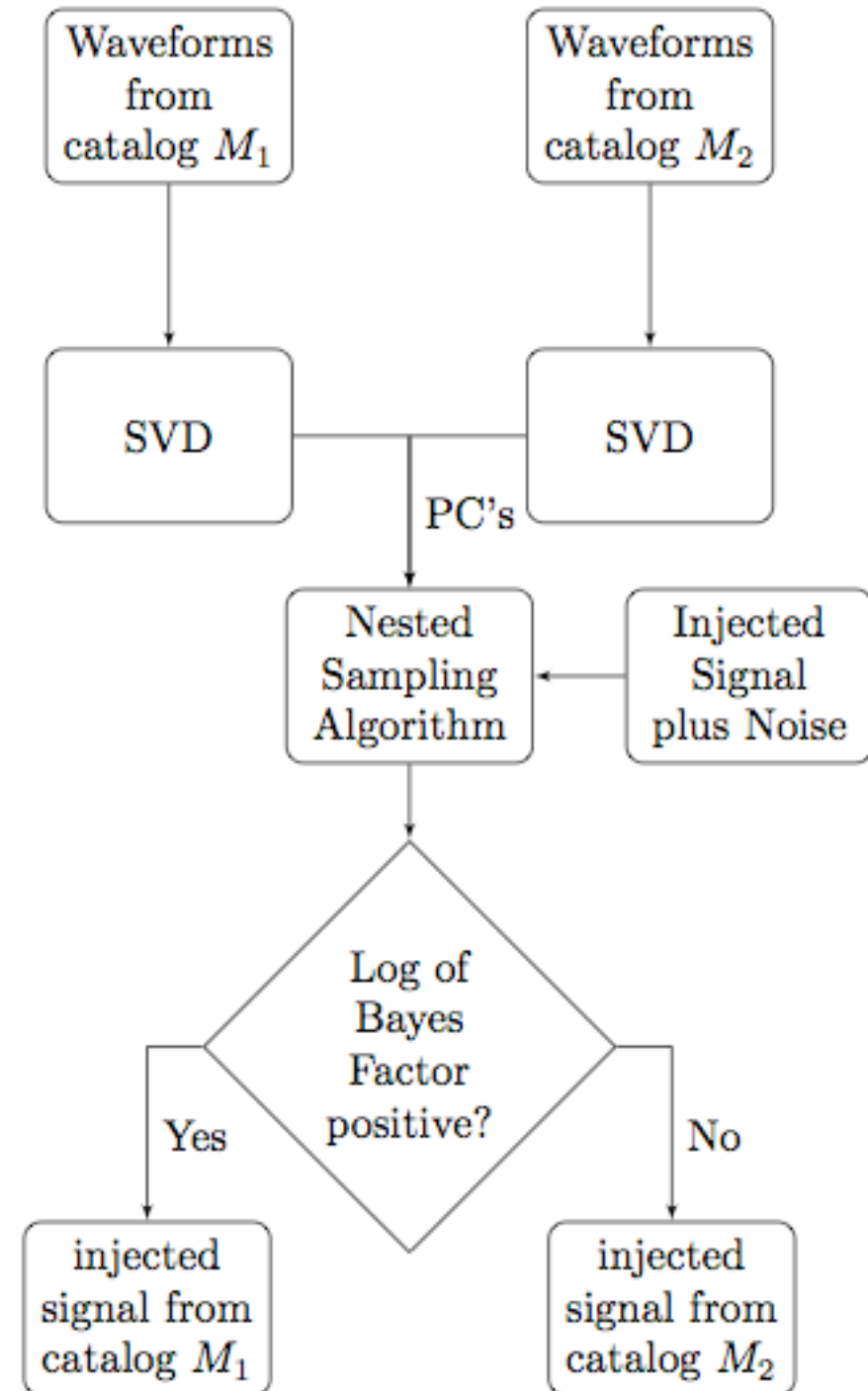


- ★ Chose 3 initial catalogues to develop analysis
- ★ **Neutrino mechanism** - use Murphy *et al.* 2009 catalogue with 16 waveforms
- ★ Magnetorotational mechanism - use Dimmelmeier *et al.* 2008 catalogue with 128 waveforms
- ★ **Acoustic mechanism** - use Ott *et al.* 2009 catalogue with 7 waveforms

SMEE

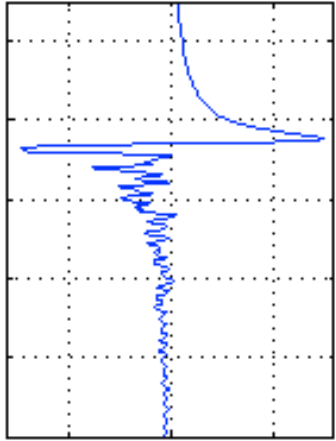
- ★ Use the **Supernovae Model Evidence Extractor (SMEE)** to distinguish between waveforms from different catalogues
- ★ SMEE:
 1. reparameterises waveforms into a set of orthonormal basis vectors
 2. uses Bayes factor to compare the likelihood that the observed signal belongs to one catalogue as opposed to another catalogue*

*the other “catalogue” could also be noise or a model for known spurious noise



Singular Value Decomposition

- ★ consider a catalogue of M waveforms, each N samples long

$$h_1 = \begin{bmatrix} h_1(t_1) \\ h_1(t_2) \\ \vdots \\ h_1(t_N) \end{bmatrix}$$


- ★ arranged into a matrix \mathbf{A} ($N \times M$) such that each column corresponds to one waveform

$$\mathbf{A} = \begin{bmatrix} h_1(t_1) & h_2(t_1) & \dots & h_M(t_1) \\ h_1(t_2) & h_2(t_2) & \dots & h_M(t_2) \\ \vdots & \ddots & \ddots & \vdots \\ h_1(t_N) & h_2(t_N) & \dots & h_M(t_N) \end{bmatrix}$$

Singular Value Decomposition

- ★ Singular Value Decomposition (SVD) states that \mathbf{A} can be factored into

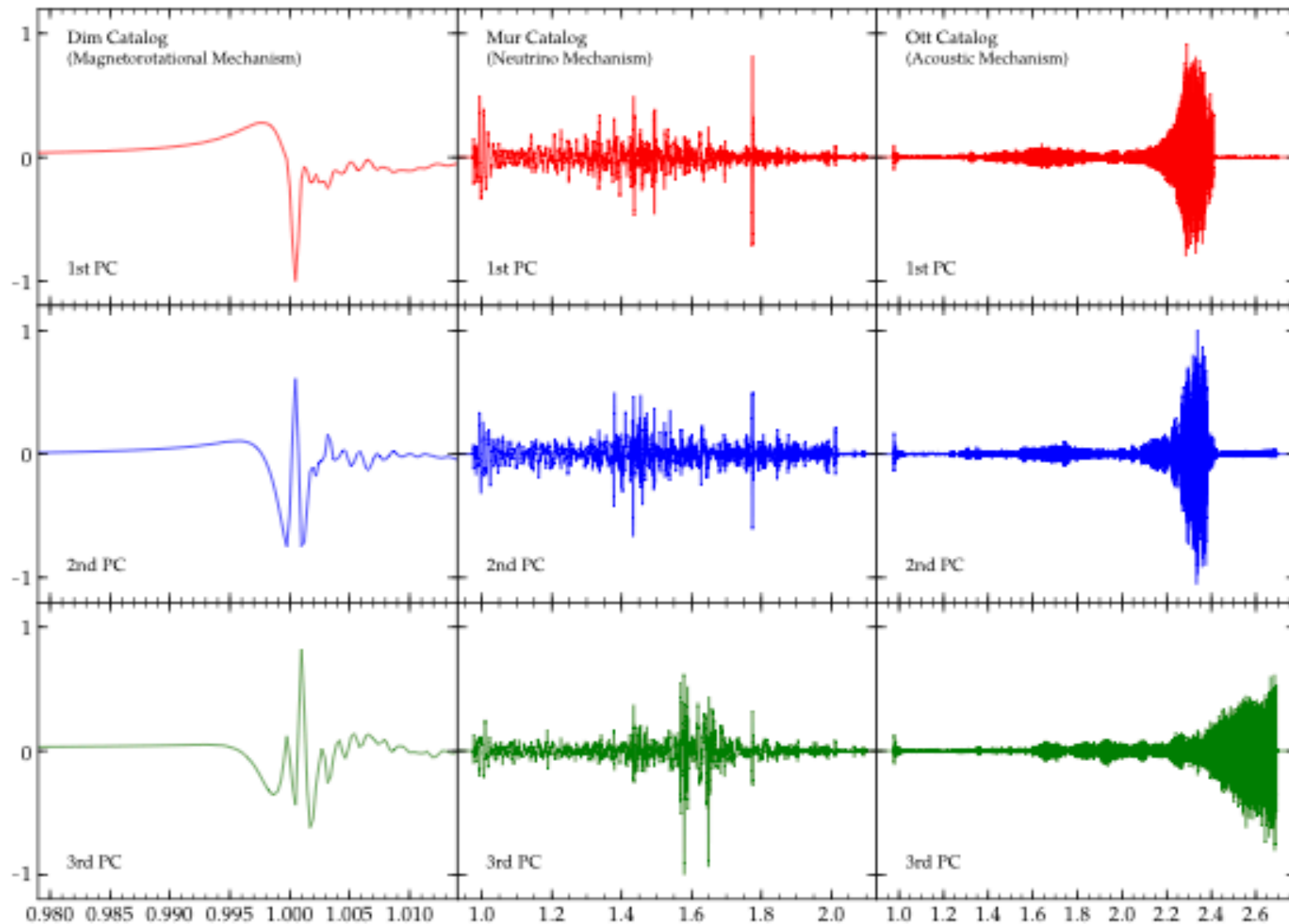
$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$$

- ★ where \mathbf{U} is $N \times N$, \mathbf{V} is $M \times M$ and $\mathbf{\Sigma}$ is $N \times M$
- ★ \mathbf{U} is a matrix where the columns are the eigenvectors of $\mathbf{A}\mathbf{A}^T$
- ★ \mathbf{V} is a matrix where the columns are the eigenvectors of $\mathbf{A}^T\mathbf{A}$
- ★ $\mathbf{\Sigma}$ has the square roots of the eigenvalues on its diagonal

Principal Component Analysis

- ★ Note that \mathbf{AA}^T is the covariance matrix for the data in \mathbf{A}
- ★ So, for our data matrix, \mathbf{A} , the eigenvectors in \mathbf{U} (Principal Components) form an orthogonal basis that spans the parameter space defined by the data
- ★ The eigenvectors are ranked by their corresponding eigenvalue
- ★ The first Principal Component is the eigenvector with the largest corresponding eigenvalue
 - direction of the largest variance in the data set
- ★ The original set of waveforms that were used to construct \mathbf{A} can now be described as a linear combination of the Principal Components in \mathbf{U}

Principal Component Analysis



Reconstructing the signal

- ★ If we use all M Principal Components, we can reconstruct all waveforms identically
- ★ The Principal Components are an efficient basis for spanning the parameter space described by the waveforms
- ★ One can approximate each waveform by taking a linear combination of k Principal Components, where $k < M$

$$h_i \approx \sum_{j=1}^k U_j \beta_j$$

- ★ Here, β_j is the scalar coefficient for the j -th Principal Component
- ★ The corresponding eigenvalues indicate how well the choice of k Principal Components will reconstruct the original waveforms

Bayesian model selection

- ★ The Bayes factor is the ratio of the marginalised likelihoods for two competing models

$$B_{12} = \frac{p(D|M_1)}{p(D|M_2)}$$

- ★ If $B_{12} > 1$, M_1 is preferred. If $B_{12} < 1$, M_2 is preferred
- ★ If $B_{12} = 1$, then there is insufficient information in the data to support either model
 - noise introduces an uncertainty which enlarges this to a “region of ambiguity”
- ★ Here, M_1 and M_2 are the different core-collapse supernova mechanisms
 - these models can also be the ratio of the likelihood that the data contains a signal versus noise only

The signal model

- ★ Since simulated noise is used, we assume a Gaussian likelihood for our signal model, M_s ,

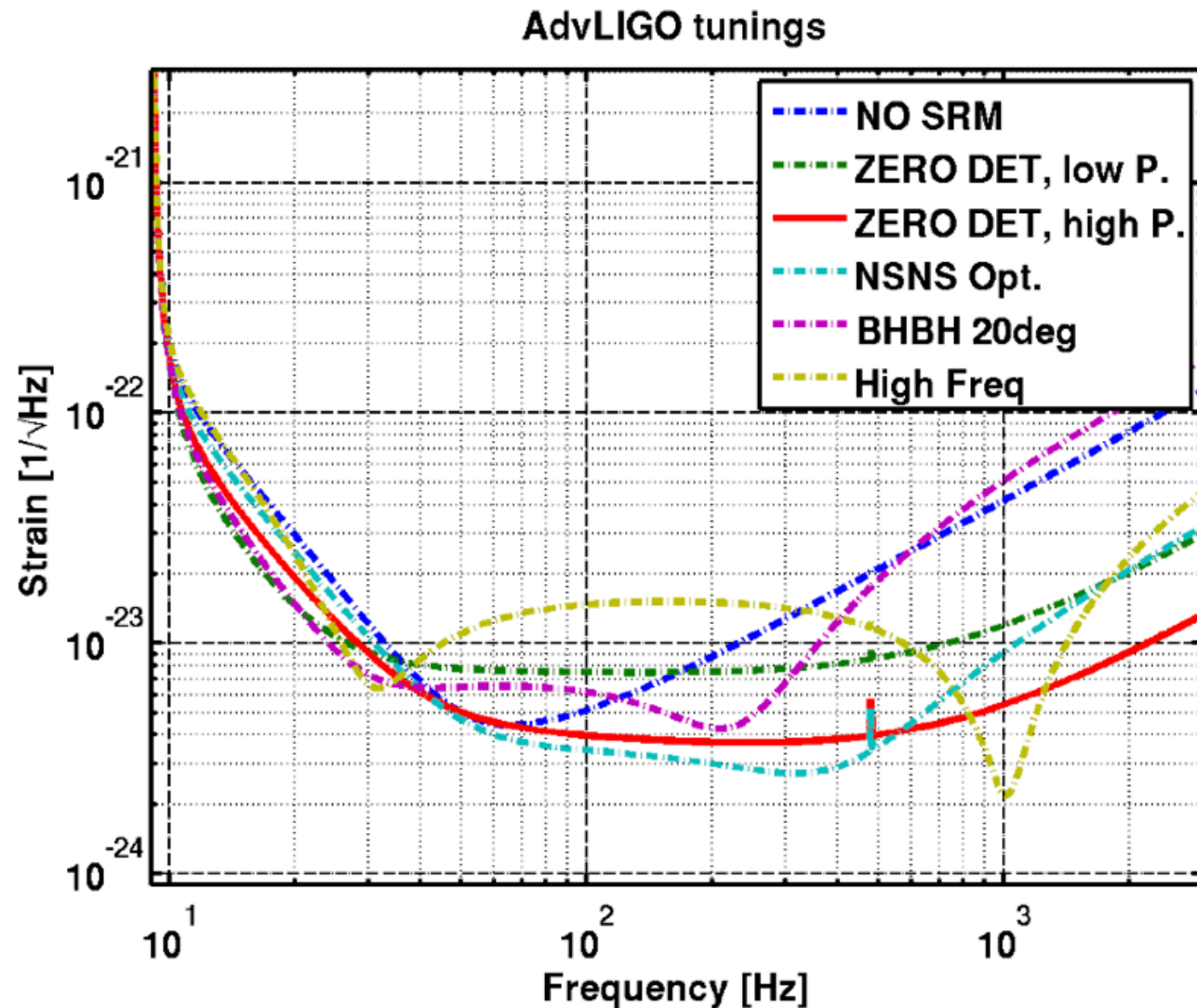
$$p(D|\beta, M_s) \propto \exp \left[- \sum_{i=1}^N \frac{(D_i - h_i(\beta))^2}{2\sigma_i^2} \right]$$

- ★ Here, σ is the expected noise, D_i is the i -th data point, h_i is the reconstructed model signal from Principal Components and β are the amplitudes or coefficients for reconstructing the signal
- ★ To obtain the evidence, marginalise over all expected values of β such that

$$p(D|M_s) = \int_{\beta_{\min}}^{\beta_{\max}} p(\beta|M_s)p(D|\beta, M_s)d\beta$$

Investigations with Advanced LIGO noise

- ★ Simulate noise for Advance LIGO in “zero detuning, high power” configuration
 - [ZERO_DET_high_P.txt](#), publicly available from LIGO DCC



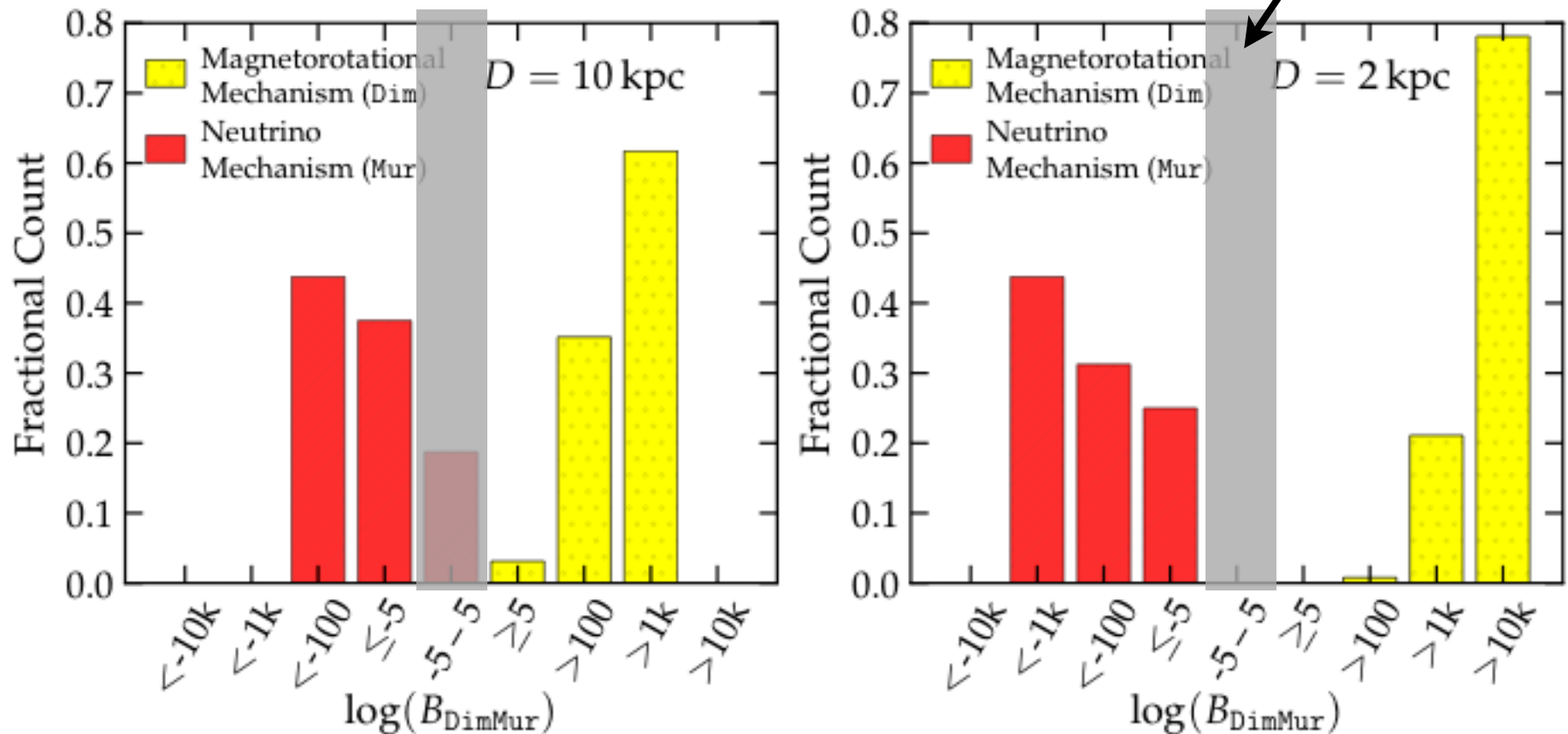
<https://dcc.ligo.org/cgi-bin/DocDB/ShowDocument?docid=T0900288>

Distinguishing SNe models

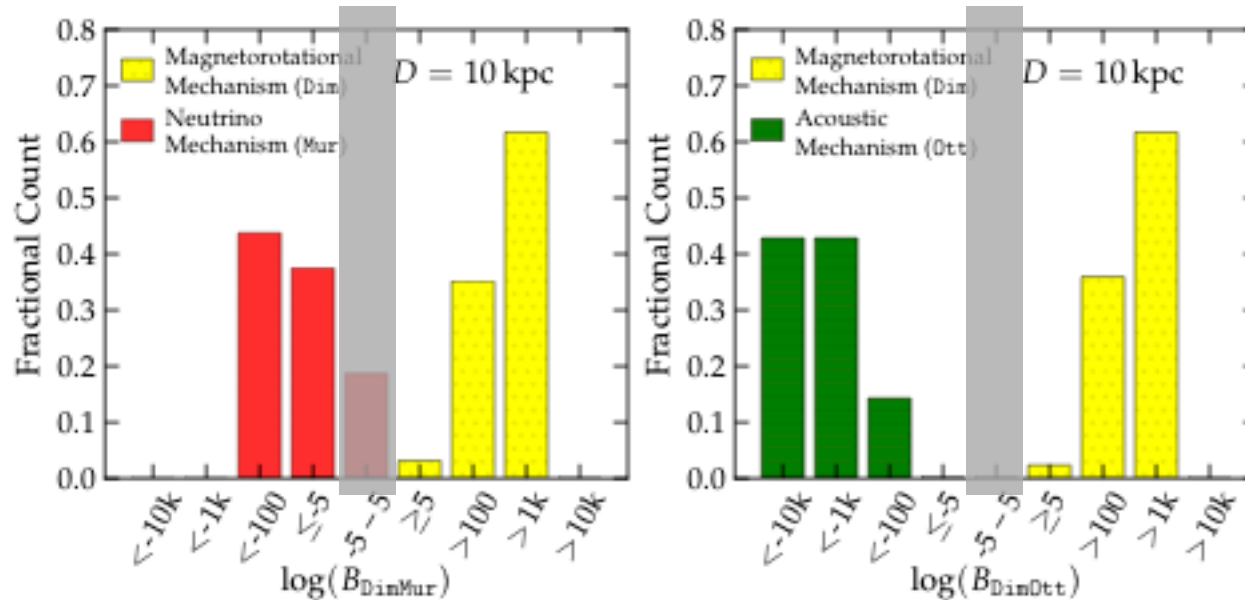
$$\log(B_{\text{DimMur}}) = \log B_{\text{Dim}} - \log B_{\text{Mur}}$$

$$B_i = p(D|M_i)$$

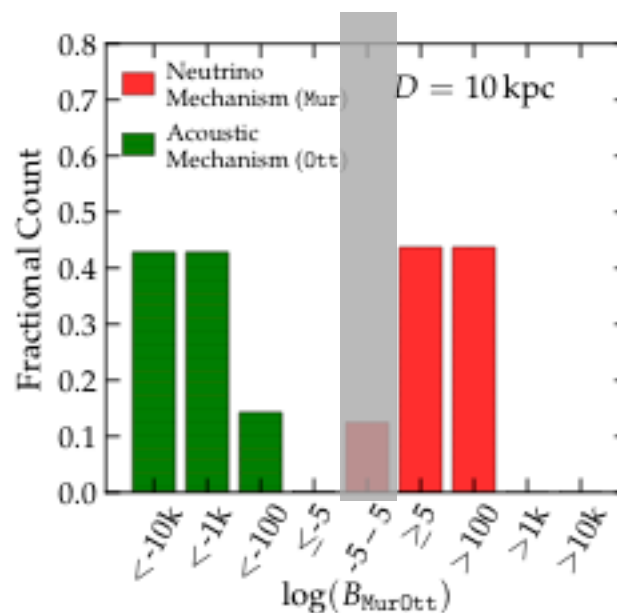
cannot distinguish
between models



Distinguishing SNe models



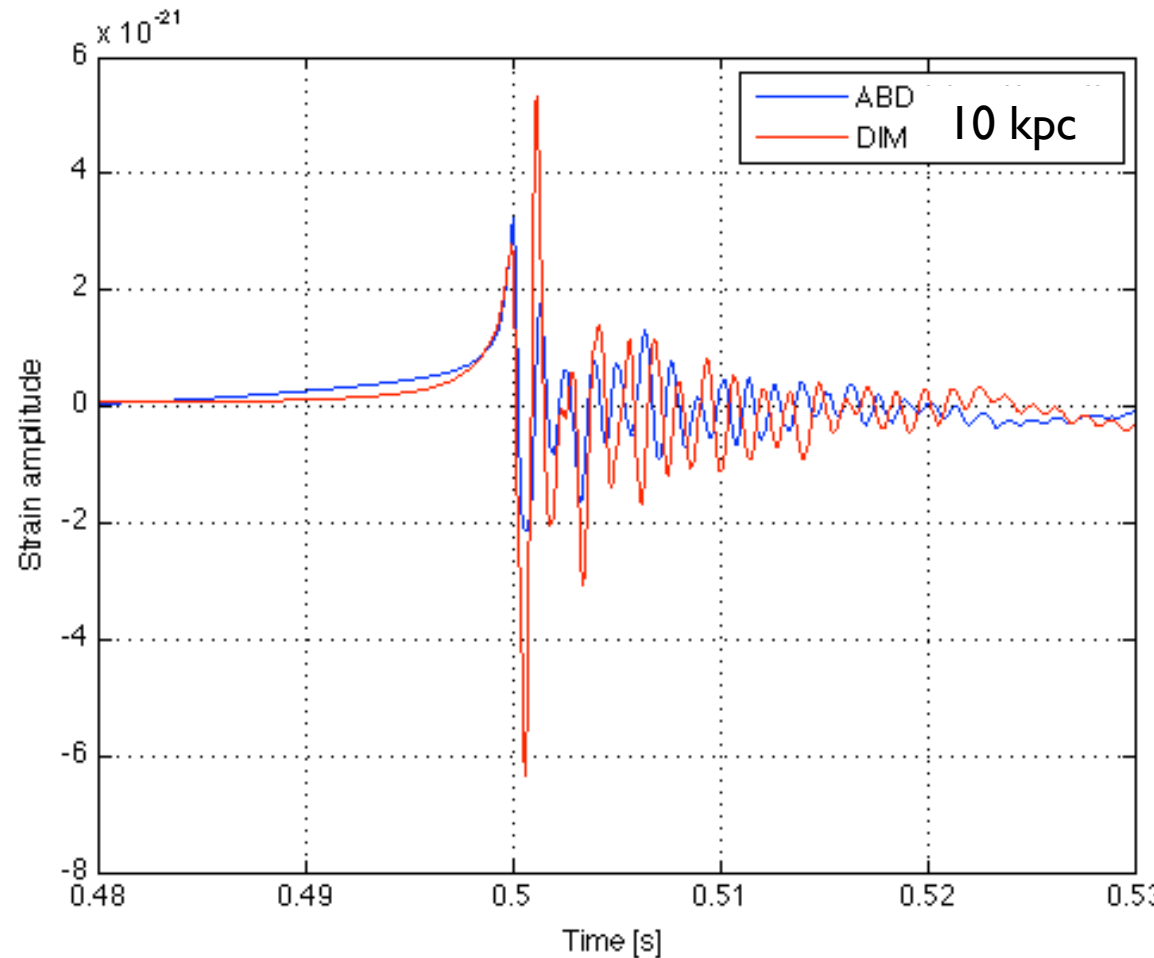
Most signals are correctly identified!
 Only a small fraction of neutrino mechanism waveforms returning an ambiguous Bayes factor



Results are published in Phys. Rev. D **86** (4) 044023 (arXiv:1202.3256)

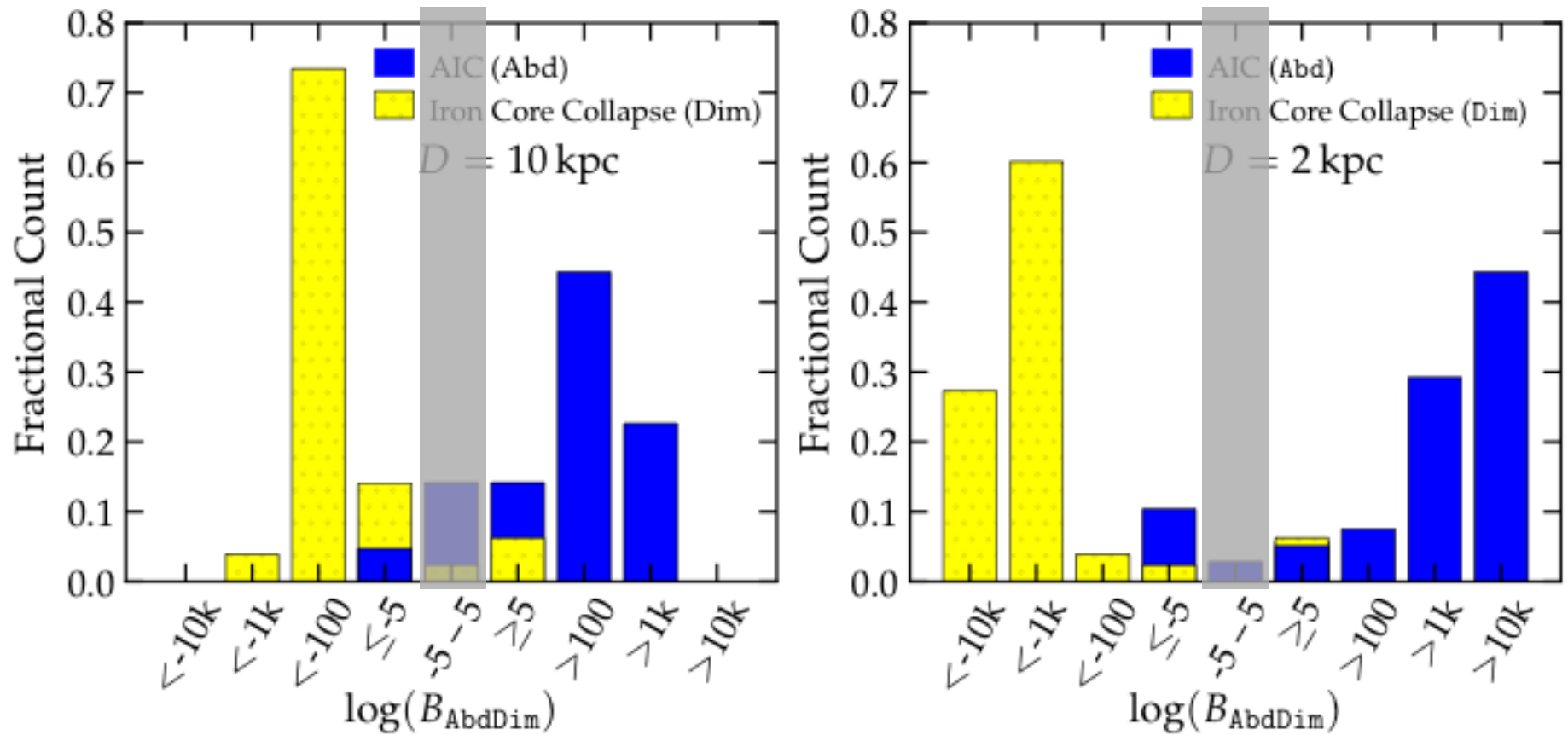
Distinguishing SNe models

- ★ The mechanisms examined so far produce quite different waveforms
- ★ Also compared Dimmelmeier waveforms to Accretion Induced Collapse (AIC) waveforms
- ★ AIC: collapse of accreting carbon white dwarfs
- ★ Use catalogue by Abdikamalov *et al.* 2010



Distinguishing SNe models

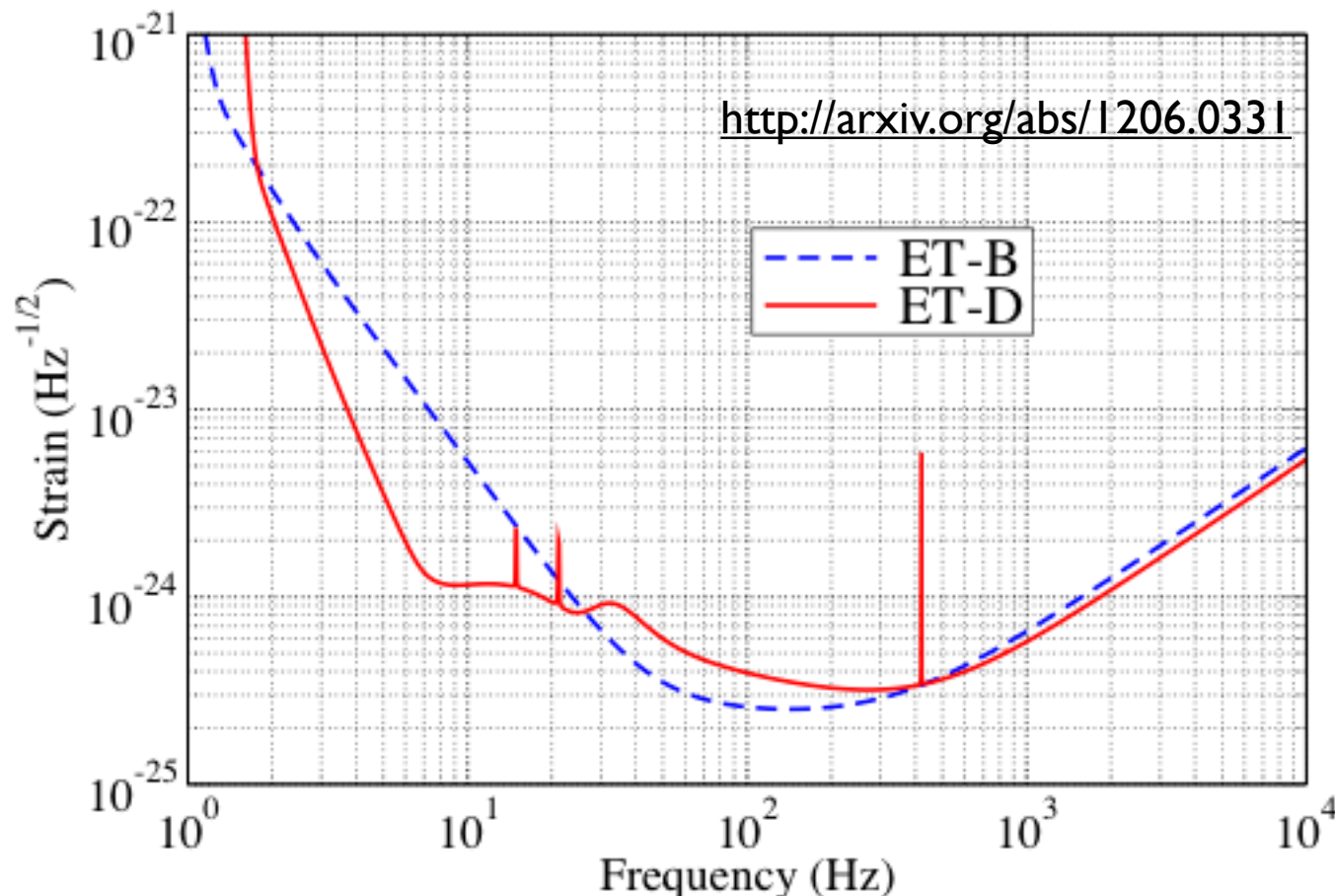
$$\log(B_{\text{AbdDim}}) = \log B_{\text{Abd}} - \log B_{\text{Dim}}$$



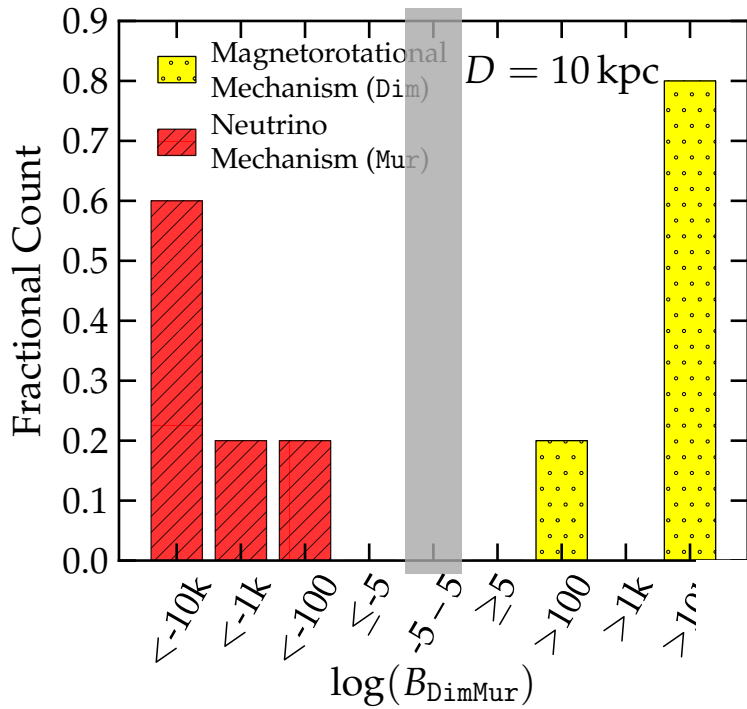
Again, most signals are correctly identified for a supernova at 10 kpc

Investigations with ET noise

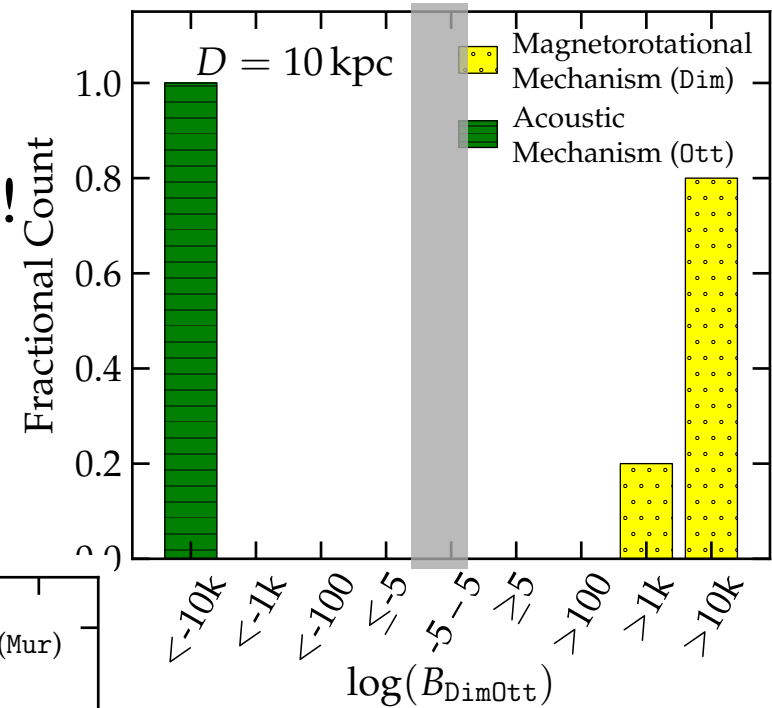
- ★ We injected the signals at 10 kpc and 778 kpc (Andromeda) into the ET-B noise curve
 - we will use something more current next time...



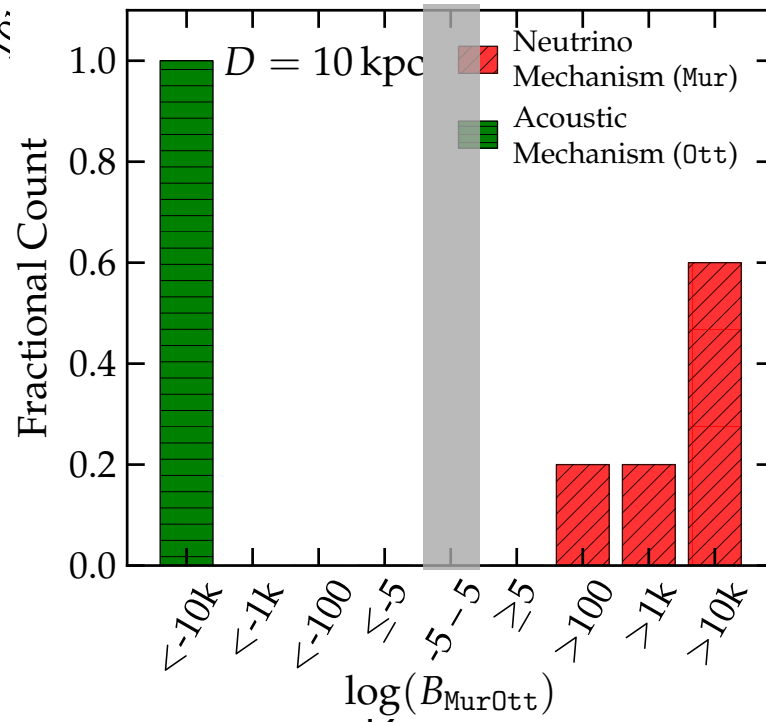
Supernova in Galactic Centre (ET)



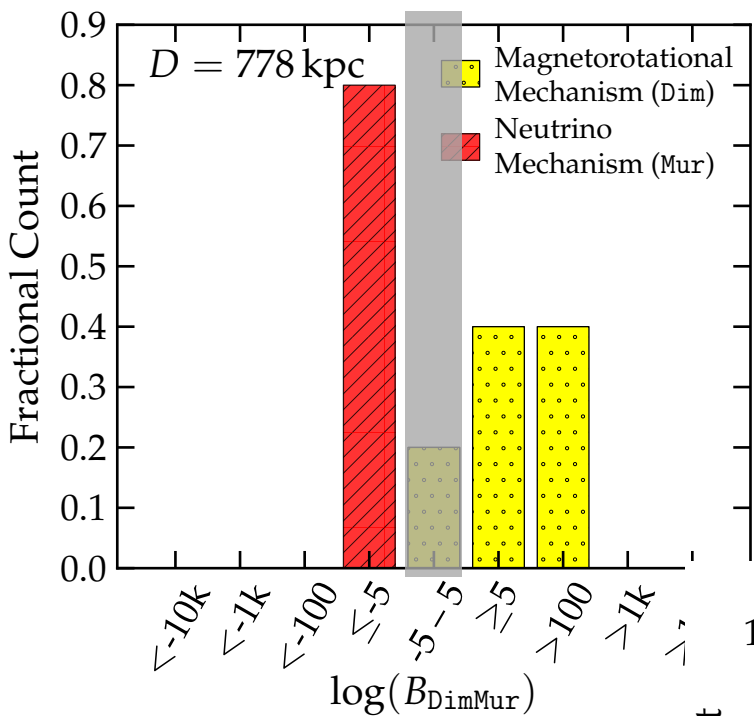
There is no ambiguity when identifying the supernova mechanisms!



SNRs: Dim ~ 1000 ,
Mur ~ 200 , Ott ~ 1500

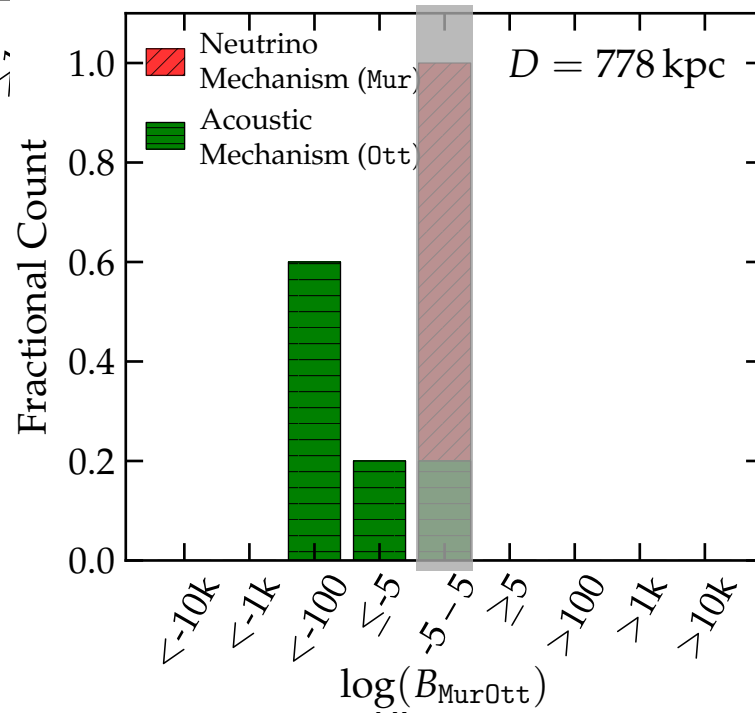
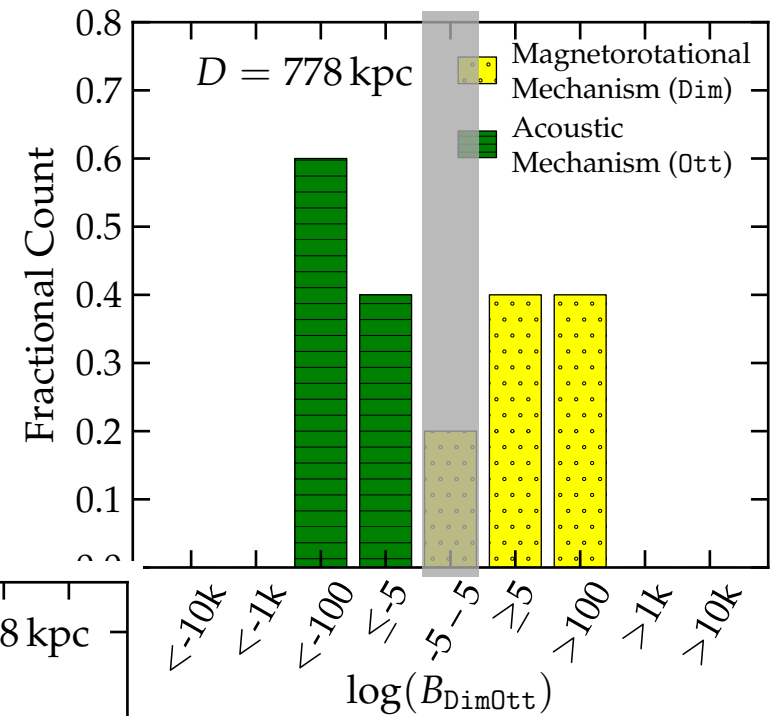


Supernova at Andromeda (ET)



SNRs: Dim ~ 13 ,
Mur ~ 2 , Ott ~ 20

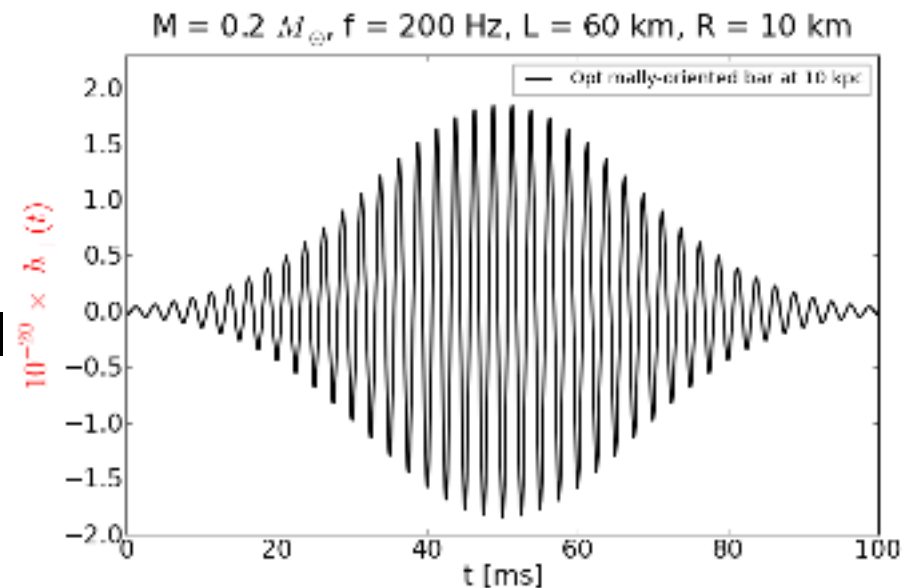
Most waveforms
can still correctly
identified.



Note: We have assumed that the signal is detected and GW signal time is known. ie false alarm rate is not considered!

ET MDC

- ★ Two supernova waveforms were injected into the latest ET Mock Data Challenge (MDC)
- ★ One from Dimmelmeier *et al.* 2008 catalogue and the other is a long bar waveform using the waveform proposed by Fryer, Hughes, Holz 2002
- ★ SMEE will be run on these injections
 - need to include long bar signal model



L. Santamaria et al., LIGO-DCC-G1100014

Summary and future work

- ★ The proposed method, SMEE, has demonstrated its ability to associate an observed core-collapse supernova gravitational wave signal with the correct waveform catalogue
- ★ This allows us to infer the astrophysics behind the core-collapse supernova from the detected gravitational wave signal
- ★ Further features are required for SMEE and work is underway to implement them
 - analyse multi-detector data, incorporate time uncertainty and antenna patterns, use power spectra or time-frequency data,.....
- ★ Investigate waveform reconstruction from SMEE outputs
- ★ Extend SMEE framework for analysis towards a broader Burst parameter estimation and glitch classification